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# Contents

*The present volume contains*

- *abstracts of invited lectures (plenary and in sections)*
- *abstracts of contributed communications*
- *abstracts of posters*
- *extended abstracts (without oral presentation)*

*provided the authors sent their manuscripts to the organizers in due time. The papers were neither refereed nor edited (except for corrections of some evident misprints). They are arranged in alphabetical order according to the names of those presenting them. All kinds of presentation—oral communications, posters and extended abstracts—are considered equivalent active contributions to the scientific program of the Conference.*

# ABSTRACTS

## SOME CONDITIONS FOR NONLINEAR THIRD DIFFERENTIAL EQUATIONS TO HAVE PERIODIC SOLUTIONS

*Zahra Afsharnezhad*, Mashhad, Iran,

*MSC 2000*: 34C

The aim is to explain some conditions for nonlinear differential equation $\dddot{x} + \omega^2\dot{x} + \mu f(x, \dot{x}, \ddot{x}) = 0$, in order to have periodic solutions. This kind of differential equation occur in many mechanical systems such as vibration system and many physical engineering problems; specially in energy and acceleration problems. Suppose that $f(x, y, z) = \frac{\partial F(x,y)}{\partial x} y + \frac{\partial F(x,y)}{\partial y} z$, where $\dot{x} = y$, $\ddot{x} = z$ and $F(0) = DF(0) = 0$; then we can prove that if the second differential equation $\ddot{x} + \omega^2\dot{x} + \mu F(x, \dot{x}) = k$ has periodic solution, so the original equation has too. Therefore we reduce the third differential equation to the second differential equation. Here we consider periodic solution for the above second differential equation. We will prove that under some condition on partial derivative of the function $f$ at the fixed point, many periodic solutions occur for the third differential equation as the parameter $k$ varies. Also a few examples will be given.

## ON AN ELLIPTIC PROBLEM ARISING IN DIFFERENTIAL GEOMETRY

*Antonio Ambrosetti*

We discuss some elliptic boundary value problem with critical exponent. The problem is related to the existence of conformal metrics on a compact $n$-dimensional manifold with boundary having a prescribed Scalar Curvature and prescribed Mean Curvature of the boundary.

## BOUNDED SOLUTIONS OF DIFFERENTIAL SYSTEMS: SEQUENTIAL VS DIRECT APPROACHES

*Jan Andres*, Olomouc, Czech Republic

*MSC 2000*: 34A60, 34B15, 34C11, 47H11

Continuation methods for the existence of entirely bounded solutions of differential systems will be presented via sequential and direct approaches. Advantages and disadvantages of both approaches will be discussed. The associated fixed-point principles are related to the well-defined notion of fixed-point indices for J-maps on ANRs in Fréchet spaces. Multiplicity results for bounded solutions of Carathéodory systems of differential equations and inclusions will be mentioned as well.

## ASYMPTOTIC METHOD FOR NONLINEAR PERIODICAL VIBRATIONS OF CONTINUOUS STRUCTURES

*Igor V. Andrianov*, Dnepropetrovsk, Ukraine, Jan Awrejcewicz, Lodz, Poland, Vladyslav V. Danishevs'kyy, Dnepropetrovsk, Ukraine

*MSC 2000*: 74 Mechanics of deformable bodies, 41 Approximations and expansions, 35 Partial differential equations

We propose the asymptotic method for determining periodic solutions of nonlinear vibration problems of continuous structures (such as rods, beams, plates, etc). We start with well-known perturbation technique and expand the independent displacement and frequency in power series of a natural small parameter. It leads to infinite systems of interconnected nonlinear algebraic equations governing relationships between modes amplitudes and frequencies. For its solution we use a non-trivial asymptotic approach, based on the introduction of an artificial small parameter. An advantage of our procedure is the possibility to take into account even a large number of vibration modes. As examples we consider free longitudinal vibrations of a rod and lateral vibrations of a beam subjected to cubically nonlinear restoring force action. Resonance interactions between different modes are investigated and asymptotic formulas for corresponding backbone curves are derived.

## NODE-CENTERED FINITE VOLUME SCHEMES AND NONCONFORMING MIXED FEM

LUTZ ANGERMANN, Magdeburg, Federal Republic of Germany

*MSC 2000*: 65N30, 65N12, 65N15

For a generalized saddle point problem with "off-diagonal terms" which are not necessarily skew-symmetric, a theory of mixed finite element methods in a nonconforming setting is formulated.

As an application, the relation between a node-centered finite volume discretization of the stationary diffusion equation in 2d and some mixed finite element method is investigated. The space of finite element functions to approximate the flux density variable is introduced as a "rotated" Raviart-Thomas space of lowest order and important properties of the corresponding interpolation operators are demonstrated. Stability results (including LBB conditions) and error estimates are given.

## QUASILINEAR ELLIPTIC DIRICHLET PROBLEM IN NONREGULAR DOMAINS

*D. E. APUSHKINSKAYA* and A. I. NAZAROV

In this talk we present the solvability result for the Dirichlet problem for nondivergent quasilinear elliptic equations of the second order in weighted Kondrat'ev spaces in the case when the boundary of a domain may include singularities—conical points or arbitrary codimensional edges (see [1] for details).

This work was partially supported by Russian Fund for Fundamental Research, grant no. 99-01-00684.

*References*

[1] *D.E. Apushkinskaya and A.I. Nazarov*: The Dirichlet problem for quasilinear elliptic equations in domains with smooth closed edges. Probl. Mat. Anal. (2000), no. 21, 3–29. (In Russian.)

## GLOBAL EXISTENCE PROBLEM FOR $q$-NONLINEAR NONDIAGONAL PARABOLIC SYSTEMS

*Arina Arkhipova*, Sankt-Petersburg

*MSC 2000*: 35K50 (primary), 35K45, 35K60 (secondary)

A class of $q$-nonlinear parabolic systems with nondiagonal main matrix is considered. We discuss the global in time solvability results of the classical initial boundary problems in the case of two spatial variables. The systems with nonlinearities $q \in (1,2)$, $q = 2$, and $q > 2$, are analyzed.

## AVERAGING, INVARIANT MEASURES AND SINGULAR PERTURBATIONS

*Zvi Artstein*

In the talk we shall see how invariant measures arise, and in turn give rise to averaging, as a result of a singular perturbation of an ordinary differential equation. Examples and concrete applications will be presented.

## THE ULTRASONIC MOTOR BASED ON A LONGITUDINAL VIBRATION OF A CANTILEVER

*Yusuke Ashida*, Yoshiaki Hata and Mikio Nakai, Kyoto, Japan

There have been far more studies about an ultrasonic motor based on a transverse vibration than those based on a longitudinal vibration. This paper proposes a cantilever as an ultrasonic motor applying a longitudinal vibration. The cantilever consists of a piezo vibrator, a guide bar and a slider. The piezo vibrator is driven by some suitable input signals, and causes a longitudinal vibration of the guide bar. Hence, the slider is actuated through the stick-slip phenomenon between the slider and the guide bar. This motor enables us to move the slider at extremely low speed, which is in the order of micro-meters per second. The vibration of this system is expressed by partial differential equations considering the stick-slip phenomenon. Predictions about the behaviors of the slider are shown in detail under various conditions of input signals into the piezo vibrator.

## ON SOME ORDINARY DIFFERENTIAL EQUATIONS WITH ADVANCED ARGUMENT

*Antoni Augustynowicz*, Poland

We discuss the following ordinary differential equation

$$u'(t) = b\,|u(\beta t)|^{1/\beta} \quad \text{for} \quad t > 0$$

with $b \in \mathbb{R}$, $\beta > 1$, and the initial condition $u(0) = a$.

Characterizing exponentialy bounded solutions and analytic solutions we show some interesting properties of the problem considered.

# VERIFICATION AND VALIDATION IN COMPUTATIONAL MECHANICS SOME MATHEMATICAL ASPECTS

*Ivo Babuška*, Austin, Texas, USA

*MSC 2000*: 35R60, 65N30, 60H15, 60H30, 65N15, 65N30

Verification addresses the reliability of the computed results in the comparison with the exact solution of the mathematical problem. Validation addresses the reliability of the mathematical formulation. One of the major mathematical aspect of the verification is the development and the analysis of the a-posteriori estimate of the error in the computed data of interest.

Validation is a much more complex subject. It relates among others to the experiments. One of the mathematical aspects of the validation is the treatment of the uncertainties in the available information.

The lecture will address the problem when the data in the PDE are obtained by digital scanning or have a stochastic character. Illustrative examples will be presented.

# ON EXISTENCE OF SINGULAR SOLUTIONS

*Miroslav Bartušek*, Brno, Czech Republic

*MSC 2000*: 34C11

Sufficient/necessary conditions will be given under which the equation

$$y^{n)} = f(t, y, y', \ldots, y^{(l)})\, g(y^{(n-1)}), \quad l \in \{0, 1, \ldots, n-1\}$$

has a singular solution $y\colon [T, \tau) \to \mathbb{R}$, $\tau < \infty$ fulfilling

$$\lim_{t \to \tau_-} y^{(i)}(t) = C_i \in \mathbb{R}, \quad i = 0, 1, \ldots, l$$

and

$$\lim_{t \to \tau_-} |y^{(j)}(t)| = \infty$$

for $j = l + 1, \ldots, n - 1$.

## ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF LINEAR DISCRETE EQUATIONS

*J. Baštinec* and J. Diblík, Brno, Czech Republic

*MSC 2000*: 39A10, 39A11

Asymptotic behaviour of a particular solution of discrete equation $\Delta u(k) = A(k)u(k) + g(k)$, $k \in N(a)$ will be studied with the aid of a formal series which satisfies this equation ($\Delta u(k) = u(k+1) - u(k)$, $N(a) = \{a, a+1, \ldots\}$, $a \in \mathbb{N}$ is fixed, $\mathbb{N} = \{0, 1, \ldots\}$ and $A, g\colon N(a) \to \mathbb{R}$.)

(This investigation was supported by the grant 201/01/0079 of Czech Grant Agency (Prague) and by the Council of Czech Government MSM 2622000 13.)

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[1] *Ravi P. Agarwal*: Differential Equations and Inequalities, Theory, Methods, and Applications. Marcel Dekker, Inc., New York, Basel, Hong Kong, 1992.

[2] *J. Diblík*: Retract principle for difference equations. Communications in Difference Equations, Proceedings of the Fourth International Conference on Difference Equations, Poznan, Poland, August 27–31, 1998 (S. Elaydi, G. Ladas, J. Popenda and J. Rakowski, eds.). Gordon and Breach Science Publ., 2000, pp. 107–114.

## USING RESIDUES TO COMPUTE THE MELNIKOV FUNCTION

*Flaviano Battelli*, Ancona, Italy and Michal Fečkan, Bratislava, Slovakia

*MSC 2000*: 34C23, 34C37

We study the Melnikov function $M(\alpha)$ associated to a periodic perturbation of a differential equation having a homoclinic orbit. We show that, in some cases, the Fourier coefficients of $M(\alpha)$ can be evaluated by means of the calculus of residues. We apply this result to construct a second order equation whose Melnikov function vanishes identically for any $C^1$, periodic perturbation $\varepsilon q(t)$.

## STERNBERG-CHEN THEOREM FOR EQUIVARIANT HAMILTONIAN VECTOR FIELDS

*Genrich R. Belitskii*, Beer Sheva, Israel,
and Alexander Ya. Kopanskii, Chisinau, Moldova

Smooth hamiltonian vector fields with linear symmetries and anti-symmetries are considered. We prove that provided the symmetry group is compact then a smooth conjugacy in Sternberg-Chen Theorem can be chosen canonical and symmetric.

## ON A MODEL OF SOLIDIFICATION WITH ADVECTION EFFECTS

*Michal Beneš*, Prague, Czech Republic

*MSC 2000*: 80A22, 82C26, 35A40

The communication deals with the phase-field model consisting of the heat equation and of the modified Allen-Cahn equation with efficient coupling and gradient terms. The equations contain advection terms with known velocity field as a function of space. We study the existence of solution, and numerical approximation. The numerical scheme based on spatial finite differencing, and Runge-Kutta time discretisation is derived. Furthermore, we present several computational studies treating mean-curvature flow as a subproblem, and microstructure growth under presence of an advecting field. Influence of advection on resulting patterns is discussed.

## SOME OSCILLATION PROPERTIES OF A LINEAR NEUTRAL DIFFERENTIAL EQUATION

*Leonid Berezansky*, Beer-Sheva

*MSC 2000*: 34K11, 34K40

For a neutral differential equation

$$\dot{x}(t) - a(t)\dot{x}(g(t)) + b(t)x(h(t)) = 0, \tag{1}$$
$$0 \leqslant a(t) < 1,\ b(t) \geqslant 0,\ g(t) \leqslant t,\ h(t) \leqslant t,$$

a connection between oscillation properties of the differential equation and differential inequalities is established.

Explicit non-oscillation and oscillation conditions and a comparison theorem are presented.

## ON INTEGRO-DIFFERENTIAL VON KÁRMÁN SYSTEM FOR VISCOELASTIC PLATES

*Igor Bock* and Ján Lovíšek, Bratislava

*MSC 2000*: 74D10, 74K20, 45K05

We shall deal with a nonlinear system of equations for a bending and Airy stress function of a viscoelastic plate made of the long memory material and acting under the perpendicular load and lateral forces. The middle surface $\Omega$ is a bounded simply connected domain in $\mathbb{R}^2$ with a Lipschitz boundary $\Gamma = \overline{\Gamma}_0 \cup \overline{\Gamma}_1$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, $|\Gamma_0| > 0$.

Considering the nonlinear strain-displacement relations we investigate a following boundary value problem for an integro-differential von Kármán system

$$\begin{aligned}
&D(0)\Delta^2 w + (D' * \Delta^2 w)(t) - [\Phi, w] = f(t,x),\\
&\Delta^2 \Phi = -\frac{h}{2}(E(0)[w,w] + (E' * [w,w])(t)), \quad D(t) = \frac{h^3 E(t)}{12(1-\mu^2)},\\
&w = \frac{\partial w}{\partial \nu} = 0 \text{ on } \Gamma_0, \quad (Mw)(t) = (Tw)(t) = 0 \text{ on } \Gamma_1,\\
&\Phi = \varphi_0, \quad \partial_\nu \Phi = \varphi_1 \text{ on } \Gamma,\\
&[u,v] = \partial_{11}u\partial_{22}v + \partial_{22}u\partial_{11}v - 2\partial_{12}u\partial_{12}v,\ u,v \in H^2(\Omega).
\end{aligned}$$

## ON A CLASS OF FORCED NONLINEAR OSCILLATORS AT RESONANCE

*D. Bonheure*, C. Fabry and D. Smets, Louvain-la-Neuve, Belgium

We present existence, non-existence and multiplicity results for periodic solutions of forced nonlinear oscillators at resonance, the nonlinearity being a bounded perturbation of a force deriving from an isochronous potential, i.e. a potential leading to free oscillations that all have the same period. The class of nonlinearities considered includes jumping nonlinearities, as well as singular forces of repulsive type. Assuming that the forcing term $p$ is $T$-periodic, we show that the existence of $T$-periodic solutions depends on the properties of the function $\Phi(\theta) = \int_0^T p(t)\psi(t+\theta)\,\mathrm{d}t$, where $\psi$ is a solution of a limiting system approaching the isochronous oscillator for large amplitudes. As particular cases of the existence results, we obtain conditions of Landesman-Lazer type.

*MSC 2000*: Primary: 70K30, 65L10

## LANDESMAN-LAZER TYPE CONDITIONS AND QUASILINEAR ELLIPTIC EQUATIONS

*Jiří Bouchala*, Ostrava, Czech Republic

*MSC 2000*: 35J20, 35P30, 47H15

We study the existence of the weak solutions of nonlinear boundary value problem

$$\begin{cases} -\Delta_p u = \lambda|u|^{p-2}u + g(u) - h(x) \text{ in } \Omega, \\ \quad u = 0 \text{ on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geqslant 1$, $p > 1$, $g\colon \mathbb{R} \to \mathbb{R}$ is continuous function, $h \in L^{p'}(\Omega)$ $(p' = \frac{p}{p-1})$, $\Delta_p$ is the $p$-Laplacian, i.e. $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ and $\lambda \in \mathbb{R}$.

Our sufficient conditions generalize all previously published results.

*Research supported by the Grant Agency of Czech Republic 201/00/0376 and Grant MOEMT J17/98: 272400019*

## SOME CONSIDERATIONS ON MIXED FINITE ELEMENT METHODS

*Jan Brandts*, Utrecht, Netherlands

In this presentation we will highlight some features of Mixed Finite Elements that have had considerable attention in the past decades. We recall stability theorems and special families of elements, point out several fields of application, and then concentrate on two major issues. First one is how to solve the sytems of linear equations related to certain mixed systems, whereas the second concentrates on superconvergence properties of mixed finite element solutions. Both topics are lively areas of recent research in which much progress has been.

## HOMOCLINIC ORBITS IN HAMILTONIAN SYSTEMS AS INTERSECTION POINTS OF TWO LAGRANGIAN MANIFOLDS

*B. Buffoni* Lausanne, Suisse

*MSC 2000*: 37J45

P. J. Holmes and C. A. Stuart have investigated homoclinic orbits for eventually autonomous planar flows by analyzing the geometry of the stable and unstable manifolds. We extend their discussion to higher-dimensional systems of Hamiltonian type by formulating the problem as the existence of intersection points of two Lagrangian manifolds. The various assumptions of Holmes and Stuart can be restated and interpreted as ensuring some complexity of the generating function of one of the Lagrangian manifold with respect to symplectic coordinates that trivialize the second Lagrangian manifold. The critical points thus obtained correspond to homoclinic orbits. The main new feature in high-dimensions is that twice as many homoclinic orbits are found as for planar flows, in analogy with results obtained for autonomous Lagrangian systems by A. Ambrosetti and V. Coti Zelati.

## A POSTERIORI ERROR ESTIMATES AND ADAPTIVE MESH REFINEMENT APPLIED TO FLOW IN A CHANNEL WITH CORNERS

*Pavel Burda*, Jaroslav Novotný, Bedřich Sousedík,
Praha, Czech Republic

In the first part of the paper we investigate a posteriori error estimates for the Stokes and Navier-Stokes equations on two-dimensional polygonal domains, as well as on three-dimensional polyhedral domains. Special attention is payed to the sources of the constants in the estimates, as these play a crucial role in practical applications to adaptive refinements, as we also show. In the second part we deal with the problem of determining accurately the constants that appear in the estimates. We develop technique for calculating the constant with high accuracy. In the third part we apply the a posteriori error estimates with the constants found numerically to the technique of adaptive mesh refinement to solve an incompressible flow problem in a domain with corners.

## EXTREMALITY RESULTS OF EXPLICIT AND IMPLICIT SINGULAR DIFFERENTIAL DIFFUSION EQUATIONS

*Alberto Cabada*, José Angel Cid
and Rodrigo L. Pouso, Santiago de Compostela, Spain

*MSC 2000*: 34A36, 34A09

In this work we prove the existence of extremal positive solutions of the following initial value problem

$$\begin{gathered}((k \circ u)u')'(x) = f(x, u(x))u'(x) + g(x, u(x))u(x) \quad \text{a.e. in } I = [0, \alpha], \\ u(0) = 0, \quad \lim_{x \to 0^+} k(u(x))u'(x) = 0.\end{gathered}$$

The regularity conditions required in this case, allow functions $k$ and $g$ to be discontinuous. Function $k$ can take the value zero in a null set.

Studying an equivalent integral representation, we obtain existence of extremal solutions by using the generalized iterative techniques of [Heikkilä, S.; Lakshmikantham, V. Monotone iterative techniques for discontinuous nonlinear differential equations] which give us extremal fixed points of discontinuous operators. Using recent results given in [Carl, S.; Heikkilä, S. Nonlinear differential equations in ordered spaces], this extremality results are applied to implicit problems.

## DISTRIBUTED DELAYED COMPETING PREDATORS

*Mario Cavani*, Cumaná, Venezuela

In this talk we consider the propose of a model of competing species where the dynamic of the predators depend on the past history of the prey by mean a distributed delay that take an average of the Michaelis-Menten functional response of the same prey population. The predators compete purely exploitatively, with no interference between rivals (no toxins are produced, for example) and the predators species have access to the prey and compete only by lowering the population of shared prey. This model consider that there significative delays in the system in the process of prey consumed into predators. We show the existence of a global atractor of the solutions of the system and some behaviour of the solutions under particular constellations of the parameters are obtained in looking what species win the competition.

## THE ASYMPTOTIC PROPERTIES OF SOLUTIONS OF A CLASS OF DELAY DIFFERENTIAL EQUATIONS

*Jan Čermák*, Brno

*MSC 2000*: 34K25, 39B22

We discuss the asymptotic properties of solutions of the differential equation

$$\dot{x}(t) = -c(t)[x(t) - L\,x(\tau(t))]$$

with a positive continuous function $c(t)$, a nonzero real scalar $L$ and unbounded lag. We present the conditions under which every solution of this equation approaches a solution of the auxiliary functional equation

$$\psi(t) = L\psi(\tau(t)).$$

Furthermore, we investigate some modifications of the studied equation and give the applications illustrating our results.

# VIABILITY FOR A CLASS OF NONCONVEX DIFFERENTIAL INCLUSIONS

*Aurelian Cernea*, Bucharest, Romania

*MSC 2000*: 34A60

Consider the Cauchy problem

$$x' \in F(x), \quad x(0) = x_0 \tag{1}$$

where $F\colon K \subset \mathbb{R}^n \to \mathcal{P}(R^n)$ is a set-valued map and $K \subset \mathbb{R}^n$ is a locally compact set. When the tangential condition

$$\forall x \in K \quad F(x) \cap T_x K \neq \emptyset \tag{2}$$

where $T_x K$ is the contingent cone to $K$ at $x \in K$ and $F$ is contained in the subdifferential of a proper convex function $V$, Rossi proved the existence of viable solution to problem (1).

We extend the result of Rossi to the case when the multifunction $F$ is contained in the Fréchet subdifferential of a $\varphi$-convex function of order two.

# IMAGE PROCESSING BY MEANS OF PARABOLIC DIFFERENTIAL EQUATIONS OF ALLEN-CAHN TYPE

*Vladimír Chalupecký* and Michal Beneš, Prague

*MSC 2000*: 35K75, 65M06, 68U10

The Allen-Cahn equation is a degenerate parabolic equation that approximates motion of a curve by its mean curvature. Areas of applications of the Allen-Cahn equation are numerous, in this paper we aim at its application in image processing. Several mathematical models based on the Allen-Cahn equation will be presented, that can be applied for tasks like noise removal, shape and boundary recovery or pattern recovery. Semi-implicit numerical schemes based on the finite difference methods will be presented together with some computational results.

**Acknowledgement.** The work of the authors has been partly supported by the projects MSM J98/210000010 of the Czech Ministry of Education and 201/01/0676 of the Grant Agency of the Czech Republic.

## EFFECTS OF UNCERTAINTIES IN THE DOMAIN ON THE SOLUTION OF NEUMANN AND DIRICHLET BOUNDARY VALUE PROBLEMS

*Jan Chleboun*, Prague

*MSC 2000*: 65N99, 35B30, 35A35

A dependence of solutions of boundary value problems (BVPs) on an uncertain domain is investigated. A stability problem is studied for BVPs defined on sequences of domains $\Omega_n$ converging in the set sense to a limit domain $\Omega$. A reformulation of the Neumann BVP is necessary to avoid unnatural loss of stability. As $\Omega$ is considered technically unattainable, i.e., uncertain, it implies the necessity to assess the unreachable solution $u$ of the limit BVP on $\Omega$ by means of a known function $u_n$, the BVP solution on $\Omega_n$. For the Neumann and Dirichlet BVPs, a norm of $u - u_n$ is estimated through solutions of BVPs defined on $\widehat{\Omega}_1$, $\widehat{\Omega}_2$, $\widehat{\Omega}_1 \subset \Omega \subset \widehat{\Omega}_2$. Some estimates open an easy way to a numerical treatment of the uncertain domain problem.

## GLOBAL ATTRACTORS IN ABSTRACT PARABOLIC PROBLEMS

*Jan W. Cholewa*, Katowice, Poland

*MSC 2000*: 35K90, 35B41

Global solvability and asymptotic behavior of solutions to (autonomous) parabolic equations are considered in our recent monograph [1], which—based on [2] and [3]—expounds the theory of global attractors. Both compact semigroups defined by equations in bounded domains and noncompact semigroups, connected with the Cauchy problems in $\mathbb{R}^n$, are discussed there. A number of examples illustrate the abstract results concerning dissipative properties of parabolic equations. Such results, presented in the main body of the book, are complemented by sections devoted to dissipativeness in two spaces, problems with monotone operators, regularity of solutions, and the results concerning backward uniqueness, linear stability and abstract maximum principle.

*References*

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[2] J. K. Hale, *Asymptotic Behavior of Dissipative Systems*, AMS, Providence, RI, 1988.
[3] D. Henry, *Geometric Theory of Semilinear Parabolic Equations*, Springer, Berlin, 1981.

## A GENERAL CONTROLLABILITY THEOREM

Veronika Chrastinová, Brno, Czech Republic

*MSC 2000*: 35A30, 58A17, 58E99

The report concerns a fundamental result from the theory of quite general systems $\Omega$ of smooth partial differential equations. The existence of a unique "composition series" of the kind $\Omega^0 \subset \Omega^1 \subset \ldots \subset \Omega$ consisting of "factorsystems" of $\Omega$ is established. Here $\Omega^k$ is the maximal system of differential equations "induced" by $\Omega$ such that the formal solution of $\Omega^k$ depends on the choice of arbitrary functions of $k$ variables (on constants if $k = 0$). This is a well-known result only in the particular case of underdetermined systems of ordinary differential equations. Then $\Omega^1 = \Omega$ and $\Omega^0$ involves all first integrals, hence $\Omega^0$ is trivial if and only if $\Omega$ is a controllable system. In full generality, we may speak of a "multidimensional controllability" composition series of $\Omega$.

## A ONE-STEP SECOND ORDER METHOD FOR FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS IN BANACH SPACES

E. Cuesta and C. Palencia, Valladolid, Spain

Integro-differential equations of fractional order

$$u'(t) = \frac{1}{\Gamma(\alpha-1)} \int_0^t (t-s)^{\alpha-2} Au(s)\,\mathrm{d}s + f(t), \qquad t \geqslant 0, \tag{1}$$

where $1 < \alpha < 2$ and $A$ is the infinitesimal generator of a holomorphic semigroup $\mathrm{e}^{tA}$, $t \geqslant 0$, of linear and bounded operators in Banach spaces are considered.

A one-step second order method for the semidiscretization in time of (1) is proposed. This method combines both the fractional quadrature rule (FQR) based on trapezoidal rule for the integral in (2) and the trapezoidal method for the derivative. An expression for the error of the FQR in terms of Peano kernel is provided.

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## APPLICATIONS OF QUADRATIC INTERPOLATION POLYNOMIALS IN VERTICES OF PLANE TRIANGULATIONS

*Josef Dalík*, Brno

*MSC 2000*: 65D05

Let $\mathcal{T} = \{\mathcal{T}_h\}$ be a strongly regular family of triangulations of a plane domain. We describe two applications of some new results concerning existence and stability of quadratic polynomials interpolating in choosen vertices of $\mathcal{T}_h \in \mathcal{T}$.

Firstly, we present a stable and accurate modification of the method of characteristics for non-stationary plane convection-diffusion problems with dominating convection. Secondly, for any inner vertex $a$ of $\mathcal{T}_h \in \mathcal{T}$ with the set of neighbours $N(a)$, we describe approximations of first partial derivatives of any smooth function $u$ by means of values of $u$ in $N(a) \cup \{a\}$ with an error $O(h^2)$.

## INFINITELY MANY SOLITARY WAVES IN THREE SPACE DIMENSIONS

*Pietro d'Avenia*, Bari, Italy

*MSC 2000*: 35Q60, 35Q51, 35A15

Benci, Fortunato and Pisani have recently introduced a model of a Lorentz-invariant nonlinear field equation in three space dimensions, which gives rise to topological solitary waves. These are finite-energy solutions which are spatially localized and can be classified by means of a topological invariant: the charge. Under some symmetry assumptions, we prove that, for every $n \in \mathbb{Z}$, there exists a solution whose charge is $n$.

Moreover, if this charge is interpreted as electric charge, Benci, Fortunato, Masiello and Pisani have introduced a model of interaction for these topological solitary waves with the electromagnetic field. Also in this case, under the same symmetry assumptions, we prove the existence of a static solution for every assigned value of the charge.

## SHARPLY LOCALIZED $L_\infty$ ESTIMATES FOR MIXED FINITE ELEMENT METHODS

*Alan Demlow*, Ithaca, NY, USA

*MSC 2000*: 65N30

Schatz has recently proven sharply localized (weighted) maximum norm estimates for Galerkin methods on unstructured meshes. These estimates, which generalize previous maximum norm stability results, show that the higher the order of polynomial used in a Galerkin method, the more local the resulting approximation. We present analogous results for a mixed method for linear elliptic problems. Our estimates are valid for the vector variable, scalar variable, and a superconvergent postprocessed approximation to the scalar variable, and hold for all typical element spaces used in this context. We also comment on the best choice of element. In particular, our estimates indicate that the lowest order Raviart-Thomas elements give a localized approximation to the vector variable. In contrast, the lowest order Brezzi-Douglas-Marini elements approximate the vector variable to one higher order than the Raviart-Thomas elements but do not appear to yield a localized approximation.

## POSITIVE AND OSCILLATING SOLUTIONS OF EQUATION $\dot{x}(t) = -c(t)x(t-\tau)$

*J. Diblík*, Brno, Czech Republic

*MSC 2000*: 34K15, 34K25

The problem of asymptotic behaviour of solutions of equation $\dot{x}(t) = -c(t) \times x(t-\tau)$ is considered ($c \in C(I,(0,\infty))$, $I = [t_0,\infty)$, $0 < \tau = \text{const}$. Results concerning existence of positive solutions, representation of solutions and asymptotic inequalities for dominant and subdominant solutions will be presented.

(The author was supported by the grant 201/99/0295 of Czech Grant Agency (Prague) and by the Council of Czech Government MSM 2622000 13.)

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## ASYMPTOTIC PROPERTIES OF INTEGRO-DIFFERENTIAL EQUATIONS

*Alexander Domoshnitsky*, Ariel, Israel

*MSC 2000*: 35K15

We propose results on oscillation and asymptotic properties of solutions to the following problem

$$v''_{tt}(t,x)+\sum_{i=1}^{m}p_i(t)v(t-\tau_i(t),x)+a(t)v''_{xx}(t,x)+b(t)\int_0^t K(s,x)v''_{xx}(s,x)\,\mathrm{d}s=0,$$
$$x\in[0,\omega],\quad t\geqslant 0,\ v'_x(t,0)=v'_x(t,\omega)=0,\quad t\in[0,+\infty),$$
$$v'_x(t,0)=v'_x(t,\omega),\quad t\in[0,+\infty),\ v(\xi,x)=0,\quad x\in[0,\omega],\quad \xi<0.$$

Here $p_i(t)$, $\tau_i(t)$ are nonnegative continuous coefficients for $i=1,\dots,m$. One of the results is the following

**Theorem.** *If $m=1$, $p_1(t)$ and $h_1(t)\equiv t-\tau_1(t)$ are nondecreasing functions, $p_1$ is a bounded function, and $\int^{\infty}\tau_1(t)\,\mathrm{d}t=\infty$, then there exist unbounded solutions of the problem.*

## ON QUASILINEAR DIFFERENTIAL AND DIFFERENCE EQUATIONS

*Zuzana Došlá*, Brno, Czech Republic

*MSC 2000*: 34C10, 39A10

We present some recent results achieved in the joint research with M. Cecchi and M. Marini of University of Florence for the nonlinear differential equation

$$(a(t)\Phi_p(x'))'=b(t)f(x) \tag{1}$$

where functions $a,b$ are continuous and positive for $t\geqslant 0$, $f:\mathbb{R}\to\mathbb{R}$ is continuous such that $uf(u)>0$ for $u\neq 0$ and $\Phi_p(u)=|u|^{p-2}u$ with $p>1$. Jointly with (1), we consider the corresponding difference equation

$$\Delta(a_n\Phi_p(\Delta x_n))=b_nf(x_{n+1}), \tag{2}$$

where $\{a_n\}$, $\{b_n\}$ are positive real sequences for $n\geqslant 1$. A-priori classification of solutions is introduced and all continuable solutions of (1) and all solutions of (2) are classifed into the disjoint subsets which are fully characterised in terms of certain integral and sum conditions. There are given conditions for the boundedness and convergence to zero of solutions. The comparison between the continuous and discrete case will be given as well.

## QUALITATIVE THEORY OF HALF-LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

*Ondřej Došlý*, Brno

*MSC 2000*: 34C10

The investigation of qualitative properties of solutions of the second order half-linear differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) := |x|^{p-2}x, \ p > 1, \tag{$*$}$$

where $r$, $c$ are continuous functions and $r(t) > 0$, has received a considerable attention in recent years. It was shown that many of the classical results for the linear Sturm-Liouville equation (which is the special case $p = 2$) can be extended to ($*$). In this contribution we are going to present some results of this investigation and also to discuss new problems concerning the question "linear versus half-linear".

## SOME QUALITATIVE PROPERTIES OF QUASILINEAR BOUNDARY VALUE PROBLEMS WITH THE $p$-LAPLACIAN

*Pavel Drábek*, Plzeň

Let $\lambda_1 > 0$ be the principal eigenvalue of the $p$-Laplacian $\Delta_p$ on the bounded domain $\Omega \subset \mathbb{R}^N$ with smooth boundary $\partial\Omega$, $\varphi_1 > 0$ be an associated eigenfunction, $1 < p < 2$. We consider the boundary value problem

$$\Delta_p u + \lambda_1 |u|^{p-2} u = h \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega \tag{1}$$

and corresponding energy functional

$$E_h(u) = \frac{1}{p}\int_\Omega |\nabla u|^p - \frac{\lambda_1}{p}\int_\Omega |u|^p + \int_\Omega hu, \qquad u \in W_0^{1,p}(\Omega). \tag{2}$$

We prove that for $h \in C(\overline{\Omega})$, $\int_\Omega h\varphi_1 = 0$, the energy functional (2) is unbounded from below and has a local saddle point geometry. Moreover, we characterize the set of all $h \in C(\overline{\Omega})$, for which the problem (1) has at least one solution.

## NUMERICAL BEHAVIOUR OF SOLITARY WAVES IN NONLINEAR DISPERSIVE EQUATIONS

*A. Durán* & M. A. López-Marcos, Valladolid, Spain

*MSC 2000*: 65M20, 70H33, 37K40

Classical analysis of numerical methods for integrating time-dependent differential equations is based on the search of small approximations errors. However, a numerical scheme can have many other important properties. In particular, conservation properties would be pointed out. Almost every problem possesses physical quantities such as mass, energy, etc. that remain constant during the evolution of the system. It is not always true that these quantities keep invariant through numerical integration. Then, we can distinguish between conservative and nonconservative numerical methods.

In this talk we show the interest of conservative integrators for the numerical simulation of solitary waves in some nonlinear dispersive equations.

## TOPOLOGICAL PROPERTIES OF NONLINEAR EVOLUTION EQUATIONS

*Vladimír Ďurikovič* and Monika Ďurikovičová, Trnava

*MSC 2000*: 35K60; 47F05, 47A53, 47H30

We deal with the general initial-boundary value problem for a nonlinear nonstationary evolution equation (possibly of non-parabolic type)

$$D_t u - A(t, x, D_x)u + f(t, x, u, D_1 u, \dots, D_n u) = g(t, x)$$

with the second order linear (differential) operator $A(t, x, D_x)u$. An associated operator equation is studied by the Fredholm and Nemitskij operator theory. Under continuity or local Hölder conditions for the nonlinear member we observe quantitative and qualitative properties of the set of solutions of the given problem. These results can be applied to the different mechanical and natural science models.

## OSCILLATION CRITERIA FOR SECOND ORDER NONLINEAR RETARDED DIFFERENTIAL EQUATIONS

*Jozef Džurina*, Košice

*MSC 2000*: 34C10

In this paper the oscillatory behaviour of nonlinear delay differential equation of the form

$$\left(|u'(t)|^{\alpha-1}u'(t)\right)' + p(t)f[u(\tau(t))] = 0$$

is investigated. The aim of this paper is to present some new oscillatory criteria which are new also for $\alpha = 1$, namely for the second order nonlinear differential equation

$$u''(t) + p(t)f[u(\tau(t))] = 0.$$

The research was supported by grant 1/7466/20 of Slovak Grant Agency.

## DESTABILIZING EFFECT OF MULTIVALUED BOUNDARY CONDITIONS FOR REACTION-DIFFUSION SYSTEMS

*Jan Eisner*, Prague

*MSC 2000*: 35B32, 35K57, 35K58, 47H04

Sufficient conditions for destabilizing effects of unilateral boundary conditions of the type

$$u = v = 0 \quad \text{on } \Gamma_D, \qquad \frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 \quad \text{on } \Gamma_N,$$
$$\frac{\partial u}{\partial n} = 0,\ \frac{\partial v}{\partial n} \in -m(v) \quad \text{on } \Gamma_U,$$

(with a certain multivalued mapping $m$) and for the existence of bifurcation points for spatial patterns to reaction-diffusion systems of the activator-inhibitor type

$$d_1\Delta u + f(u,v) = 0, \quad d_2\Delta v + g(u,v) = 0 \quad \text{in } \Omega$$

are studied. The conditions are related with the mollification method employed to overcome difficulties connected with empty interiors of appropriate convex cones.

## A NONLINEAR DISSIPATIVE WAVE EQUATION

*Jorge A. Esquivel-Avila*, Tamaulipas, Mexico

*MSC 2000*: 35L70, 35B35

We consider the following wave equation, in a bounded domain $\Omega \in \mathbb{R}^n$,

$$u_{tt} - \alpha\Delta u + g(u_t) = f(u),$$

with homogeneous boundary Dirichlet condition, where

$$f(s) = \mu s|s|^{r-2}, \quad \mu > 0,\ r > 2,$$

and

$$g(s) = \delta s|s|^{\lambda-2}, \quad \delta > 0, \quad \lambda \geqslant 2.$$

We give conditions for globality, boundedness, nonglobality, blow-up, and convergence to equilibria of solutions. For nonlinear dissipation, $\lambda > 2$, there exist gobal and unbounded solutions, in contrast with the linear case, $\lambda = 2$, where any global solution is bounded and any nonglobal solution blows-up in a finite time.

## INVARIANT FOLIATIONS UNDER NUMERICS

*Gyula Farkas*, Győr, Hungary

In this talk we investigate invariant center-unstable foliations about equilibria under numerical approximations. Our main result shows that the foliations are preserved in the $C^j$-topology. We give estimates on the order of convergence as well. A numerical fold-bifurcation theorem and results on partial linearization are also given.

# FORCED VIBRATIONS OF ABSTRACT UNDAMPED WAVE EQUATIONS

*Michal Fečkan*, Bratislava

*MSC 2000*: 35B10, 35B20, 11A55

Existence results of periodic solutions of certain abstract nonlinear wave equations are given when eigenvalues of linear parts of those equations are incommensurable to the time period of forcing terms. An equation of the form

$$u_{tt} + Au = f(u, t)$$

is studied, where $A$ is a self-adjoint, unbounded linear operator with eigenvalues $\lambda_1 \leqslant \lambda_2 \leqslant \ldots \to \infty$ and $f$ is $T$-periodic in $t \in \mathbb{R}$. Resonant and nonresonant cases are investigated by using the Leray-Schauder fixed point theory together with a Diophantum-like inequality for the positive eigenvalues of $A$. An application is given to a one-dimensional forced beam equation.

# ON SOME RECENT RESULTS ON THE EXISTENCE AND THE LONG-TIME BEHAVIOUR OF SOLUTIONS TO THE NAVIER-STOKES EQUATIONS OF COMPRESSIBLE VISCOUS FLUIDS

*Eduard Feireisl*, Prague

The time evolution of the density $\varrho = \varrho(t, x)$ and the velocity $\vec{u} = \vec{u}(t, x)$ of a compressible viscous fluid is governed by the Navier-Stokes system

$$\varrho_t + \operatorname{div}(\varrho\vec{u}) = 0,$$
$$(\varrho\vec{u})_t + \operatorname{div}(\varrho\vec{u} \otimes \vec{u}) + \nabla p = \mu\Delta\vec{u} + (\lambda + \mu)\nabla(\operatorname{div}\vec{u}) + \varrho\vec{f}$$

complemented by suitable boundary conditions as the case may be.

We present some recent results on the existence and the long-time behaviour of the weak solutions provided the pressure is given by the adiabatic state equation

$$p = p(\varrho) = a\varrho^{\gamma},\ a > 0,\ \gamma > \frac{N}{2}$$

where $N$ is the dimension of the space variable. In particular, the existence of global in time weak solutions is shown with no restriction on the size of the initial data.

## DISCONTINUOUS GALERKIN METHODS FOR CONVECTION-DIFFUSION PROBLEMS

*Miloslav Feistauer*, Prague

The subject-matter of this paper is the theory and applications of the Discontinuous Galerkin Finite Element Methods for nonlinear convection-diffusion problems and compressible flow. The sought solution is approximated in space by piecewise linear discontinuous functions over convex polyhedra. The convective fluxes are approximated with the aid of the finite volume approach and the approximation of the diffusion terms is carried out by the method proposed by I. Babuska, E. Baumann and T. Oden. Time is considered either continuous (the method of lines) or a suitable Runge-Kutta discretization is used. An important problem is the "limiting" avoiding spurious oscillations in the numerical solution. We propose two new methods and analyze them theoretically, as well as with the aid of numerical experiments. The method is applied to the solution of high speed compressible flow and a technically relevant results will be presented.

The presented results were obtained in cooperation of the author with V. Dolejší and C. Schwab. The research was supported under the grant No. 201/99/0267 of the Czech Grant Agency and grant No. MSM 113200007.

## HOMOGENIZATION OF A BOUNDARY CONDITION FOR THE HEAT EQUATION

*Ján Filo*, Bratislava, Slovakia

*MSC 2000*: 35K20, 35C20, 35B27

In the recent paper [FL], an asymptotic analysis was given for the heat equation with mixed boundary conditions rapidly oscillating between Dirichlet and Neumann type. A general framework where deterministic homogenization methods can be applied to calculate the second term in the asymptotic expansion with respect to the small parameter characterizing the oscillations was stated.

The aim of my contribution is to present some applications of the above result.

*References*

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# ON THE SINGULARITIES OF SOLUTIONS TO STOCHASTIC NAVIER-STOKES EQUATIONS

*Franco Flandoli*, Pisa, Italy

Denote by $S(u)$ the set of singular points of a suitable weak solution $u$ of the Navier-Stokes equations. Caffarelli, Kohn and Nirenberg proved that $\mathcal{H}^1(S(u)) = 0$. We approach this problem by a probabilistic technique. We consider both deterministic and stochastic Navier-Stokes equations. For stationary statistical solutions $\mu$ (probability measures on the space of suitable weak solutions that are invariant for the time shift) we prove that for every $t > 0$, the set $S_t(u)$ is *empty* for $\mu$-a.e. $u$. Here $S_t(u)$ is the section of $S(u)$ at time $t$. At present we do not know whether $S(u)$ itself is empty for $\mu$-a.e. $u$. Moreover, if $\nu$ is the projection at time 0 of a stationary statistical solution $\mu$, we prove a similar result for $\nu$-a.e. given initial conditions $u_0$. Under suitable non-degeneracy assumptions on the white noise, the support of $\nu$ is the whole space of square integrable initial conditions.

# TOPOLOGICAL TECHNIC FOR THE GEODESIC CONNECTEDNESS OF SOME LORENTZIAN MANIFOLDS

*J.L. Flores* and M. Sánchez, Granada, España

*MSC 2000*: 53C50

The problem of the geodesic connectedness in Lorentzian manifolds which can be written as a global product $I \times M$, $I \subseteq \mathbb{R}$ interval, implies a topological problem related with the Brouwer's topological degree. In some interesting spacetimes this relation can be explicitely done by means of a system of ordinary differential equations. As a consequence of this approximation, the geodesic connectedness of some importants spacetimes (multiwarped spacetimes) is obtained. Moreover, the non geodesic connectedness of other spacetimes (as some natural regions of Kerr spacetime) can be proved.

## RECTIFIABILITY IN THE HEISENBERG GROUP

*Bruno Franchi*, Bologna, Italy, Raul Serapioni, Milano, Italy,
Francesco Serra Cassano, Trento, Italy

*MSC 2000*: 28A75

A classical result due to E. De Giorgi states that, if $E \subset \mathbb{R}^d$ is a set of locally finite perimeter (a Caccioppoli set, according to De Giorgi's terminology) then the associated perimeter measure $|\partial E|$ is concentrated on a portion of the boundary, the so-called reduced boundary $\partial^* E \subset \partial E$ and that $\partial^* E$ is $\mathcal{H}^{d-1}$-countably rectifiable. This means that $\partial^* E$, up to a set of $(d-1)$-Hausdorff measure zero, is a countable union of compact subsets of $C^1$ submanifolds. Through Riesz' representation theorem, if $E$ is a Caccioppoli set then $|\partial E|$ defines a Radon measure in $\mathbb{R}^d$, the so-called perimeter measure, that, when the set $E$ is sufficiently regular, coincides with the $(d-1)$-dimensional Hausdorff measure concentrated on the boundary of $E$. However, the notion of perimeter is much more natural in the context of geometric problems in Calculus of Variations, since the perimeter, unlike the Hausdorff measure, is lower semicontinuous with respect to the convergence of sets meant as the $L^1$-convergence of their characteristic functions.

In this paper we prove an exact counterpart of De Giorgi's result in the setting of the Heisenberg group $\mathbb{H}^n = \mathbb{C}^n \times \mathbb{R}$ endowed with a left-invariant metric $d$ that is equivalent to its Carnot-Carathéodory metric.

## HOMOGENIZATION OF HEAT EQUATION WITH HYSTERESIS

*Jan Franců*, Brno, Czech Republic

*MSC 2000*: 35B27, 47J40, 74Qxx

The contribution deals with a heat equation in the form $e_t = \operatorname{div}(k\,\nabla u) + f$, where the linear constitutive law $e = c\,u$ is replaced by a nonlinear relation $e = \mathcal{G}[u]$ with a scalar hysteresis operator $\mathcal{G}$ in $t$. The initial boundary value space-dependent problem is studied, i.e. the coefficient $k$ and the operator $\mathcal{G}$ depends on $x$.

Further the homogenization problem for the equation is studied. A sequence of problems of the above type with space-periodic data $\mathcal{G}^\varepsilon$ and $k^\varepsilon$ are considered while the space-period $\varepsilon$ tends to 0. The data $\mathcal{G}^*$ and $k^*$ in the homogenized problem are identified and convergence of the corresponding solutions $u^\varepsilon \to u^*$ while $\varepsilon \to 0$ is proved.

## A GLOBAL APPROACH TO HILBERT'S SIXTEENTH PROBLEM

*Valery Gaiko*, Minsk, Belarus

*MSC 2000*: 34C05 (34C23)

We suggest a new global approach to solving Hilbert's Sixteenth Problem on the maximum number and relative position of limit cycles in two-dimensional quadratic systems. This approach can be applied to arbitrary polynomial systems and to the global qualitative analysis of higher-dimensional dynamical systems. Namely: 1) to use five-parameter canonical systems with field-rotation parameters; 2) to prove in every concrete case of finite singularities that the maximal one-parameter family of multiple limit cycles is not cyclic; 3) using Bautin's result and the Wintner-Perko termination principle, to prove by contradiction the nonexistence of four limit cycles around a singular point; 4) to control simultaneously bifurcations of limit cycles around different singular points and to prove that the maximum number of limit cycles in a quadratic system is equal to four and the only possible their distribution is $(3:1)$.

## ON STABILITY OF THE EULER SCHEME FOR AFFINE STOCHASTIC DELAY DIFFERENTIAL EQUATIONS

*Hagen Gilsing*, Humboldt University at Berlin

*MSC 2000*: 60Hxx, 60H10

Stochastic Delay Differential Equations (SDDE) play an important rôle in models, e.g. in the fields of biology, economics and physics. As explicit analytical solutions are known for a quite restricted class of SDDE only, the numerical treatment of SDDE is important. But along the numerical simulations of SDDE typical questions for consistency and stability arise. In this talk we consider the affine SDDE $\mathrm{d}X(t) = \mu X(t-\tau)\,\mathrm{d}t + \mathrm{d}W(t)$ with a real scalar parameter $\mu$, a positive delay time $\tau$, additive noise and the Euler scheme applied to this SDDE with the step width $h$ of the arithmetic lattice. We compare the stationarity region of those parameters $\mu$, for which the affine SDDE has a stationary solution, with the stability region of the Euler scheme containing those $\mu$, which are stable in a sense to be defined. In particular, the stability region depends on $h$, $\mu$ and $\tau$.

## SUFFICIENT CONDITIONS FOR A DEGENERATE CENTER

JAUME GINÉ, Lleida, Spain

*MSC 2000*: Primary 34C05; Secondary 34C23, 37G15

Consider the two-dimensional autonomous systems of differential equations of the form

$$\dot{x} = P_3(x,y) + P_4(x,y), \quad \dot{y} = Q_3(x,y) + Q_4(x,y),$$

where $P_3(x,y)$ and $Q_3(x,y)$ are homogeneous polynomials of degree 3, and $P_4(x,y)$ and $Q_4(x,y)$ are homogeneous polynomials of degree 4. The origin is a completely degenerated critical point of this system. In this work we give sufficient conditions in order to have a center at the origin. The two fundamental ideas to find these sufficient conditions are based on a method of construction of integrable systems, a generalization of the Darboux method, and imposing that the system has an isolated minimum at the origin.

## FLOQUET-LYAPUNOV THEOREMS FOR INTEGRO-DIFFERENTIAL EQUATIONS

YA. M. GOLTSER and ALEXANDER DOMOSHNITSKY, Ariel, Israel

*MSC 2000*: 35K15

Periodic systems of integro-differential equations

$$\begin{aligned} v'(t,x) &= A(t)v(t) + f(t,v(t)) \\ &\quad + B(t)\int_0^t K(t,s)(C(s)v(s) + r(s,v(s)))\,\mathrm{d}s = 0, \quad t \geqslant 0, \end{aligned} \tag{1}$$

where $v \in \mathbb{R}^n$, $f(t,v)$, $r(t,v)$ are nonlinear vector-functions of more than first order.

For linear systems ($f(t,v) = 0$, $r(t,v) = 0$) the problem of solution's representation is considered. Analogs of Floquet's representation are obtained for kernels of the form

$$K(t,s) = F(t)\exp[l(t-s)]M(s), \tag{2}$$

where $n \times n$ matrices $A(t)$, $B(t)$, $F(t)$, $M(s)$, $C(s)$ are periodic matrices and $l$ is a constant matrix. For nonlinear systems the problem of stability of the trivial solution is considered. An approach is based on the reduction method developed by the authors for several classes of systems.

## MEMORY DRIVEN INSTABILITY IN A DIFFUSION PROCESS

*Michael Grinfeld*, Glasgow, UK

*MSC 2000*: 35B40, 35K47, 35L70

We consider the $n$-dimensional version of a model proposed by Olmstead et al. for the flow of a non Newtonian fluid in the presence of memory. We prove the existence of a global attractor and obtain conditions for the existence of a Lyapunov functional which allows us to give a full description of this attractor in a certain region of the parameter space in the bistable case. We then study the stability and bifurcation of stationary solutions and, in particular, prove that for certain values of the parameters it is not possible to stabilize the flow by increasing a Rayleigh-type number. The existence of periodic and homoclinic orbits is also shown, by studying the Bogdanov-Takens singularity obtained from a centre manifold reduction. This is joint work with Pedro Freitas (IST, Portugal) and Brian Duffy (Strathclyde, UK).

## THERMAL EFFECTS IN AN ELASTIC PLATE-BEAM STRUCTURE

*Marié Grobbelaar-van Dalsen*, Pretoria, South Africa

With the development of "smart materials technology" well-posedness results for interactive structures have become of interest. In this talk we consider a linear plate-beam problem which serves as a model for the deflections of a thermo-elastic plate which has a beam attached to its free end. We show that the initial-boundary-value problem for the interactive system of partial differential equations which take account of the mechanical strains/stresses and the thermal stresses in the plate and the beam, can, with no additional mechanical dissipation in the boundary conditions, be associated with a uniformly bounded evolution operator.

We then show that the interplay of parabolic dynamics due to the thermal effects in the plate-beam structure and the hyperbolic dynamics associated with the elasticity of the structure, yields analyticity for the entire structure. This result furnishes a unique solution for the plate-beam problem under optimal initial freedom of the displacement, velocity and temperature both within the plate and the beam, while uniform stability of the structure is readily available.

The abstract form into which the initial-boundary-value problem is cast, is an implicit evolution equation with cause and effect in two different spaces. By constructing for this problem an analytic double family of evolution operators "in empathy" has the advantage that once the analytic property is accomplished, the displacement, velocity and temperature in the plate and the beam may be decoupled from one another, i.e. need not match initially.

## UPPER AND LOWER SOLUTIONS FOR SOME HIGHER ORDER BOUNDARY VALUE PROBLEMS

Maria do Rosário Grossinho, Lisboa, Portugal

*MSC 2000*: 34B15

We study the existence of solutions for the higher order boundary value problem

$$\begin{cases} u^{(n)}(t) = f(t, u(t), u'(t), \dots, u^{(n-1)}(t)), \quad n \geqslant 2, \\ u^{(i)}(0) = 0, \quad i = 0, \dots, n-3, \\ au^{(n-2)}(0) - bu^{(n-1)}(0) = A, \\ cu^{(n-2)}(1) + du^{(n-1)}(1) = B, \end{cases}$$

with $f\colon [0,1] \times \mathbb{R}^n \to \mathbb{R}$ a continuous function, $a, c, A, B \in \mathbb{R}$ and $b, d \geqslant 0$ such that $a^2 + b^2 > 0$ and $c + d > 0$.

The main theorem is obtained by the topological degree method and lower and upper solutions technique. Moreover, bounds for $u^{(i)}$ derivatives are given, for $i = 0, \dots, n-2$.

*This is a joint work with Feliz Minhós*

## NON EXISTENCE OF TRAVELLING WAVE SOLUTIONS FOR A DAVEY-STEWARTSON SYSTEM

Marisela Guzmán Gómez, Azcapotzalco, México

*MSC 2000*: 35Q55, 76B15

We consider the problem of existence of travelling wave solutions of the form $u(x,y,t) = e^{i\omega t} v(x,y)$ for the Davey-Stewartson (DS) system that reads,

$$\begin{aligned} &iu_t + \delta u_{xx} + u_{yy} = \lambda |u|^2 u + bu\varphi_x, \quad (x,y) \in \mathbb{R}^2,\ t \in \mathbb{R}, \\ &\varphi_{xx} + m\varphi_{yy} = (|u|^2)_x \end{aligned}$$

for the (complex) wave amplitude $u(x,y,t)$ and the (real) mean velocity potential $\varphi$. The coefficients $(\delta, \lambda, m, b)$ depend on the fluid depth, surface tension and gravity and can take both signs. It is known that for the pair $\delta > 0$ and $m > 0$ travelling wave solutions exist and decay when $x^2 + y^2 \to +\infty$. If $\delta = -1$ and $m < 0$, $u(x,y,t) = e^{i\omega t} v(x,y)$ is a solution of the DS system only if $b = -1$. If the second equation is hyperbolic ($m < 0$) the approach shall be different. We show that no travelling wave solutions exist for a class of boundary conditions for $\varphi$.

## HIERARCHICAL MATRICES

*Wolfgang Hackbusch*, Leipzig, Germany

Hierarchical matrices ($\mathcal{H}$-matrices) are a tool for approximating dense matrices associated with elliptic pseudo-differential operators. In particular, they are suited for the discretisation of integral operators as they appear, e.g., in the boundary element method. But $\mathcal{H}$-matrices can also be used to represent the (fully populated) inverse of a finite-element stiffness matrix.

The storage of an $\mathcal{H}$-matrix of size $n \times n$ amounts to $O(n \log^{\alpha} n)$, where $\alpha$ is a small number depending an the variant of the $\mathcal{H}$-matrix (there are even variants with $\alpha = 0$).

The important feature of $\mathcal{H}$-matrices is the fact that not only the matrix-vector multiplication but also all matrix operations (matrix-matrix addition, matrix-matrix multiplication and inversion) can be performed approximately. Again the operation cost is linear in $n$ up to logarithmic factors.

## ON THE ASYMPTOTIC BEHAVIOUR OF CONSERVATION LAWS WITH DEGENERATE SOURCE TERMS

*Jörg Härterich*, Berlin, Germany

We are concerned with hyperbolic balance laws of the form

$$u_t + f(u)_x = \frac{g(u)}{\varepsilon}, \quad u \in \mathbb{R}, \ x \in \mathbb{R}$$

with convex flux $f$. For a dissipative reaction term $g$ which has a degenerate zero we study separately the asymptotic behavior for large $t$ and for small $\varepsilon$. Surprisingly, it turns out that for $t \to \infty$ and periodic initial data it may happen that solutions converge to some spatially homogeneous equilibrium which is (weakly) unstable with respect to the pure reaction dynamics.

For the zero reaction time limit $\varepsilon \searrow 0$ all solutions tend to piecewise constant functions which are separated by classical shocks or by non-shock discontinuities. While the speed of the shocks depends on the flux $f$ only, the propagation of the non-shock discontinuity involves the interplay between flux and the source term.

The talk is based on joint work with Haitao Fan (Georgetown University).

## ASYMPTOTIC STABILITY OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH STATE-DEPENDENT DELAYS

*Ferenc Hartung*, Veszprém, Hungary

*MSC 2000*: 34K, 34D

In this talk, which is a joint work with I. Győri, we study asymptotic stability of certain classes of functional differential equations with state-dependent delays. We show results when stability (including asymptotic and exponential stability) of a certain associated state-independent FDE implies that of the state-dependent FDE. We also present some cases when, with this technique, we get not only a sufficient condition, but necessary conditions, as well. As an application of our technique, we formulate stability theorems for threshold-type state-dependent FDEs.

## ON THE DIRICHLET PROBLEM FOR PRESCRIBED MEAN CURVATURE EQUATIONS IN SOME NON-CONVEX DOMAINS

*Kazuya Hayasida*, Fukui-city, Japan

*MSC 2000*: 35J60 and 35J65

Recently the author and the coworker have solved the Dirichlet problem for the prescribed mean curvature equation and have given a few examples in the following paper:

K. Hayasida and M. Nakatani, On the Dirichlet problem of prescribed mean curvature equations without H-convexity condition, Nagoya Math. J., 157(2000), 177–209.

Here some portion $\Gamma$ of the boundary does not need to have the H-convexity condition. Then we have proved that the required solution satisfies the Dirichlet boundary condition on $\Gamma$, in some weak sense. But it is assumed that for each point $P$ of $\Gamma$ there is a one-to-one $\mathbb{C}^3$ mapping $(x_1, \ldots, x_n) \to (y_1, \ldots, y_n)$ such that

$$D_x y_i \cdot D_x y_j = 0,$$

if $i \neq j$.

In this talk the author removes this assumption.

## ON THE RADIALLY SYMMETRIC SOLUTIONS OF A CLASS OF NONLINEAR, NONLOCAL ELLIPTIC PROBLEMS

*Jenő Hegedűs*, Szeged, Hungary

*MSC 2000*: 35B30, 35B65, 35J60, 35J65

Existence, uniqueness, monotonicity, comparison and concavity results will be presented for the solutions of the Problems:

(0) $u \in C^2(B^0_R) \cap C(\overline{B^0_R})$,

(1) $\Delta u(x) + f(|x|, u, |\nabla u|) = 0 \qquad x \in B^0_R$,

(2) $(\alpha_{1,0} u + \alpha_{1,1} u_r)|_{r=R_1} + (\alpha_{2,0} u + \alpha_{2,1} u_r)|_{r=R_2} = A$,

(3) $\exists v \in C([0,R])\colon v(|x|) = u(x) \qquad x \in \overline{B^0_R}$.

Here $B^0_R := \{x \in \mathbb{R}^n \,|\, r \equiv |x| < R\}$, $2 \leqslant n \in \mathbb{N}$, $R \in (0,\infty)$, $f \in C(G_{a_0}, (0,\infty))$ where $G_{a_0} := [0,R] \times (a_0, \infty) \times [0,\infty)$ and $a_0 \geqslant -\infty$; $\alpha_{ij} \in \mathbb{R}$, $i = \overline{1,2}$; $j = \overline{0,1}$; $A$ is a real variable; $0 < R_1 < R_2 \leqslant R$. Continuous and mixed type nonlocal conditions instead of (2) also will be considered.

## MULTIPLICITY OF SOLUTIONS FOR NONHOMOGENEUOUS NONLINEAR ELLIPTIC EQUATIONS WITH CRITICAL EXPONENTS

*Norimichi Hirano*, Yokohama, Japan

*MSC 2000*: 35J60, 35J65

Let $N \geqslant 3$ and $\Omega \subset \mathbb{R}^N$ be a bounded domain with a smooth boundary $\partial\Omega$. In this talk we consider the existence and multiplicity of solutions of problem

$$\begin{cases} -\Delta u = |u|^{2^*-2} u + f & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{P}$$

where $2^* = 2N/(N-2)$ and $f \in C(\overline{\Omega})$ with $f \not\equiv 0$ and $f \geqslant 0$ on $\Omega$. In this talk, we give a multiplicity result for (P) by using the homology groups of $\Omega$.

## WORST SCENARIO APPROACH FOR ELASTOPLASTICITY WITH HARDENING AND UNCERTAIN DATA

*I. Hlaváček*, Prague

*MSC 2000*: 49J40, 65M60, 74C05, 93C41

Coefficients of stress-strain relations, yield function and hardening parameters in a model of isotropic hardening are considered to be uncertain. Using simplest finite elements in $\mathbb{R}^3$ and backward differences in time, approximate solution is defined and its continuous dependence on the uncertain input data investigated. Three different criteria are chosen to find the worst input data in a given set of admissible data. Some existence and convergence results are proven.

## ON BLACK- AND WHITE HOLE SOLUTIONS OF SECOND ORDER NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS

*Jaroslav Jaroš*, Bratislava, Slovak Republic

*MSC 2000*: 34C10

We consider the nonlinear ordinary differential equation

$$(|y'|^{\alpha})' + q(t)|y|^{\beta} = 0 \tag{A}$$

where $\alpha, \beta \in \mathbb{R}$, $\alpha \neq 0$, are constants and $q\colon [a, \infty) \to \mathbb{R}$, $a \geqslant 0$, is a continuous function which is not identically zero in any neighborhood of infinity.

Our main objective is to investigate the structure of the solution set of (A) and to show that nonlinear equations of the form (A) with $\alpha < 0$ may have singular solutions of a new type, called *black hole solutions*, which satisfy

$$\lim_{t\to T_y-0} |y(t)| < \infty \quad \text{and} \quad \lim_{t\to T_y-0} |y'(t)| = \infty$$

at the (finite) right end-point of the maximal interval of existence. The question of the existence of singular solutions of Eq. (A) (with $\alpha > 0$) satisfying

$$\lim_{t\to T_y-0} |y(t)| < \infty \quad \text{and} \quad \lim_{t\to T_y-0} y'(t) = 0$$

as $t$ approaches the maximal existence time $T_y < \infty$ of $y$, called *white hole solutions*, is also discussed.

## NON-REGULARITY OF PARABOLIC SYSTEMS WITH BOUNDED AND MEASURABLE COEFFICIENTS

*Oldřich John*, Jana Stará, Praha

*MSC 2000*: 35B65, 35K40

We consider the parabolic systems

$$u_t^i = D_\alpha(A_{ij}^{\alpha\beta}(x,t)D_\beta u^j), \quad i = 1,\dots,N$$

on $\mathbb{R}^n \times \mathbb{R}$ with bounded and measurable coefficients satisfying ellipticity condition. In case of $N = n \geqslant 3$ we can prove—by the construction of the example—the following assertion: For any $F \subset \mathbb{R}^n \times \mathbb{R}$, $F$-closed, there exists a system and its weak solution $u \in L_{2,\mathrm{loc}}(\mathbb{R}; W^1_{2,\mathrm{loc}}(\mathbb{R}^n))$ such that $u$ is discontinuous at each point of $F$ and continuous at each point of the complement of $F$. From here it follows that the solution of BVP with "good" initial and boundary functions on the parabolic boundary of the cylinder $Q = B_R \times (0,T)$ can develop its singularities in the interior of $Q$. (We can also prove that the isolated singularities of the weak solutions of BVP can occur even in case of the quasilinear system.)

## ON THE LOWER/UPPER SOLUTIONS TECHNIQUE FOR MULTIVALUED BOUNDARY VALUE PROBLEMS

*Libor Jüttner*, Olomouc-Hejčín, Czech Republic

*MSC 2000*: 34A60, 34B15, 34B40, 34C25

The lower and upper functions technique for second order upper-Carathéodory differential inclusions is investigated in this paper. We extend the results in [RT], for multivalued mappings. The results are based on the method of lower and upper functions, associated with the problem

$$x'' \in \varphi(t,x,x'), \quad x(a) = x(b),\ x'(a) = w(x'(b)),$$

and their relation to the Leray-Schauder degree for corresponding multivalued operator. As usual in the multivalued setting, two different concepts of lower and upper functions, the strong one and the weak one, are treated. We show that these concepts lead to similar existence results. We also give the existence result of a bounded solution on the whole line.

*References*

[RT] *I. Rachůnková and M. Tvrdý*: Nonsmooth lower and upper functions and solvability of certain nonlinear second order BVP's.. To appear in J. of Inequal. & Appl.

## ASYMPTOTIC PROPERTIES OF A SYSTEM OF TWO DIFFERENTIAL EQUATIONS WITH DELAY

*Josef Kalas*, Lenka Baráková, Brno, Czech Republic

*MSC 2000*: 34K15, 34K20

This lecture is devoted to asymptotic behaviour and stability questions of solutions for the real system

$$x' = \mathsf{A}(t)x(t) + \mathsf{B}(t)x(t-r) + h(t, x(t), x(t-r)),$$

where $\mathsf{A}(t) = (a_{jk}(t))$, $\mathsf{B}(t) = (b_{jk}(t))$ $(j, k = 1, 2)$ are real square matrix-functions, $x = (x_1, x_2)$, and $h(t, x, y) = (h_1(t, x, y), h_2(t, x, y))$ is a real vector function, $r$ being a positive constant. Since the plane has special topological properties different from those of $n$-dimensional space, where $n \neq 2$, it is interesting to study asymptotic behaviour of two-dimensional system by using tools which are typical and efficient for two-dimensional systems. Our approach is based on the combination of the technique of complexification and the second Lyapunov method. It allows to simplify some results, calculations, arrangements and estimations of expressions and formulate results in a relatively simply and lucid form.

## ON TOPOLOGICAL DEGREE TO MULTI-VALUED SOLUTION MAP IN A SEMILINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATION

*Takashi Kaminogo*, Sendai, Japan

*MSC 2000*: 35B99

We defined the topological degree to multi-valued Poincaré (solution) map for a semilinear parabolic partial differential equation with Neumann boundary condition in 1997. In order to do that we constructed an approximate sequence of single-valued continuous and completely continuous mappings, and defined the degree by using the degree to those mappings.

In this article, we shall give a different construction of single-valued mappings which approximate the Poincaré map from the above, and we shall also define the degree by using those mappings. Furthermore, it will be seen that the degree obtained in this article coincides with that given in the above.

## PROPERTIES OF THE SET OF SOLUTIONS FOR NONLINEAR BOUNDARY VALUE PROBLEM

*Mária Kečkemétyová*, Bratislava, Slovakia

*MSC 2000*: 34B15

We consider the system of nonlinear differential equations with linear boundary conditions

$$x'(t) - A(t)x(t) = f(t,x),$$
$$Tx = r$$

on unbounded interval $[a,\infty)$. The topological structure of the set of solutions for this boundary value problem is investigated. By using the theorem based on N. Aronszajn's idea we have established sufficient conditions that the set of solutions is an $R_\delta$-set.

## CONSTRUCTING WEAK SOLUTIONS IN A DIRECT VARIATIONAL METHOD AND AN APPLICATION OF VARIFOLD THEORY (A RECENT RESULT)

*Koji Kikuchi*, Shizuoka, Japan

In recent several years the method of semidiscretization in time and minimizing variational functionals is applied to constructing weak solution to various partial differential equations. This approximating method is firstly applied to constructing weak solutions to linear parabolic equations by Rektorys and Norio Kikuchi has independently rediscovered this method, and there are many works in applying this method after his work. Usually difficulty lies in showing the convergence of nonlinear terms. One of the ideas of overcoming this difficulty is to employ the convergence in the space of varifolds, which are a kind of generalized surfaces. Each function can be identified with its graph and hence regarded as a varifold. It is not difficult to show the convergence of varifolds corresponding to approximate solutions up to a subsequence. Then the goal could be reached by investigating the structure of the limit varifold.

## BIFURCATIONS OF HOMOCLINIC ORBITS TO A SADDLE-CENTER FOR REVERSIBLE SYSTEMS

*Jenny Klaus*, Ilmenau, Germany

We consider two-parameter families of $R$-reversible vector fields having (at the critical parameter value) a homoclinic orbit to a non-hyperbolic fixed point. The non-hyperbolicity is due to a pair of purely imaginary eigenvalues. We give a complete description of the bifurcating 1-homoclinic orbits to the center manifold. For that purpose we adapt Lin's method. We have to consider several cases distinguished by the relative position of the center-stable manifold and the fixed space of $R$.

*MSC 2000*: 37G25, 37G40, 37C29, 37C80, 34C23, 34C37

## NEW TYPE OF COMPLEX DYNAMICS IN THE $1:2$ SPATIAL RESONANCE

J. Porter, Evanston, USA, and *E. Knobloch*, Leeds, UK

The amplitude equations describing the $1:2$ spatial resonance in $O(2)$ symmetric systems are reexamined. Complex dynamical behavior associated with a new class of global connections is identified and analyzed. These connections form further from the mode interaction point where the structurally stable heteroclinic cycles described by Armbruster, Guckenheimer and Holmes[a] no longer exist. The resulting behavior is characteristic of systems with $O(2) \times Z_2$ symmetry in which the symmetry $Z_2$ is weakly broken[b], and related behavior occurs in $1:n$ resonances ($n > 2$) as well[c].

*References*

[a]D. Armbruster, J. Guckenheimer and P. Holmes, Heteroclinic cycles and modulated travelling waves in systems with $O(2)$ symmetry, Physica D 29 (1988) 257–282.

[b]I. Mercader, J. Prat, and E. Knobloch, The 1 : 2 mode interaction in Rayleigh-Bénard convection with weakly broken midplane symmetry, Int. J. Bif. Chaos. 11 (2001) 27–41.

[c]J. Porter and E. Knobloch, Complex dynamics in the 1 : 3 spatial resonance, Physica D 143 (2000) 138–168.

# INERTIAL MANIFOLDS FOR NONAUTONOMOUS DYNAMICAL SYSTEMS

*Norbert Koksch*, Dresden

*MSC 2000*: 34C30, 34D45, 34G20, 35B42, 37L25

Assuming suitable generalizations of the cone invariance and squeezing properties of the nonautonomous dynamical system $\varphi$ and two additional technical assumptions on $\varphi$, we are in position to show the existence of an (nonautonomous) inertial manifold for $\varphi$. In particular, under suitable assumptions on the family of linear operators $A(t)$ on $Y$ and on the nonlinearity $f\colon \mathbb{R}\times X\to Y$ with dense $X\subseteq Y$, a nonautonomous evolution equation

$$\dot{u}+A(t)u=f(t,u) \tag{1}$$

in a Banach space $Y$ generates a two-parameter semiflow over $X$ and hence a nonautonomous dynamical system $\varphi$ over $X$. Using exponential dichotomy assumptions on the linear part of (1) and Lipschitz and boundedness assumptions on $f$, we find a generalization of the well-known spectral gap condition ensuring the existence of a time-depending inertial manifold for (1).

# POLYNOMIAL FUNCTIONALS GIVING $L^p$-BOUNDS AND GLOBAL EXISTENCE FOR SOLUTIONS OF REACTION-DIFFUSION SYSTEMS

*Said Kouachi*
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*MSC 2000*: Primary 35K45, 35K57; Secondary 35K45

Generally authors that work on systems of coupled reaction-diffusion equations impose conditions on reactions terms so that the solution, on its interval of existence, remains positive and that one of its components is uniformly bounded, then they deduct that the other one is too. Here one tries to demonstrate the boundedness of the solution without supposing it for the one of its components. For this purpose we construct polynomials (according to solutions of the coupled reaction-diffusion equations) functionals. These last give often $L^p$-bounds for solutions and if the reaction terms are sufficiently regular, using the well known effect regularizing, we deduce global existence.

## PERIODIC SOLUTIONS IN A GIVEN SET OF DIFFERENTIAL SYSTEMS

*Bohumil Krajc*, Ostrava

*MSC 2000*: 34C25

The study of periodic solutions to differential systems which remain in a given set is mostly related to the flow invariance of this set. On the other hand, the more complicated case when no invariance is assumed is still far from to be obvious.

In our contribution we investigate this case by means of the Ważewski-type approach combined with degree arguments. The transversality behaviour on the boundary of a given domain is required to obtain the existence of periodic solutions of the Carathéodory systems of ordinary differential equations.

## OSCILLATION OF DIFFERENTIAL AND DIFFERENCE SYSTEMS

*Werner Kratz*, Ulm, Germany

*MSC 2000*: 39A12, 15A18

In the first part of this talk we present an oscillation theorem on differential systems. We consider an eigenvalue problem, which consists of a linear, Hamiltonian differential system and self-adjoint boundary conditions. The oscillation theorem relates the number of eigenvalues, which are less than a given number, to the number of focal points ("zeros") of solutions of the corresponding differential system.

In the second part we state analogical results for corresponding *discrete* eigenvalue problems on symplectic difference system. Moreover, we consider Sturm-Liouville difference equations of even order, which are special symplectic systems. Finally, we discuss applications to the numerical analysis of *banded matrices*, because every banded matrix corresponds uniquely to a Sturm-Liouville difference operator.

# ON APPROXIMATE SOLUTION TO SEMI-LINEAR ELLIPTIC BOUNDARY PROBLEM WITH SMALL PARAMETER AT ONE HIGHER DERIVATIVE

*Sergiy V. Kravchuk*, Mawson Lakes, Australia

*MSC 2000*: 35B25, 35J25

The current work studies a boundary problem for a singularly perturbed semi-linear elliptic equation with a small parameter at one higher derivative: $\mu^2\partial^2\psi/\partial\xi^2 + \partial^2\psi/\partial\eta^2 + \omega(\psi) = 0$. The problem is considered for an infinite semi-strip $\mathcal{D} = \{0 < \xi < +\infty,\ 0 < \eta < h(\xi)\}$. An extension of boundary functions method is developed and justified. The exact solution, $\psi(\xi,\eta,\mu)$, is approximated by a sum of a far field term, $\varphi(\xi,\eta)$, and a boundary layer term, $\Phi\,(\xi/\mu,\eta)$. The existence of $\psi$, $\varphi$, and $\Phi$ is proven, the far-field and boundary layer terms are constructed, and the order of approximation is verified: $\psi(\xi,\eta,\mu) = \Phi\,(\xi/\mu,\eta) + \varphi(\xi,\eta) + O(\mu)$.

# COLOURING AND REFINING SIMPLICIAL PARTITIONS IN $\mathbb{R}^n$

*Michal Křížek*, Prague

*MSC 2000*: 05C15, 65N30

We consider face-to-face partitions of $n$-dimensional polyhedra into simplexes for an arbitrary $n \geqslant 1$. First, we prove that any simplicial partition can be coloured by at most $n+1$ colours so that each adjacent simplexes sharing an $(n-1)$-dimensional face have different colours. The number of colours cannot be reduced, in general. In particular, every planar triangulation can be coloured by at most 3 colours, every space tetrahedralization by at most 4 colours, etc. This fact has applications in visualization of the finite element method.

Second, we show how to treat vertex and edge singularities when solving partial differential equations by the finite element method. We recall yellow, red, green and blue refinement techniques of planar triangulations and introduce their analogues in $\mathbb{R}^3$. We show how to generate refinements of tetrahedral partitions into nonobtuse tetrahedra, which guarantees the validity of the discrete maximum principle.

## BIFURCATION FOR VARIATIONAL INEQUALITIES BASED ON IMPLICIT FUNCTION THEOREM

*Milan Kučera*, Prague

*MSC 2000*: 47J15, 35J85, 47J20

Joint results with L. Recke and J. Eisner will be presented. Implicit Function Theorem is applied in a nonstandard way to variational inequalities depending on a parameter which can be multidimensional in general. In this way, smooth branches of nontrivial solutions bifurcating from eigenvalues of the homogenized variational inequality satisfying certain particular assumptions are obtained. Analogously, a smooth continuation of solutions in dependence on a parameter as well as a smooth continuation of eigenvalues for variational inequalities is shown. Theoretical results can be applied to the model of a beam compressed by a force given by the bifurcation parameter and supported from one side by fixed obstacles, to second order eigenvalue problems with a coefficient in $L_\infty$ as a parameter, and to a bifurcation of spatial patterns in reaction-diffusion systems with simple nonlocal unilateral boundary conditions.

## RIEMANNIAN APPROACH TO INTEGRATION REDIVIVUS

*Jaroslav Kurzweil*, Praha

The comeback of the Riemannian approach to integration started in 1957 in connection with an attempt to obtain a unified treatment of convergence phænomena in ODE's. In the new approach a positive function $\delta$ is sought (instead of a positive constant $\delta$) to a positive $\varepsilon$. A brief review of some important developments will be given with emphasis on topologization of the vector space of integrable functions.

## THE QUALITATIVE ANALYSIS OF AN INITIAL VALUE PROBLEM $F(t,x,x')=0,\ x(0)=0$

Yuliya V. Kuzina, Oleksandr E. Zernov, Odessa, Ukraine

MSC 2000: 34A08, 34C11

The following initial value problems are under consideration:

$$\sum_{0\leqslant i+j+k\leqslant m} a_{ijk}t^i x^j (x')^k + f_1(t,x,x') = 0, \quad x(0)=0,$$

$$t^\alpha x^\beta (x')^\gamma = f_2(t,x,x'), \quad x(0)=0,$$

where $x\colon (0,\tau)\to\mathbb{R}$ is an unknown function, $m$ is a natural number, $i,j,k\in\{0,1,\dots,m\}$, $f_s\colon\mathcal{D}\to\mathbb{R}$ are continuous functions, $s\in\{1,2\}$, $\mathcal{D}\subset(0,\tau)\times\mathbb{R}\times\mathbb{R}$, $|f_1(t,x,y)|\leqslant K(t^{m+1}+|x|^{m+1}+|y|^{m+1})$, $(t,x,y)\in\mathcal{D}$, $\alpha$, $\beta$, $\gamma$ are integer nonnegative numbers, $\gamma\geqslant 1$. Existence of continuously differentiable solutions $x\colon(0,\varrho)\to\mathbb{R}$ is being proved ($\varrho$ is small enough). Asymptotic properties each of these solutions is being plotted. If certain conditions are fulfilled then the number of such solutions is being determined.

## ON GENERAL DIFFERENTIAL-ALGEBRAIC AND INTEGRO-ALGEBRAIC SYSTEMS

Marian Kwapisz, Bydgoszcz, Poland

MSC 2000: 34A50, 65L05

In the paper we consider the following systems of equations:

$$\begin{aligned} x'(t) &= f(t,x(\cdot),\lambda(\cdot)), \quad && x_0 = x^0\\ \lambda(t) &= g(t,x(\cdot),\lambda(\cdot)), \quad && t\in J=[0,T], \end{aligned}$$

as well as the systems of the form

$$\begin{aligned} x(t) &= x_0 + \int_0^t f(t,s,x(\cdot),\lambda(\cdot))\,\mathrm{d}s\\ \lambda(t) &= g(t,x(\cdot),\lambda(\cdot)), \end{aligned}$$

where the operators $f$ and $g$ are given; $f\in C(J\times J\times C_p\times C_q, R^p)$, $g\in C(J\times C_p\times C_q, R^q)$, $C_p=C(J,R^p)$, $C_q=C(J,R^q)$. It is assumed that the operators $f$ and $g$ are Lipschitz continuous with respect to functional arguments and that they are not of the Volterra type operators. Under suitable conditions we are able to prove results concerning the existence and uniqueness of solutions to the systems considered as well as the convergence of various iterative methods for construction of approximate solutions to the systems mentioned above.

## A GLOBAL STABILITY THEOREM IN A SYSTEM OF P.D.E.S FOR ANISOTROPIC HYDROMAGNETIC FLOWS

M. Maiellaro, *A. Labianca*, Bari, Italy

*MSC 2000*: 76E-05-25-30

We prove, by the direct Liapunov method, a theorem of global stability for a system of P.D.E.s which governs some steady hydromagnetic flows in isothermal evolution in a bounded domain and in presence of anisotropic electrical currents. By applying suitably a stability estimate deduced in this theorem, we obtain asymptotic stability with respect to the $L^2$-measure of 3-D perturbations and we prove that:

*high levels of the anisotropic currents can give stability as well.*

## PAINLEVÉ DIFFERENTIAL EQUATIONS IN THE COMPLEX PLANE

*Ilpo Laine*, Joensuu, Finland

*MSC 2000*: 34M05, 34M10

We offer a survey on the recent developments concerning the meromorphic nature of solutions of Painlevé differential equations in the complex plane as well as of their growth and value distribution. In particular, we shall consider the equations PI, PII, PIV and a modified form of PIII and PV. The recent results to be treated are mostly due to Hinkkanen, Shimomura, Steinmetz and the present speaker.

## APPROXIMATION OF ATTRACTORS FOR MULTIVALUED RANDOM DYNAMICAL SYSTEMS

T. Caraballo, *J. A. Langa*, Sevilla, and J. Valero, Alicante, Spain

*MSC 2000*: 58F39, 35B40, 35K55, 35K22, 35R60, 60H15

The concept of global attractor for stochastic partial differential inclusions has been recently introduced as a joint generalization of the theory of random attractors for random dynamical systems and global attractors for multivalued semiflows. We present a general result on the upper semicontinuity of attractors for multivalued random dynamical systems. In particular, our theory shows how the random attractor associated to a small random perturbation of a (deterministic) partial differential inclusion approximates the global attractor of the limiting problem. Some applications illustrate the results.

## HYPERBOLIC STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS DRIVEN BY BOUNDARY NOISES

*Olivier Lévêque*, Lausanne

*MSC 2000*: 60H15

The framework of this presentation is the study of wave propagation phenomena, where the waves considered are generated by noise sources which are random both in time and space. More precisely, we are interested in finding real-valued solutions of linear hyperbolic partial differential equations (typically the wave equation) driven by additive Gaussian noise sources which are white in time and concentrated on surfaces in space. It is a well known fact that when the spatial dimension is greater than one, the wave equation driven by a space-time Gaussian white noise admits a solution which takes its values in a distribution space. If we want the solution to be function-valued, it is natural to consider noise with some spatial correlation. In this presentation, we are going to see that this phenomemon happens for the wave equation driven by a Gaussian noise concentrated on a surface, in particular the sphere or the plane. In both cases, we give minimal conditions on the regularity of the spatial covariance of the noise which guarantee the existence of a function-valued solution of the linear equation. This allows us afterwards to study non-linear equations of the same type.

## OSCILLATORY PROFILES OF BALANCE LAWS NEAR BIFURCATIONS ALONG MANIFOLDS OF EQUILIBRIA

*Stefan Liebscher*, Berlin

*MSC 2000*: 35L67, 34C23, 34C37

Looking for travelling waves alias viscous or non-viscous profiles of hyperbolic balance laws, we encounter bifurcations along manifolds of equilibria. Profiles with oscillatory tails can emerge near such bifurcations.

This behaviour is generated by the interaction of flux and source similarly to the Turing instability in reaction-diffusion systems. In fact, the strictly hyperbolic flux can be chosen as a gradient. The pure kinetics, on the other hand, can be chosen to be stabilising. However, the interaction of flux and source can provide oscillatory profiles. Possible bifurcations include Hopf- and Takens-Bogdanov bifurcations.

The structure and the pde stability of these oscillatory profiles shall be discussed in the talk.

## ASYMPTOTIC AND NUMERICAL ANALYSIS OF A SINGULAR BOUNDARY-VALUE PROBLEM OF EMDEN-FOWLER TYPE

N.B. Konyukhova, Moscow, Russia,
*P.M. Lima* and M.P. Carpentier, Lisboa, Portugal

In this work, we consider a singular boundary-value problem for a nonlinear second-order differential equation of the form

$$g''(u) = ug(u)^q/q, \tag{1}$$

where $0 < u < 1$ and $q$ is a known parameter, $q < 0$. We search for a positive solution of (1) which satisfies the boundary conditions

$$g'(0) = 0, \tag{2}$$

$$\lim_{u\to 1-0} g(u) = \lim_{u\to 1-0} (1-u)g'(u) = 0. \tag{3}$$

This problem arises in the study of boundary layer equations for the stationary flow of an incompressible fluid over an impermeable, semi-infinite plane.

In this work we analyse the asymptotic properties of the solution near the singularity, depending on the value of $q$. We show the existence of a one-parameter family of solutions of equation (1) which satisfy the boundary condition (2a). By means of variable substitutions, asymptotic expansions are obtained for this family of solutions. Using these asymptotic expansions we can find by the shooting method the solution that satisfies the boundary condition (2). Numerical results are obtained and compared with the ones presented in other papers.

# EXTERIOR PROBLEM FOR A CLASS OF SEMILINEAR EQUATIONS

*Vitali Liskevich*, Bristol, United Kingdom

We study the existence and non-existence of positive weak solutions to the equation

$$\sum_{i,j=1}^{n} \partial_{x_i} a_{ij}(x)\partial_{x_j} u(x) + \sum_{j=1}^{n} b_j(x)u(x) + V(x)u(x) + u(x)^q = 0$$

outside a compact set $K \subset \mathbb{R}^n$ ($n \geqslant 3$), where $(a_{ij})$ is a uniformly elliptic matrix, $(b_j)$ and $V$ are measurable (in general singular) functions. If $b_j = 0$, $V = 0$, the critical value of $q = \frac{n}{n-2}$ separates the cases of existence and non-existence of positive solutions. We reveal classes of perturbations $b_j$ and $V$ guaranteeing the same critical value. The critical case is shown to belong to the non-existence case.

# ON SOME EXTENSIONS OF 3/2-STABILITY CONDITIONS BY WRIGHT AND YORKE

*Eduardo Liz*, Vigo, Spain, Victor Tkachenko, Kiev, Ukraine,
and Sergei Trofimchuk, Santiago, Chile

*MSC 2000*: 34K20

We extend the well-known 3/2-stability condition established by Wright in 1955 to a general delay differential equation

$$x'(t) = -ax(t) + f(x(t-h)), \quad a \geqslant 0, h > 0, \tag{1}$$

where $f$ satisfies the nonnegative feedback condition ($xf(x) < 0$ for $x \neq 0$), is bounded below, has at most one critical point $x^*$ which is a local extremum and $(Sf)(x) < 0$ for all $x \neq x^*$, being $Sf$ the Schwarz derivative of $f$. When $a = 0$ and $f(x) = b(e^{-x} - 1)$, we obtain the condition $bh < 3/2$ established by Wright. Our condition for the global attractivity of the equilibrium $x \equiv 0$ in (1) improves previous results in the literature, and it is exact for the exponential stability if we consider nonconstant delay $h(t)$ ($0 \leqslant h(t) \leqslant h$) in Eq. (1), as it was recently proved in the paper [A. Ivanov, E. Liz and S. Trofimchuk, *Halanay inequality, Yorke* 3/2 *stability criterion, and differential equations with maxima*, to appear in Tohoku Math. J.].

## ON CAUCHY PROBLEM FOR THE FIRST ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS

A. Lomtatidze

Nonimprovable, in a certain sense, sufficient conditions are established for the solvability and unique solvability of the initial problem

$$u'(t) = F(u)(t),$$
$$u(a) = c,$$

where $F\colon C([a,b];\mathbb{R}) \to L([a,b];\mathbb{R})$ is a continuous operator satisfying the Carathéodory conditions.

## MULTI-DIMENSIONAL SCHEMES FOR SYSTEMS OF HYPERBOLIC EQUATIONS

Mária Lukáčová-Medviďová, Brno, Czech Republic

This contribution deals with genuinely multidimensional numerical schemes for solving systems of hyperbolic equations. In recent years the most commonly used methods for hyperbolic problems were finite volume methods which were based on a quasi dimensional splitting using one-dimensional Riemann solvers. It turns out that in special cases this approach leads to structural deficiencies in the solution.

The aim is to construct a method which takes into account all of the infinitely many directions of propagation in the flow. The main idea of the evolution Galerkin methods is the following: the initial function is evolved by means of approximate evolution operators along the bicharacteristic cone and then projected onto a finite element space. The described approach has been fully exploited for the linear hyperbolic systems such as the wave equation system or the Maxwell equations as well as nonlinear systems such as the shallow water equations and the Euler equations.

## ON A SECOND ORDER SINGULAR BOUNDARY VALUE PROBLEM

*Luisa Malaguti*, Modena and Reggio Emilia, Italy

*MSC 2000*: 34B16

We deal with the boundary value problem

$$u'' = f(t, u, u'), \quad u(a+) = u(b-) = 0, \tag{P}$$

where $f\colon ]a, b[ \times \mathbb{R}^2 \to \mathbb{R}$ is a Carathéodory function on $]a+\varepsilon, b-\varepsilon[ \times \mathbb{R}^2$ for each sufficiently small $\varepsilon$. We do not exclude the possibility for $f$ to have non-integrable singularities, in its first argument, at the points $t = a$ and $t = b$. By means of a one-side growth condition and a comparison type argument, we obtain an existence result for (P). We also propose some models, for the dynamic $f$, satisfying the required existence conditions. The results come from a joint research (A. Lomtatidze, L. Malaguti, *"On a two-point boundary value problem for the second order ordinary differential equations with singularities"*, to appear in Nonlinear Anal.).

## OSCILLATION CRITERIA FOR SECOND ORDER NONLINEAR DIFFERENTIAL EQUATIONS INVOLVING INTEGRAL AVERAGES

*Jelena Manojlović*, Niš, Yugoslavia

We present new oscillation criteria for second order nonlinear differential equation

$$\bigl[a(t)\psi(x(t))x'(t)\bigr]' + q(t)f(x(t)) = 0, \quad t \geqslant t_0 > 0, \tag{$E$}$$

where $a \in C^1\bigl(([t_0, \infty)\bigr)$ is positive function, $q \in C([t_0, \infty))$ has no restriction on its sign, $\psi, f \in C(\mathbb{R}; \mathbb{R})$ have continuous derivative on $\mathbb{R} \setminus \{0\}$, $\psi(x) > 0$, $xf(x) > 0$, $f'(x) \geqslant 0$ for $x \neq 0$. We are concerned with the oscillation of the differential equation (E) in both case where it is either strongly sublinear or strongly superlinear.

Established oscillation criteria have been motivated by classical averaging criterion of Kamenev, for the linear differential equation $x''(t) + q(t)x(t) = 0$ and they involve integral averages conditions. Under appropriate additional assumptions on $\psi(x)$ and $a(t)$, we extend and improve known oscillation criteria for the nonlinear second differential equation $x''(t) + q(t)f(x(t)) = 0$, to the nonlinear equation $(E)$, by using more general kernel function from a class of the parameter functions

$$H\colon \mathcal{D} = \{(t, s)\colon t \geqslant s \geqslant t_0\} \to \mathbb{R},$$

in the integral averages techniques. With the appropriate choice of the function $H$, it is possible to derive a great number of oscillation criteria for the equation $(E)$.

## A CLASS OF COMPETING MODELS WITH DISCRETE DELAYS

*Julio Marín* and Mario Cavani, Cumaná, Venezuela

*MSC 2000*: Primary: 39B82, 34K60

In this talk we consider a class of predator-prey models with discrete time lag. The prey, is assumed to regenerate in the absence of predators by logistic growth with carrying capacity $K$. By other hand, it is assumed that there exist proportionality between the individual death and the birth in the predator population. The functional response of the predator population is given by the Michaelis-Menten kinetics. We show the existence of the global attractor for the 2-dimensional predator-prey system. The system is improved as to consider two predators feeding on the same prey population, being established a competence between the two predators population. In this case we concern with look for the specie that win the competition.

## INCOMPRESSIBLE NEWTONIAN FLOW THROUGH A NETWORK OF THIN PIPES

*Eduard Marušić-Paloka*, Zagreb, Croatia

*MSC 2000*: 35B25, 76D30, 76D05

We study the fluid flow through a network of intersected thin pipes with prescribed pressure at their ends. Pipes are either thin or long and the ratio between the length and the cross-section, denoted as usual by $\varepsilon$, is considered as the small parameter. It is well known that the stationary Navier-Stokes system describing the viscous flow in straight pipes with impermeable walls governed by the prescribed pressure drop has a solution in the form of the Poiseuille flow with perfectly parabolic velocity profile. In real-life situations, often two (or several) pipes are interconnected (watering systems, water-works, system of blood vessels). Also, pipes can be curved and constricted (particularly in case of blood vessels). In such situation the flow is not so simple any more and the velocity profile is not necessarily parabolic. We study the above situations using the asymptotic analysis with respect to $\varepsilon$

# ZERO CONVERGENT SOLUTIONS OF CERTAIN ORDINARY NONLINEAR SYSTEMS

*Serena Matucci*, Florence, Italy

*MSC 2000*: 34C11, 34B15

We present some results recently obtained in a joint work with M. Marini of the University of Florence and P. Rehák of the Masaryk University of Brno concerning the coupled nonlinear system

$$[r(t)\Psi_p(x')]' = -f(t,y),$$
$$[q(t)\Psi_k(y')]' = g(t,x)$$

where $\Psi_\sigma(u) = |u|^{\sigma-2}u$ with $\sigma > 1$. System (1) arises in searching radial solutions for partial differential systems with $p$-laplacians operators. Here we study the existence of positive solutions of (1) asymptotically decreasing towards zero, under mild assumptions for the functions $r$, $q$, $f$, $g$ involved. Concerning the nonlinearities $f$ and $g$ neither monotonicity nor sublinearity/superlinearity with respect to the second variable is supposed, and this fact allows us to consider also the singular case and the forced one. Some special cases as Emden-Fowler type systems are also considered.

# LAPPO-DANILEVSKI SYSTEMS

*S. A. Mazanik*, Minsk

*MSC 2000*: 34A30

We consider the linear system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = A(t)x, \quad x \in \mathbf{R}^n, \quad t \geqslant 0, \tag{1}$$

where $A(t)$ is an $n \times n$ matrix of real-valued continuous and bounded functions of real variable $t$ on $[0, +\infty[$. We say that (1) is a Lappo-Danilevski system if $A(t)\int_s^t A(u)\,\mathrm{d}u = \int_s^t A(u)\,\mathrm{d}u A(t)$ for some $s$ and $t$. The main goal of our research is to investigate Lyapunov's reducibility of (1) to the Lappo-Danilevski systems.

We also present some results on the distribution of the Lappo-Danilevski systems among linear systems.

## CONTINUOUSLY EXTENDIBLE SOLUTIONS OF THE ROBIN PROBLEM FOR THE LAPLACE EQUATION

*Dagmar Medková*, Prague

The problem $\Delta u = 0$ in $\Omega$, $\partial u/\partial n + \lambda u = f$ on $\partial\Omega$ is studied on relatively wide class of domains $\Omega \subset \mathbb{R}^m$, including domains with piecewise-smooth boundary. The necessary and sufficient condition on $f$ is given for the existence of a solution $u$ of this problem (in the sense of distribution), which is continuous on the closure of $\Omega$. Under additional condition, that the boundary of $\Omega$ is locally Lipschitz, we study problem $\Delta u = g$ in $\Omega$, $\partial u/\partial n + \lambda u = f$ on $\partial\Omega$. If $g \in L_2(\Omega) \cap L_{p,\mathrm{loc}}(\Omega)$ then the necessary and sufficient condition on $f$ is given for the weak solution $u$ of this problem (in the Sobolev space $W^{1,2}(\Omega)$) to be continuous on the closure of $\Omega$.

## NONLINEAR INTEGRAL AND DIFFERENCE INEQUALITIES WITH SINGULAR KERNELS

*Milan Medveď*, Bratislava, Slovakia

*MSC 2000*: 34A40 (35K, 45D05)

The inequalities of the form

$$u(t) \leqslant a(t) + \int_0^t (t-s)^{-\alpha} F(s)\omega(u(s))\,\mathrm{d}s,\ \ t \geqslant 0,$$

$$v(x,y) \leqslant b(x,y) + \int_0^x \int_0^y (x-s)^{-\alpha}(y-\tau)^{-\beta} G(s,\tau)\omega(v(s,\tau))\,\mathrm{d}s\,\mathrm{d}\tau,\ \ x,y \geqslant 0,$$

$$w_n \leqslant c_n + \sum_{k=0}^{n-1} (t_n - t_k)^{-\alpha} \tau_k H_k \omega(w_k),\ \ n \geqslant 1$$

are solved, where $0 < \alpha < 1$, $0 < \beta < 1$, $\omega\colon \langle 0,\infty) \to R$ is a continuous, nondecreasing function with $\omega(u) > 0$, $u > 0$, $a(t)$, $F(t)$, $b(x,y)$, $G(x,y)$, $v(x,y)$ are continuous, nonnegative functions, $c_n \geqslant 0$, $H_n \geqslant 0$, $w_n \geqslant 0$, $\tau_k = t_{k+1} - t_k > 0$ with $\sup_{k\geqslant 0} \tau_k < \infty$. The obtained results are related to the Gronwall-Bihari inequality for functions in one variable, to the Wendroff inequality for functions in two variables and to the Henry inequality, convenient for the study of partial differential equations of parabolic type.

## NUMERICAL MODEL OF THERMAL FLOW IN POROUS MEDIA

*Jiří Mikyška* and Michal Beneš, Prague

*MSC 2000*: 35K05, 35J25, 65M25, 65M60, 76S05

Our contribution deals with the mathematical model of groundwater flow in porous media, including the thermal effects and heat transport. At first, the system of equations which describes the mentioned problem is set together. Next, the discretization of the equations is done and one of the possible numerical algorithms is proposed. During the discretization, the finite element method is combined with the method of characteristics. At the end, the problem of approximation of density in the individual elements is discussed and there are also refered some results of the model computed for several situations. Obtained results could be used for answering the question, how much the changes of temperature in groundwater can influence the flow.

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## STABLE SOLUTIONS TO GINZBURG-LANDAU EQUATION IN A THIN DOMAIN

*Yoshihisa Morita*, Ryukoku University

We consider the functional

$$E(\Psi, A) := \frac{1}{2}\int_{\Omega}\left\{|(\nabla - iA)\Psi|^2 + \frac{\alpha}{2}(1-|\Psi|^2)^2\right\}\mathrm{d}x + \frac{\beta}{2}\int_{\mathbb{R}^3}|\mathrm{rot}\, A|^2\,\mathrm{d}x$$

and the Euler equation that is called the Ginzburg-Landau equation, where $\Psi$ is a complex-valued function, $A$ is a magnetic potential and $\Omega$ is a bounded domain in $\mathbb{R}^3$. We assume that the thickness of the domain is controlled by a small parameter $\varepsilon > 0$ as $\Omega = \Omega(\varepsilon)$,

$$\Omega(\varepsilon) := \{x = (x', z) \in \mathbb{R}^3 : 0 < z < \varepsilon a(x'),\ x' \in D\}$$

where $D$ is a planar domain and $a(x')$ is a smooth positive function. We show the reduction of the equation as $\varepsilon \to 0$ and the existence of non-trivial stable solutions for small $\varepsilon$, which are approximated by solutions to the reduced equation.

## A MISSING TERM IN THE ENERGY INEQUALITY FOR WEAK SOLUTIONS TO THE NAVIER-STOKES EQUATIONS

Takeyuki Nagasawa, Sendai, Japan

*MSC 2000*: 35Q30, 76D05

We consider the initial-boundary value problem of the three-dimensional non-stationary Navier-Stokes equations. It is well-known that there exists a weak solution satisfying the energy inequality. It, however, is still open that the solution satisfies the energy identity. It is also uncertain that any weak solution satisfies the energy inequality. The author has already shown that some weak solutions satisfy an energy inequality with an "additional term" ([1]), and that an energy identity with an "additional term" holds under some a posteriori assumption ([2]). In this talk it is shown that every weak solution satisfies the identity with an "additional term" without a posteriori assumption.

*References*

[1] *Nagasawa, T.*: preprint, to appear in J. Math. Fluid Mech.
[2] *Nagasawa, T.*: preprint, to appear in Nonlinear Anal.

## $L_p$-ESTIMATES FOR SOLUTIONS OF DIRICHLET AND NEUMANN PROBLEMS TO HEAT EQUATION IN THE WEDGE WITH EDGE OF ARBITRARY CODIMENSION

A. I. Nazarov, St.-Petersburg

In this talk $L_p$-estimates in anisotropic weighted spaces are presented for solutions of Dirichlet and Neumann problems to the heat equation in the wedge with arbitrary codimensional edge.

This work was partially supported by Russian Fund for Fundamental Research, grant no. 99-01-00684.

## LOCALIZATION EFFECTS FOR EIGENFUNCTIONS NEAR TO THE EDGE OF A THIN DOMAIN

*Serguei A. Nazarov*, London, UK

*MSC 2000*: Primary 74B05, 74E10, 35B40

For certain shapes of the edge of a thin plate-like domain, boundary value problems for the Laplacian possess eigenfunctions which are localized near to either the edge itself, or a point on the edge. These functions decay exponentially on a distance of the whole edge and the point respectively. Any combination of the Dirichlet and Neumann conditions on the edge and the bases of the plate provide those effects while in the case of the Dirichlet conditions on the bases this localization attributes even to the first eigenfunction. The effect originates in a trapped mode appearing in the model problem posed on a strip-like domain and intended to describe boundary layers in asymptotic expansions.

Mixed boundary value problems for other differential operators are considered as well. In particular, eigen-oscillations of an isotropic cylindrical plate (a gasket) is discussed in the case that it's bases are clamped and it's lateral side is free of stresses.

## ASYMPTOTIC PROPERTIES OF THE STEADY FALL OF A BODY IN A VISCOUS LIQUID

*Š. Nečasová*, Praha, Czech Republic

We are interested in the following problem. We say that a body undergoes a steady falling motion in an infinite viscous fluid if the motion of the fluid is attached to the body is independent of time. We prescribe the shape and downward orientation of the body we think of the body as a hollow one inside which we are free to move masses about, and we seek a position result in a steady falling motion with the given downward orientation. In general the body must undergo a rotation about the vertical axis as well as a translation in this motion. We consider the fall under its own weight of a bounded connected rigid body in an infinite Stokes and Oseen fluid which is at rest at infinity.

## ON THE NEW PHENOMENA OF COMPLEX WKB METHOD FOR HIGHER ORDER ORDINARY DIFFERENTIAL EQUATION: CONNECTION FORMULAS

*T. Nishimoto*, K. Matsubara, and M. Nakano

The complex WKB theory for $N$-th order ordinary differential equations, $N > 2$, is quite incomplete. Before treating the general equations, we study a certain third order equation, which was analyzed by Berk, Nevins and Roberts in the paper, New Stokes lines in WKB theory, J. Math. Phys. 23(1982), 988–1002. By applying the so-called extended Fedoryuk theory and the saddle point method, we succeeded to obtain the WKB solutions on the whole complex plane, the connection formulas and new phenomena that don't exist in the secound order equations. In these analysis, the Stokes curve configuration and the canonical domains on the Riemann surface associated with the differential equation play the fundamental role. The Riemann surface of our equation consists of 6 sheets of complex plane, but is combined into one sheet of plane by a suitable transformation, so that the Stokes curve configuration can easily be visualized using computer graphics. This greatly helps us understand the whole feature of analysis.

## AN ADAPTIVE UZAWA FEM FOR STOKES: CONVERGENCE WITHOUT THE INF-SUP

*Ricardo H. Nochetto*

We introduce and study an adaptive finite element method for the Stokes system based on an Uzawa outer iteration to update the pressure and an elliptic adaptive inner iteration for velocity. We show linear convergence for the pairs of spaces consisting of continuous finite elements of degree $k$ for velocity and either continuous or discontinuous elements of degree $k-1$ and $k$ for pressure. The popular Taylor-Hood family is the sole example of stable elements included in the theory, which in turn relies on the stability of the continuous problem and thus makes no use of the discrete inf-sup condition. We provide consistent computational evidence that the resulting meshes are quasi-optimal for *any* pair of spaces.

## ON A PHASE FIELD MODEL WITH MEMORY

A. Novick-Cohen

We discuss a phase field model with memory:

$$u_t + \frac{l}{2}\varphi_t = \int_{-\infty}^{t} a_1(t-s)\,\triangle u(s)\,\mathrm{d}s \quad (x,t)\in\Omega\times(0,T),$$

$$\tau\varphi_t = \int_{-\infty}^{t} a_2(t-s)[\xi^2\triangle\varphi + \frac{\varphi-\varphi^3}{\eta} + u](s)\,\mathrm{d}s \quad (x,t)\in\Omega\times(0,T),$$

for $T>0$, with Neumann boundary conditions and initial data $(u_0,\varphi_0)\in L^2\times H^1$, which has been proposed to describe phase transitions in the presence of slowly relaxing internal variables. We demonstrate the existence of a solution $(u,\varphi)\in\mathcal{C}([0,T];L^2\times H^1)$ assuming that $\Omega\subset\mathbb{R}^n$, $n=1$, 2, or 3 is a bounded smooth domain and $a_1,a_2\in L^1(\mathbb{R}^+)$ are of positive tpe. We indicate the index of stability of the equilibria and construct a Morse decomposition based on energy. (Joint work with M. Grinfeld).

## OSCILLATION IN NONCANONICAL SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

Jan Ohriska, Košice

*MSC 2000*: 34C10

The report is devoted to the problem of oscillation of the second order linear differential equation

$$(r(t)u'(t))' + p(t)u(t) = 0 \tag{1}$$

in so called noncanonical form, i.e. in the case $\int^{\infty}\mathrm{d}t/r(t)<\infty$. Using oscillation criteria of E. Hille, A. Wintner, P. Hartman et al., known for the equation

$$u''(t) + q(t)u(t) = 0$$

we present oscillation criteria for the equation (1) as an analogy of previous ones. Obtained results are compared with some other ones known for the equation (1).

*The research was supported by grant 1/7466/20 of Slovak Grant Agency.*

## A NUMERICAL TREATMENT OF THIN FILM MOVEMENT WITH FREE BOUNDARY

*Seiro Omata* and Hroshi Iwasaki, Kanazawa, Japan

We treat a movement of thin film (soapy film) with free boundary. The physical phenomena is the following: Put the wire frame into soapy water and pull up in the vertical direction. Then we may find the soapy film which touches into water surface in some part. Our aim is to treat the movement of this film. The place that the film touches the water surface is said to be a free boundary. We describe the film as a graph of a function which would approximately be a stationary point of the following Lagrangean:

$$J(u) = \int_0^T \int_\Omega \big(|\nabla u|^2 + 1 - (u_t)^2\big)\chi_{u>0}\,\mathrm{d}x\,\mathrm{d}t.$$

Equations should be derived as the Euler-Lagrange equation of $J(u)$. We show existence of a solution for a one dimensional problem under some conditions and show results of numerical experiments.

## A DOUBLY DEGENERATED ELLIPTIC SYSTEM

María Teresa González Montesinos, *Francisco Ortegón Gallego*,
Cádiz, Spain

*MSC 2000*: 35J70

We analyze the steady state of the thermistor problem in divergence form, namely

$$\left.\begin{array}{lr} -\nabla\cdot(a(u)\nabla u) = \nabla\cdot(\sigma(u)\varphi\nabla\varphi) & \text{in } \Omega \\ \nabla\cdot(\sigma(u)\nabla\varphi) = 0 & \text{in } \Omega, \\ u = 0, \quad \varphi = \varphi_0 & \text{on } \partial\Omega, \end{array}\right\}$$

which governs the temperature and the electrical potential in a device $\Omega \subset \mathbb{R}^N$. We study the existence of weak solutions under the assumption that the diffusion coefficients are not bounded below far from zero, arising to a degenerated system. This situation is encountered in practical cases where the Wiedemann-Franz law is verified.

## ON A FULL VON KÁRMÁN SYSTEM FOR VISCOELASTIC MINDLIN-TIMOSHENKO PLATES

Igor Bock and *Dávid Pancza*, Bratislava

*MSC 2000*: 74D10, 74K20, 45K05

We shall deal with a nonlinear system of integro-differential equations describing plane displacements, a bending and angles of rotation of a thin viscoelastic plate made of the long memory material and acting under the perpendicular load and plane forces. The nonlinear strain-displacements relations

$$\varepsilon_{ij} = \frac{1}{2}[\partial_i u_j + \partial_j u_i + z(\partial_i \varphi_j + \partial_j \varphi_i) + \partial_i w \partial_j w],\ i,j = 1,2,$$
$$\varepsilon_{i3} = \frac{1}{2}(\partial_i w + \varphi_i),\ i = 1,2, \quad \varepsilon_{33} = \frac{1}{2}(\varphi_1^2 + \varphi_2^2)$$

are considered.

Using the Rothe's method with respect to time we obtain a finite sequence of stationary nonlinear problems possessing a weak (variational) solution. A resulting solution can be obtained as a limit of the corresponding segment-line functions in the case of sufficiently small right-hand sides.

## ANALYTIC SOLUTIONS OF THE PAINLEVÉ EQUATIONS IN THE BANACH SPACE $H_1(\Delta)$

*Eugenia N. Petropoulou* and Panayiotis D. Siafarikas

*MSC 2000*: 34A12, 34A25, 34A34, 34C11, 34M55

For each one of the well-known six Painlevé equations, it is proved that there exists a unique solution which together with its first two derivatives belong to the Banach space $H_1(\Delta)$. Moreover, we give a bound of the solution for all six Painlevé equations and a bound of the first two derivatives of the solution for the last four Painlevé equations. Finally for all of them we give a region of attraction, depending on the initial conditions and the parameters of the equations, in which the solution holds.

*Supported by the Greek National Foundation of Scholarships.*

## TOPOLOGICAL SOLITONS AND BORN-INFELD TYPE ELECTROMAGNETIC FIELD

*Lorenzo Pisani*, Bari, Italy

*MSC 2000*: 35Q40, 35Q60, 35J70, 35J50

In a recent paper Benci Fortunato Masiello and Pisani have studied a system: soliton-electromagnetic field. The soliton a solution of a field equation with a particle-like behaviour; the form of the quasilinear field equation is the same suggested by Derrick in his celebrated paper in 1964. The electromagnetic field $(\mathbf{E}, \mathbf{B})$, which is unknown, is generated by the soliton itself. The system is described by a total lagrangian density which is the sum of three terms: $\mathcal{L}_1$ is the Lagragian density of the free soliton; $\mathcal{L}_2$ is the classical Lagragian density of the electromagnetic field; $\mathcal{L}_3$ is the Lagragian density of the interaction.

We present the same system, substituting $\mathcal{L}_2$ with a suitable non-linear Lagrangian density obtained from the Born-Infeld model for the electron. In particular we obtain a static solution $(u, \varphi)$, where $u$ is the soliton and $\varphi$ the electric potential.

From the mathematical point of view, the functional we study is strongly unbounded, moreover there is lack of compactness since the domain $\mathbb{R}^3$ is unbounded.

## CONVERGENCE TO EQUILIBRIA IN A DIFFERENTIAL EQUATION WITH SMALL DELAY

*Mihály Pituk*, Veszprém, Hungary

*MSC 2000*: 34K25

It is shown that if the delay is small, then the solutions of certain nonlinear scalar delay differential equations have the same convergence properties as the solutions of the corresponding ordinary differential equation obtained by ignoring the delay.

## WEAK SOLUTIONS OF A PHASE-FIELD MODEL FOR AN ALLOY WITH THERMAL PROPERTIES

José Luiz Boldrini and *Gabriela Planas*, Campinas, Brazil

*MSC 2000*: 35K65, 80A22, 35K55, 82B26, 82C26

The phase-field method provides an alternative mathematical description for free-boundary problems corresponding to physical processes with phase transitions. It postulates the existence of a function, called the phase-field, whose value identifies the phase at a particular point in space and time, and it is particularly suitable for cases with complex growth structures occurring during phase transitions.

The mathematical model studied in this work describes the solidification process occurring in a binary alloy with temperature dependent properties. It is based on a highly nonlinear degenerate parabolic system of partial differential equations with three independent variables: phase-field, solute concentration and temperature.

Existence of weak solutions of such system is obtained via the introduction of a regularized problem, followed by the derivation of suitable estimates and the application of compactness arguments.

## REGULARITY CRITERION FOR SMOOTHNESS OF AXISYMMETRIC NAVIER-STOKES EQUATIONS

*Milan Pokorný*, Praha

*MSC 2000*: 35Q30, 76D05

We consider the Navier-Stokes equations in the entire three-dimensional space under the additional assumption that the data $(\mathbf{f}, \mathbf{u_0})$ are axisymmetric and (written in cylindrical coordinates) the angular component of the velocity belongs to $L^t((0,T); L^s(R^3))$ with $2/t + 3/s < 1$. We show that this already implies that the weak solution to the axisymmetric data satisfying the above mentioned regularity condition and the energy inequality is already a smooth axisymmetric solution to the Navier-Stokes equations. This in particular improves the conditions which guarantee the full regularity of the axisymmetric flow given in Neustupa J., Pokorný, M.: *An Interior Regularity Criterion for an Axially Symmetric Suitable Weak Solutions to the Navier-Stokes Equations*, JMFM **2** (2000) 381–399.

## CENTER MANIFOLDS IN THE STUDY OF PARABOLIC PDES

*Peter Poláčik*, Bratislava

*MSC 2000*: 37L10, 35K57, 35B40

The center manifold reduction has become a basic tool in the study of evolution partial differential equations. While its most common applications are found in bifurcation and stability analyses, it has also proved very useful for exploration of possible behavior of solutions. We shall review several results on parabolic equations that arose from such considerations.

## LOGISTIC EQUATION ON TIME SCALES

*Zdeněk Pospíšil*, Brno, Czech Republic

*MSC 2000*: 39 A 10

The following initial value problem for dynamical equation is considered:

$$x^{\Delta} = \left[ r(t)x\left(1 - \frac{x}{K(t)}\right)\right] \Big/ \left[1 + \mu^*(t)\frac{r(t)}{K(t)}x\right], \quad x(t_0) = x_0 \geqslant 0.$$

The equation generalizes both Verhulst continuous and Pielou discrete logistic equations:

$$x' = rx\left(1 - \frac{x}{K}\right), \quad x_{k+1} = \frac{rx_k}{1 + ((r-1)/K)x_k};$$

hence it can be taken for a general model of one population growth. Limit properties of solution are studied. In particular, the statement

$$\lim_{t\to\infty} \frac{1}{\mu(t,t_0)} \int_{t_0}^{t} \xi_{\mu^*(s)}(r(s))\Delta s > 0 \;\Rightarrow\; \liminf_{t\to\infty} x(t) > 0$$

is proved; $\mu$ and $\mu^*$ denote growth callibration and graininess function, respectively, and $\xi_h(z) = \frac{\log(zh+1)}{h}$ for $h > 0$, $\xi_h(z) = z$ for $h = 0$.

## ON FIRST ORDER DISCONTINUOUS SCALAR INITIAL VALUE PROBLEMS

*Rodrigo L. Pouso*, Santiago de Compostela

*MSC 2000*: 34A36

We derive sufficient conditions for the existence of Carathéodory-type solutions for the problem

$$x'(t) = f(t, x(t)) \text{ for a.e. } t \in [0, T], \quad x(0) = 0,$$

where $f$ needs not be continuous with respect to any of its variables. In fact we show that if $f$ is positive, then nondecreasingness in $t$ and measurability in $x$ (*cf.* [1]) is sufficient for the existence of nonnegative solutions.

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## NONLOCAL BOUNDARY VALUE PROBLEM FOR PENDULUM-LIKE EQUATION

*Bogdan Przeradzki*, Łódź, Poland

*MSC 2000*: 34B15

We study BVP:

$$x'' + p\left(\int_0^1 x \,\mathrm{d}t\right)(x + f(x)) = g(t),$$

where $p\colon \mathbb{R} \to \mathbb{R}$ is continuous and monotone function, $f\colon \mathbb{R} \to \mathbb{R}$ is continuous and bounded one and $g\colon [0,1] \to \mathbb{R}$ is integrable. The existence of at least one or infinite number of solutions are proved under some conditions on the range of $p$ and Fourier coefficients of $h$ with respect to $\sin mt$, $m = 1, 2, \ldots$.

## MATHEMATICAL AND NUMERICAL MODELS IN MULTIPHYSICS

*Alfio Quarteroni*, Milano

In this talk I will introduce some models to describe heterogeneous phenomena in continuum mechanics. Applications will concern fluid dynamics and cardiovascular flow problems. The role of interface condition will be discussed, and suitable numerical methods based on domain decomposition will be addressed.

## A PRIORI BOUNDS OF SOLUTIONS OF SUPERLINEAR PARABOLIC PROBLEMS AND APPLICATIONS: CONTINUITY OF THE BLOW-UP TIME, EXISTENCE OF STATIONARY AND PERIODIC SOLUTIONS

*Pavol Quittner*, Bratislava

*MSC 2000*: 35B10, 35B30, 35B35, 35B45, 35J65, 35K55, 35K60

We prove a priori estimates of solutions of various classes of superlinear parabolic problems. The bounds are of the form $\|u(t,u_0)\| \leqslant C(\|u_0\|,\delta)$ for any $t < T_{\max}(u_0) - \delta$, where $\delta > 0$, $u(\cdot,u_0)$ is the solution with initial condition $u_0$ and $T_{\max}(u_0) \leqslant \infty$ is its maximal existence time. Using these estimates we show that $T_{\max}(u_0)$ depends continuously on $u_0$. The results are also applied to the proof of existence of nontrivial stationary or periodic solutions. The applications include problems on bounded and unbounded domains. The nonlinearities in the equations and/or boundary conditions are subcritical and they may be nonlocal. Our proofs are based on energy and maximal regularity estimates. Optimality of our results and some open problems are discussed.

# RELAXATION OF QUASILINEAR ELLIPTIC SYSTEMS VIA A-QUASICONVEX ENVELOPES

ULDIS RAITUMS, Riga

MSC 2000: 49J45

We consider the weak closure $WZ$ of the set $Z$ of all feasible pairs (solution, cogradient) of the family of potential elliptic systems

$$\operatorname{div}\Bigl(\sum_{s=1}^{l}\sigma_s(x)F_s'(\nabla u(x)+g(x))-f(x)\Bigr)=0 \text{ in } \Omega,$$
$$u=(u_1,\dots,u_m)\in H_0^1(\Omega;\mathbb{R}^m),\ \sigma=(\sigma_1,\dots,\sigma_l)\in S,$$

where $\Omega\subset\mathbb{R}^n$ is a bounded domain, $F_s$ are strongly convex smooth functions with quadratic growth and $S=\{\sigma \text{ measurable} \mid \sigma_s(x)=0 \text{ or } 1,\ \sigma_1(x)+\ldots+\sigma_l(x)=1\}$.

We show that $WZ$ is the zero level set for an integral functional with the integrand $Q\mathcal{F}$ being the A-quasiconvex (for $A=(\mathrm{curl},\mathrm{div})^m$) envelope for a certain function $\mathcal{F}$. If $F_s$ are isotropic then on the characteristic cone $\Lambda$ (defined by $A$) $Q\mathcal{F}$ coincides with the A-polyconvex envelope of $\mathcal{F}$ and can be computed by means of rank-one laminates.

# DYNAMICS OF THE SYSTEM WITH CONSTRAINTS

RAFAEL RAMIREZ, Tarragona, and NATALIA SADOVSKAIA, Barcelona, Spain

We state and solve the inverse on dynamics problem of constructing the Lagrangian equations from given partial integrals (Dainelli-Suslov's problem).

We applied the obtained results to develop a Lagrangian and Cartesian approach to describe the behaviour of nonholonomic systems with constraints which are linear with respect to velocity.

## THE COUPLING OF NONLINEAR HYPERBOLIC MODELS: MATHEMATICAL AND NUMERICAL ASPECTS

*P.-A. Raviart*, Palaiseau Cedex, France

When modeling complex physical problems,one is often led to use different models in different geometric domains depending on the dominant physics which must be taken into account. Just as a simple illustration, for modeling a compressible inviscid fluid flow, one can use the Euler equations of gas dynamics in some domain and the isentropic ones in another neighbouring domain where the flow is smooth. The difficulty now stems from the coupling of these different models. In this talk, we will discuss the coupling of two nonlinear hyperbolic models. In a first part, we will present various physical examples of such a situation. Next, we will analyze the coupling of two scalar conservation laws from both theoretical and numerical points of view. Finally, we will turn to the more realistic case of systems and give some first results.

## BOUNDARY STABILIZATION OF THE SCHROEDINGER EQUATION IN ALMOST STAR-SHAPED DOMAIN

*S.E. Rebiai*, Batna, Algeria

*MSC 2000*: 35B45, 35J10, 35B05, 93C20, 93D15

The question of uniformly stabilizing the solution of the Schroedinger equation $y' - i\Delta y = 0$ in $\Omega\ \times(0,+\infty)$ ($\Omega$ is a bounded domain of $\mathbb{R}^n$) subject to boundary conditions $y = 0$ on $\Gamma_0 \times (0,+\infty)$ and $\partial y/\partial\nu = F(x, y')$ on $\Gamma_1 \times (0,+\infty)$, $(\Gamma_0, \Gamma_1)$ being a partition of the boundary, is studied. We prove by means of a particular choice of the feedback $F$ that, if $(\Omega, \Gamma_0, \Gamma_1)$ is an almost star-shaped domain, then the solution decays exponentially in the energy space $H^1_{\Gamma_0}(\Omega)$. The approach adopted is based on multipliers technique.

# APPLICATIONS OF THE IMPLICIT FUNCTION THEOREM TO ELLIPTIC BOUNDARY VALUE PROBLEMS WITH NON-SMOOTH DATA

*Lutz Recke*, Berlin

The communication concerns boundary value problems for quasilinear elliptic equations and systems with non-smooth data ($L^\infty$-coefficients, Lipschitz domains, mixed boundary conditions). The results of [1] will be generalized to arbitrary space dimensions by means of a new theorem [2] on Fredholmness of the corresponding linearized operators on Sobolev-Campanato spaces.

*References*

[1] *L. Recke*: Applications of the Implicit Function Theorem to quasilinear elliptic boundary value problems with non-smooth data. Commun. in PDE *20* (1995), 1457–1479.

[2] *J. A. Griepentrog, L. Recke*: Linear elliptic boundary value problems with non-smooth data: Normal solvability on Sobolev-Campanato spaces WIAS Preprint 466 (1998). To appear in Math. Nachrichten.

*MSC 2000*: 35B30, 35J55, 35J65, 35R05, 47J07

# ERROR ANALYSIS OF ABSORBING BOUNDARY CONDITIONS FOR A SPATIAL DISCRETIZATION OF SCHRÖDINGER-TYPE EQUATIONS

I. Alonso Mallo, Valladolid, and *Nuria Reguera*, Burgos, España

When solving numerically a partial differential equation defined in an infinite domain, it is essential to reduce the problem to a bounded domain and to use appropriate boundary conditions. If the solution of this new problem is equal to the restriction of the original solution to the subdomain, the boundary condition is called transparent (TBC). When the TBC are nonlocal it is convenient to use local absorbing boundary conditions (ABC), allowing only small reflections. We consider a Schrödinger-type equation discretized in space with finite differences. By considering rational approximations to the nonlocal TBC, we develop a class of local ABC for this semidiscrete problem. We study in detail one of the simplest cases and we make a complete analysis of the error of the full discrete problem with these ABC. We also present several numerical experiments.

## DECAYING SOLUTIONS OF DISCRETE SYSTEMS

*P. Řehák*, Brno, Czech Republic

*MSC 2000*: 34C10, 39A10, 93C70

We investigate the nonlinear difference system

$$\Delta(r_k\Phi_\alpha(\Delta x_k)) = \sigma f(k, y_{k+1}),$$
$$\Delta(q_k\Phi_\beta(\Delta y_k)) = g(k, x_{k+1}),$$

where $r_k$, $q_k$ are positive sequences on $\{m, m+1, \ldots\}$, $f$, $g$ are positive continuous functions on $\{m, m+1, \ldots\} \times (0, \delta]$, $\delta > 0$, $\Phi_\lambda(s) = |s|^{\lambda-2}s$ with $\lambda > 1$, and $\sigma \in \{-1, 1\}$. There will be shown necessary and sufficient conditions for the system to have positive solutions approaching zero. We will discuss also possible extensions of our results, comparison with the continuous case, and indicate future directions of the research.

*Joint research with M. Marini and S. Matucci. Research supported by the Grants No. 201/01/P041 and No. 201/01/0079 of the Czech Grant Agency, and by C.N.R. of Italy.*

## PLASTIC WRINKLING AND FLUTTER IN SHEET METAL SPINNING

*Volker Reitmann* and Holger Kantz, Dresden, Germany

*MSC 2000*: 35K60

In this report a short mathematical and mechanical description of the sheet metal spinning process is given. The representation is based on nonlinear shell theory, elasto-plastic material laws and perturbation theories. Various plastic buckling and plastic flutter situations are analyzed. The influence of the angular velocity of a spinning disc on the plasticity process is investigated.

We discuss some mathematical and mechanical basic problems arising in the investigation of instabilities in metal spinning processes. Plastic and dynamical wrinkle formations in metal forming processes for quasi-static deformations such as deep-drawing are intensively investigated in the last time. The central theoretical approach is based on bifurcation functionals which go back to Hill and Hutchinson. In most of these investigations it is assumed that the forces are conservative. Inertia forces and general gyroscopic forces are not considered.

## REGULAR HALF-LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

*Jana Řezníčková*, Brno, Czech republic

The concept of the regular (nonoscillatory) half-linear second order differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) := |x|^{p-2}x,\ p > 1 \tag{$*$}$$

is introduced and it is shown that if $(*)$ is regular, a solution $\tilde{x}$ of this equation is principal if and only if

$$\int^{\infty} \frac{\mathrm{d}t}{r(t)\tilde{x}^2(t)|\tilde{x}'(t)|^{p-2}} = \infty.$$

Conditions on the functions $r$, $c$ are given which guarantee that $(*)$ is regular.

The presented results were achieved in the joint investigation with O. Došlý (Brno).

## CERTAIN VARIATIONAL INEQUALITY IN A PLASTICITY MODEL FOR SHAPE-MEMORY ALLOYS

*Tomáš Roubíček*, Praha

*MSC 2000*: 35K85, 49S05, 74C15, 74N20, 80A17

Stored energy density $\varphi$ governing the steady-state configuration in metalic crystals that exhibit shape-memory phenomena due to at least two stable configurations (= phases) is necessarily nonconvex. Evolutionar inclusion of the form

$$\frac{\partial^2 u}{\partial t^2} - \operatorname{div}\left(\operatorname{sign}\left(\frac{\partial}{\partial t}\lambda(\nabla u)\right)\lambda'(\nabla u) + \varphi'(\nabla u)\right) + \varepsilon_1 \frac{\partial}{\partial t}\Delta^2 u + \varepsilon_2 \Delta^2 u \ni f$$

with $u = (u_1, \ldots, u_n)(x,t)$ being a displacement, $\lambda$ a function which distinquishes particular phases, and $\varepsilon_1 > 0$, $\varepsilon_2 \geqslant 0$, describes plastic, hysteretic response characteristic for shape-memory effects. Rigorous existence analysis will be outlined, as well as an anisothermal, thermodynamically consistent extension. Illustrative numerical experiments will be presented, too.

## ON ELECTRORHEOLOGICAL FLUIDS

*Michael Růžička*, Freiburg, Germany

*MSC 2000*: 35, 76

Electrorheological fluids change dramatically their mechanical properties in dependence on an applied electrical field. The elliptic or parabolic system of PDE's, which describes this phenomenon, possesses so called non-standard growth conditions. We discuss the underlying structure of this system, which naturally leads to generalized Sobolev spaces $W^{1,p(x)}(\Omega)$ and present first existence, uniqueness and regularity results for the steady and the unsteady problem.

*References*

[1] *M. Růžička*: Electrorheological Fluids: Mathematical Modelling and Existence Theory. LNM 1748, Springer, 2000.

## GLOBAL BIFURCATIONS OF PERIODIC SOLUTIONS OF THE RESTRICTED THREE BODY PROBLEM

*Sławomir Rybicki*, Toruń, Poland

*MSC 2000*: 37J45, 37G40

The Restricted Three Body Problem provides a good approximation for a real physical situations and was taken as the model problem in numerous studies.

The aim of my talk is to study global bifurcations of periodic orbits of the planar Restricted Three Body Problem. As the main tool, we use degree theory for $SO(2)$-equivariant gradient maps. We prove that the only admissible families of non-stationary periodic solutions of the Restricted Three Body Problem can be short period families bifurcating from the Lagrangean stationary solutions. We show that there are possible only six admissible families of periodic solutions of the planar Restricted Three Body Problem.

## NONLINEAR SINGULAR EIGENVALUE PROBLEMS FOR SECOND ORDER EQUATIONS

Felix Sadyrbaev, Riga

*MSC 2000*: 34B15

A nonlinear eigenvalue problem

$$x'' + \lambda q(t)|x|^{\gamma} \operatorname{sign} x = 0, \tag{1}$$

$$x(a) = 0, \quad x'(a) = 1, \tag{2}$$

$$\int_a^{+\infty} \frac{\mathrm{d}s}{x^2(t)} = +\infty \tag{3}$$

is considered, where $x \in \mathbb{R}$, $\lambda > 0$, $\gamma > 1$, $q \in C([a, +\infty), (0, +\infty))$, $a \geqslant 0$. Solutions, which satisfy the condition (3), are called *principal.* It is shown that under the basic condition

$$\int_a^{+\infty} s^{\gamma} q(s)\,\mathrm{d}s < +\infty$$

there exists a sequence $\{\lambda_n\}$ such that

$$0 < \lambda_0 < \lambda_1 < \ldots < \lambda_n < \ldots, \qquad \lim_{n\to+\infty} \lambda_n = +\infty$$

and for any $i$ a principal solution $x(t;\lambda_i)$ to the problem (1), (2) exists, which has exactly $i$ zeros in $(a, \infty)$.

## UNCONDITIONALLY STABLE C1-CUBIC SPLINE COLLOCATION METHOD FOR SOLVING PARABOLIC EQUATIONS

S. Sallam, Safat, Kuwait

This paper aims to introduce a new approach, based on $C^1$-cubic splines, for the time integration of parabolic equations (heat equation). The method of results in is the so-called “method of lines” (the solution through is consisting of space discretization and time integration). The main objective of this paper is to prove the unconditional stability of the proposed approach as well as to show that the method is convergent and has order $O(h^2) + O(k^4)$, i.e., it is a fourth order in time and second order in space and it is relevant for long time interval problems. Moreover, the method can be regarded as extension of some fully discrete methods, in the sense that it provides approximations, which reproduce the values given by the fully discrete methods at the grid points.

## HETEROCLINICS FOR A CLASS OF FOURTH ORDER CONSERVATIVE DIFFERENTIAL EQUATIONS

L. SANCHEZ, Lisbon

*MSC 2000*: 34C37, 58E30

We study the functional $\mathcal{J}(u) = \int_{-\infty}^{+\infty}[\frac{1}{2}(u''^2) + f(u)]\,\mathrm{d}x$ and its corresponding Euler-Lagrange equation

$$u^{iv} + f'(u) = 0. \tag{1}$$

This can be seen as a special case of the extended Fisher-Kolmogorov equation related to a Ginzburg-Landau model for ternary mixtures.

Our potentials $f \in C^2(\mathbb{R})$ are positive and have two nondegenerate minima $\pm 1$ such that $f(\pm 1) = 0$, these being the only zeros of $f$. We look for heteroclinic connections between these equilibria. These are found as minimizers of $\mathcal{J}$ in a suitable space of functions.

We also consider the case where $f$ is a $C^2$ even function with three minima at the same level, say $\pm 1$ and 0. If $f$ is increasing in some interval $(0, a)$ we show that there still exists a heteroclinic of (1) going from $-1$ to 1.

*This is a joint work with P. Habets, M. Tarallo and S. Terracini.*

## CONVERGENCE, VIA SUMMABILITY, OF FORMAL POWER SERIES SOLUTIONS TO A CERTAIN CLASS OF COMPLETELY INTEGRABLE PFAFFIAN SYSTEMS

JAVIER SANZ, Valladolid, Spain

*MSC 2000*: 35C10, 35C20, 40C15

A theory of **k**-summability in a direction ($\mathbf{k} = (k_1, k_2, \ldots, k_n)$) has been put forward for formal power series of several complex variables. It involves the study of multidimensional Laplace and Borel transforms, and their effect on series (resp. functions) subject to Gevrey-like bounds (resp. admitting Gevrey strongly asymptotic expansion). As an example of the application of this tool to some questions in PDE's, a new proof is given of a result of R. Gérard and Y. Sibuya stating the convergence of the formal power series solutions to certain completely integrable Pfaffian systems.

## FEEDBACKS FOR NON-AUTONOMOUS REGULAR LINEAR SYSTEMS

*Roland Schnaubelt*, Halle, Germany

*MSC 2000*: 93C25, 47D06

We introduce non-autonomous well-posed and (absolutely) regular linear systems as quadrupels consisting of an evolution family and output, input and input-output maps subject to natural hypotheses. In the spirit of G. Weiss' work these maps are represented in terms of admissible observation and control operators (the latter in an approximative sense) in the time domain. In this setting the closed-loop system exists for a canonical class of 'admissible' feedbacks, and it inherits the absolute regularity and other properties of the given system. In particular, one can iterate feedbacks. Applications to controlled partial differential equations are given.

## NON-UNIQUENESS OF SOLUTION TO QUASI-1D COMPRESSIBLE EULER EQUATIONS

P. Šolín, *K. Segeth*, Prague

*MSC 2000*: 35A05

The study presented is aimed at the analytical solution of the stationary one-dimensional compressible Euler equations with varying cross-section. We go back to results obtained for simple and Laval nozzles and generalize them for multiple ones. It turns out that for multiple nozzles, more exact solutions corresponding to a given set of boundary data can exist. We develop an algorithm for the construction of all analytical solutions at multiple nozzles. Examples of non-uniqueness are presented. These analytical quasi one-dimensional results are supported numerically, using a quasi one-dimensional finite volume scheme. Moreover, numerical results indicating the non-uniqueness of solution of three-dimensional compressible Euler equations at axi-symmetric nozzles are presented.

## A DIRECT METHOD FOR SOLVING AN ANISOTROPIC MEAN CURVATURE FLOW OF PLANAR CURVE WITH AN EXTERNAL FORCE

*Daniel Ševčovič*, Bratislava

*MSC 2000*: 35K65, 65N40, 53C80

A new method for solution of the evolution of plane curves satisfying the geometric equation $v = \beta(x, k, \nu)$, where $v$ is the normal velocity, $k$ and $\nu$ are the curvature and tangential angle of the plane curve $\Gamma \subset \mathbb{R}^2$ at a point $x \in \Gamma$ is proposed. We derive a governing system of partial differential equations for the curvature, tangential angle, local length and position vector of an evolving family of plane curves and prove local in time existence of a classical solution. The equations include a nontrivial tangential velocity functional governing a uniform redistribution of grid points and thus preventing numerically computed solutions from forming various instabilities. We discretize the governing system in order to find a numerical solution for 2D anisotropic interface motions in thermomechanics and medical image segmentation problems. This is a joint work with K. Mikula. A preprint is available at `www.iam.fmph.uniba.sk/institute/sevcovic`

## THE SET OF SOLUTIONS OF NONLINEAR INTEGRAL EQUATIONS IN BANACH SPACES AND HENSTOCK-KURZWEIL-PETTIS INTEGRAL

Ireneusz Kubiaczyk and *Aneta Sikorska-Nowak*, Poznań, Poland

We prove two existence theorems of solutions of nonlinear integral equation of Urysohn type:

$$x(t) = \varphi(t) + \lambda \int_0^\alpha f(t, s, x(s))\,\mathrm{d}s \tag{1}$$

and Volterra type:

$$x(t) = \varphi(t) + \int_0^t f(t, s, x(s))\,\mathrm{d}s, \qquad t \in I_\alpha =< 0, \alpha > \tag{2}$$

with the Henstock-Kurzweil-Pettis integral. This integral is a generalization of previous integrals. Moreover, we prove that the set of solutions of equations (1) and (2) is weakly compact and connected in $(C(I_\beta, E), \omega)$, where $0 < \beta < \alpha$. The assumptions about the function $f$ are really weak: scalar measurability and weak sequential continuity with respect to the third variable. We suppose that the function $f$ satisfies some conditions expresses in terms of the measure of weak noncompactness.

# ON PARABOLIC FUNCTIONAL DIFFERENTIAL EQUATIONS IN UNBOUNDED DOMAINS

L. SIMON, Budapest, Hungary

*MSC 2000*: 35R10

We shall consider weak solutions of initial-boundary value problems for nonlinear parabolic differential equations with delay in $(0,T)\times\Omega$ where $\Omega\subset\mathbb{R}^n$ is an unbounded domain with sufficiently smooth boundary and the equation contains terms which are discontinuous with respect to the unknown function. There will be formulated conditions for the existence of solutions, for the boundedness of solutions and the stabilization of the solutions as $t\to\infty$. We shall show that the solutions of problems considered in bounded domains $\Omega_r\supset B_r=\{x\in\mathbb{R}^n : |x|<r\}$ converge to a solution of the problem considered in $\Omega$ as $r\to\infty$. The problem was motivated by the climate model considered by J. I. Díaz and G. Hetzer where a particular case of our equation was investigated on the unit sphere in $\mathbb{R}^3$ (instead of $\Omega$).

# ON THE STABILITY OF THE ROSSBY-HAURWITZ WAVE

YURI N. SKIBA, México, D.F., Mexico

An ideal incompressible fluid on a rotating sphere is considered. Stability of the Rossby-Haurwitz wave (RHW) of subspace $H_1\oplus H_n$ is analyzed ($n\geqslant 2$). $H_k$ is the Laplacian eigen subspace for the eigenvalue $\lambda_k=k(k+1)$. By a conservation law, all real perturbations to the RHW are divided into invariant sets $M_+^n$, $M_0^n$ and $M_-^n$ defined by the sign of $f(\psi)=\chi-\lambda_n$ where $\chi=\eta(t)/K(t)$, and $\eta(t)$ and $K(t)$ are the perturbation energy and entropy. It is proved the Liapunov instability of a non-zonal RHW in $M_-^n$ caused by asynchronous oscillations of waves (typical of nonlinear pendulum). It is obtained a condition for the exponential instability of a RHW stating that the amplitude of an unstable normal mode must belong to $M_0^n$. The bounds of the mode growth rate are estimated. The orthogonality of the unstable mode amplitude to the RHW is shown. The new instability condition is useful on trials of a numerical linear stability study algorithm.

## NONLINEAR HEMIVARIATONAL INEQUALITIES

*G. Smyrlis*, Athens, Greece

*MSC 2000*: 35J

In this work we study nonlinear hemivariational inequalities. Using the method of upper and lower solutions, variatonal methods based on nonsmooth critical point theory and a generalized version of the multivalued Leray-Schauder alternative theorem, we establish the existence of solutions, of multiple solutions and of positive solutions for problems driven by the $p$-Laplacian or even more general differential operators.

## ASYMPTOTIC BEHAVIOUR OF NONOSCILLATORY SOLUTIONS OF THE FOURTH ORDER DIFFERENTIAL EQUATIONS

*Monika Sobalová*, Brno, Czech Republic

In the paper the fourth order nonlinear differential equation

$$y^{(4)} + (q(t)y')' + r(t)f(y) = 0,$$

where $q \in C^1([0,\infty))$, $r \in C^0([0,\infty))$, $f \in C^0(\mathbb{R})$, $r \geqslant 0$ and $f(x)x > 0$ for $x \neq 0$ is considered. We investigate the asymptotic behaviour of nonoscillatory solutions and give sufficient conditions under which all nonoscillatory solutions either are unbounded or tend to zero for $t \to \infty$.

## THE NUMBER OF SOLUTIONS FOR THE SECOND ORDER NONLINEAR BOUNDARY VALUE PROBLEM VIA THE ROOT FUNCTIONS METHOD

*Peter Somora*, Bratislava, Slovakia

*MSC 2000*: Primary 34B15

We consider a second order nonlinear differential equation with homogeneous Dirichlet boundary conditions. Using the root functions method we prove a relation between the number of zeros of some variational solutions and the number of solutions of our boundary value problem. We get a lower bound of the number of its solutions as well as the exact number of solutions for special type of our boundary value problem. We also use the root functions method for a proof of existence of infinitely many solutions of the boundary value problem with strong nonlinearity.

The author would like to thank Assoc. Prof. Milan Gera and Zbyňek Kubáček for their valuable advices and suggestions.

## BOUNDED SOLUTIONS OF NONLINEAR ELLIPTIC EQUATIONS IN UNBOUNDED DOMAINS

*Robert Stańczy*, Łódź

We prove here the existence of a bounded solution in unbounded domain $\Omega \subset \mathbb{R}^n$ of the nonlinear elliptic problem at resonance

$$\begin{aligned} \Delta u &= f(x,u) \quad \text{for } x \in \Omega, \\ u(x) &= 0 \quad \text{for } x \in \partial\Omega, \end{aligned}$$

under some asymptotic and sign condition on $f$. We apply some perturbation technique together with some topological methods to establish the existence of classical solutions.

## EXISTENCE OF SMOOTH FLOWS FOR A CLASS OF NON NEWTONIAN FLUIDS IN PLANE

*J. Stará*, J. Málek, P. Kaplický, Praha, Czech Republic

*MSC 2000*: 76F10, 35Q35, 35K65

We study a nonlinear evolutionary fluid model in two space dimensions characterized by the viscosity being a decreasing function of the modulus of the symmetric velocity gradient with a power growth $p-1$. For sufficiently large $p$ we establish the existence of solutions with the locally Hölder continuous velocity gradients. This result enables to continue the classical regularity ladder known for elliptic and parabolic systems and get solutions so smooth as the data of the problem allow. Moreover, in case of technically more simple space periodic boundary value problem we can show that a smooth solution is unique in the class of weak solutions.

## $n$-WIDTHS FOR SINGULARLY PERTURBED PROBLEMS

*Martin Stynes*, R. Bruce Kellogg, South Carolina

Kolmogorov $n$-widths are an approximation theory concept that, for a given problem, yields information about the optimal rate of convergence attainable by any numerical method applied to that problem. We survey sharp bounds recently obtained for the $n$-widths of certain singularly perturbed convection-diffusion and reaction-diffusion problems.

# ON SOME NONLINEAR VIBRATION EQUATIONS

*Barbara Szomolay*, Bratislava, Slovakia

1) Study of the existence, uniqueness and regularity of weak local solutions (Galerkin method, theory of abstract evolution equations and semigroups for linear operators) to the nonlinear vibration equation:

$$u_{tt} + M(\|u\|^2_{H^1(\Omega)})(-\Delta u + u) + |u|^\alpha u = f, \quad x \in \Omega, 0 < t < T,$$
$$\gamma_1 \frac{\partial u}{\partial n} + \gamma_2 u = 0, \quad x \in \partial\Omega,\ 0 \leqslant t \leqslant T,$$
$$u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x), \quad x \in \Omega,$$

where $\Omega \in C^{0,1}$, $\delta > 0$, $\alpha \geqslant 0$, $T > 0$ are given constants, $M(r)$ is a positive $C^1$-function on $[0,\infty)$ and $\gamma_1$, $\gamma_2 \in L^\infty(\partial\Omega)$ are of the same sign.

2) Study of asymptotic behavior of solutions to the initial value problem:

$$u_{tt} + \gamma u_t + M(\|u\|^2_{H^1(\Omega)})(-\Delta u + u) = f, \quad x \in \Omega, t \geqslant 0,$$
$$\gamma_1 \frac{\partial u}{\partial n} + \gamma_2 u = 0,\ x \in \partial\Omega, t \geqslant 0,\ u(x,0) = u^0(x),\ u_t(x,0) = u^1(x),\ x \in \Omega,$$

where $\gamma$ is a positive constant.

# ARONSZAJN TYPE THEOREMS FOR AN $M^{\text{th}}$ ORDER DIFFERENTIAL EQUATION IN BANACH SPACES

*Aldona Szukała*, Poznań, Poland

*MSC 2000*: 34G20

We investigate the Cauchy problem:

$$x^{(n)} = f(t,x), \quad x(0) = \eta_1,\ x'(0) = \eta_2, \ldots, x^{(n-1)}(0) = \eta_n \tag{1}$$

in a Banach space. First, assuming that $f$ satisfies $\alpha$-Osgood condition $\alpha(f(t,X)) \leqslant w(\alpha(X))$, when $\alpha$ is the measure of noncompactness and $w$ is a nondecreasing function $\mathbb{R}_+ \mapsto \mathbb{R}_+$ such that $\int\limits_{0+} \mathrm{d}r/\sqrt[n]{r^{n-1}w(r)} = \infty$, we shall show that the set of all local solutions of (1) is an $R_\delta$-set. Similar result is true if $f$ satisfies $\alpha$-Nagumo condition. Next, we shall prove that if $f$ is locally $\alpha$-Lipschitzean and satisfies the Wintner condition $\|f(t,x)\| \leqslant w(\|x\|)$ with $\int_1^\infty \mathrm{d}r/\sqrt[n]{r^{n-1}w(r)} = \infty$, then the set of all global solutions of (1) is an $R_\delta$-set.

## AN ASYMPTOTICALLY PERIODIC SCHRÖDINGER EQUATION WITH INDEFINITE LINEAR PART

*Andrzej Szulkin*, Stockholm

*MSC 2000*: 35J60, 35Q55, 58E05

Consider the Schrödinger equation

$$-\Delta u + V(x)u = f(x,u), \tag{$*$}$$

$u \in H^1(\mathbb{R}^N)$, where $V$ is periodic in $x_j$ for $1 \leqslant j \leqslant N$, $0 \notin \sigma(-\Delta + V)$ and $f$ is superlinear (but subcritical) or asymptotically linear. Moreover, suppose that $f$ is asymptotically periodic in the sense that $f(x,u) - g(x,u) \to 0$ as $|x| \to \infty$ for some function $g$ which is periodic in $x_j$. It is known that if $V > 0$ and $0 < ug(x,u) < u^2 g_u(x,u)$ as $u \neq 0$, then, under some additional hypotheses, $(*)$ possesses a solution $u \neq 0$. This can be shown by comparing the Palais-Smale sequences for the asymptotic problem

$$-\Delta u + V(x)u = g(x,u) \tag{$**$}$$

with those of $(*)$. We shall show that if 0 is in a spectral gap of $-\Delta + V$, then the same result remains valid by a similar comparison argument. However, if $V > 0$, then the functionals corresponding to $(*)$ and $(**)$ have the mountain pass geometry while in our case they have a geometry of linking type.

## BOUND SETS FOR DIFFERENTIAL INCLUSIONS WITH FLOQUET BOUNDARY CONDITIONS

*Valentina Taddei*, Modena and Reggio Emilia, Italy

*MSC 2000*: 34A60, 34B15

The object of this joint research with J. Andres and L. Malaguti is the existence of solutions of

$$\begin{cases} x' \in F(t,x), \ t \in [a,b], \ x \in \mathbb{R}^n \\ x(b) = Cx(a), \end{cases}$$

where $F$ is an upper semi-continuous map and $C$ a non-singular $n \times n$ matrix.

The problem is investigated by means of a continuation principle due to Andres, Gabor and Górniewicz, where an abstract boundary value problem is reduced to a fixed point one via a Schauder linearization. We extend to inclusions the usage of not necessarily smooth Lyapunov-like bounding functions, already considered by the author for differential equations. We combine this tecnique with the viability theory to obtain the transversality condition required by the topological method.

An application to an anti-operiodic problem is given.

# ON THE SINGULARITIES OF SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS IN THE COMPLEX DOMAIN

*Hidetoshi Tahara*, Tokyo, Japan

*MSC 2000*: 35A20

Let us consider the following nonlinear first order partial differential equation (E) $\partial u/\partial t = F(t, x, u, \partial u/\partial x)$ in the complex domain $C_t \times C_x^n$. The structure of holomorphic solutions of (E) is completely understood by the Cauchy-Kowalevski theorem. But the structure of the singular solutions (that is, the solutions with some singularities) of (E) has not yet been studied well. In this talk I will consider the following problem:

Problem. Investigate the structure of solutions of (E) which possess singularities only on the hypersurface $S = \{t = 0\}$.

Result. Under suitable conditions we can find such a negative real number $\sigma$ that the following (1) and (2) are satisfied: (1) (E) has no singular solutions with growth order $o(|t|^\sigma)$ (as $t \longrightarrow 0$), but (2) (E) has a solution which has really singularities on $S$ with growth order $O(|t|^\sigma)$ (as $t \longrightarrow 0$).

# STABILISED FINITE ELEMENT APPROXIMATIONS FOR THE INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

*Lutz Tobiska*, Magdeburg, Germany

We present a comprehensive overview of stabilised finite element methods developed recently for solving the stationary incompressible Navier-Stokes equations. In particular, the relationship between various approaches like the streamline-diffusion method, the Galerkin-least squares technique, the residual-free bubble stabilisation and the subgrid-scale approach is clarified. Stability and error estimates for both conforming and nonconforming finite element approximations are given.

## REGULARITY OF MINIMIZERS IN OPTIMAL CONTROL

Delfim F. M. Torres, Aveiro, Portugal

*MSC 2000*: 49J15, 49N60

We consider the Lagrange problem of optimal control with unrestricted controls —given a Lagrangian $L$, a dynamical equation $\dot{x}(t) = \varphi\left(t, x(t), u(t)\right)$, and boundary conditions $x(a) = x_a$, $x(b) = x_b \in \mathbb{R}^n$, find a control $u(\cdot) \in L_1\left([a, b]\,;\, \mathbb{R}^r\right)$ such that the corresponding trajectory $x(\cdot) \in W_{1,1}\left([a, b]\,;\, \mathbb{R}^n\right)$ of the dynamical equation satisfies the boundary conditions, and the pair $\left(x(\cdot), u(\cdot)\right)$ minimizes the functional $J[x(\cdot), u(\cdot)] := \int_a^b L\left(t, x(t), u(t)\right)\,\mathrm{d}t$. We address the question: under what conditions we can assure optimal controls are bounded? This question is related to the one of Lipschitzian regularity of optimal trajectories, and the answer to it is crucial for closing the gap between the conditions arising in the existence theory and necessary optimality conditions. Rewriting the problem in parametric form, we obtain some new results. The results generalize previous Lipschitzian regularity conditions proved for the basic problem of the calculus of variations.

## ASYMPTOTICS OF PSEUDODIFFERENTIAL PARABOLIC EQUATIONS

Andrzej W. Turski, Katowice, Poland

*MSC 2000*: 35S15, 35B40, 35K90

We discuss certain examples covered by the general theory of global attractors for abstract parabolic equations presented in the monographs [1], [2] and [3]. Inside the class of sectorial equations of the form

$$\dot{u} + Au = F(u),\ t > 0,\ u(0) = u_0, \tag{1}$$

we cover pseudodifferential parabolic problems

$$u_t = -(-\Delta)^\alpha u + f(u),\ \alpha \in \left(\tfrac{1}{2}, 1\right), \tag{2}$$

studied with suitable initial-boundary conditions; also their generalizations to problems with the main part being a finite sum of the fractional powers.

*References*

[1] J. W. Cholewa, T. Dlotko, *Global Attractors in Abstract Parabolic Problems*, Cambridge University Press, Cambridge, 2000. .

[2] J. K. Hale, *Asymptotic Behavior of Dissipative Systems*, AMS, Providence, RI, 1988. .

[3] D. Henry, *Geometric Theory of Semilinear Parabolic Equations*, Springer, Berlin, 1981. .

# ON SOME PERIODIC BOUNDARY VALUE PROBLEMS WITH SINGULARITIES

*Milan Tvrdý*, Praha

The contribution is based on the joint work with Irena Rachůnková and Ivo Vrkoč. It deals with periodic boundary value problems for nonlinear scalar differential equations of the second order and with the method of lower and upper functions. For example, we obtain new existence and multiplicity results for the problem

$$u'' + k\,u = g(u) + e(t), \quad u(0) = u(2\pi), \quad u'(0) = u'(2\pi),$$

where $k \in \mathrm{R}$, $e \in \mathrm{L}[0, 2\pi]$, $g \in \mathrm{C}(0, \infty)$, $\liminf_{x\to\infty} g(x)/x > k - \frac{1}{4}$ and $g$ has a strong singularity at the origin, i.e.

$$\lim_{x\to 0+} \int_x^1 g(s)\,\mathrm{d}s = \infty.$$

# OSCILLATION PROPERTY FOR SEMILINEAR WAVE EQUATIONS

*Hiroshi Uesaka*, Tokyo

*MSC 2000*: 35B05

In this talk we shall speak on a change of signs of real valued solutions of the following semilinear wave equations:

$$Pu = \partial_t(\alpha\partial_t u) - \beta\partial_x^2 u + \gamma\partial_t u + \delta\partial_x u + \varepsilon u + f(u) = 0.$$

We consider the Cauchy problem or the mixed problem for $Pu = 0$ in one space dimension. First under some suitable assumptions we can show that a solution for the Cauchy problem does not change its sign if initial values satisfy some conditions. Next we consider the interior mixed problem for $Pu = 0$ in a finite interval $(0, l)$. Under some suitable assumptions we can show that a solution changes its sign infinitely many times for each fixed $x$ as $t \to \infty$ if initial data do not change their signs.

We call it pointwise oscillation property.

## MANY PERIODIC ORBITS FOR DISSIPATIVE, FORCED, ENTIRE PENDULUM-LIKE EQUATIONS

*Antonio J. Ureña*, Granada, Spain

*MSC 2000*: 34C25

Our talk deals with the periodic problem for the forced, damped pendulum-like equation:

$$\begin{cases} u'' + cu' + g(u) = h(t) \\ u(0) = u(T); \quad u'(0) = u'(T) \end{cases} \tag{1}$$

Here, $c$ and $T$ are given positive numbers, and the periodic function $g\colon \mathbb{R} \to \mathbb{R}$ is assumed to be the restriction to the real line of an entire one on the complex plane.

We investigate the number of solutions of (1) for $h \in L^1[0,T]$. In particular, we show that for any real number $n$ there exists an open subset $\mathcal{O}_n \subset L^1[0,T]$ such that for any $h \in \mathcal{O}_n$, problem (1) has at least n solutions.

This result extends a previous work by Ortega on the conservative pendulum equation.

## QUADRATIC SPLINE DIFFERENCE SCHEMES FOR SINGULAR PERTURBATION PROBLEMS OF CONVECTION-DIFFUSION TYPE

*Zorica Uzelac*, Ljiljana Pavlović, Novi Sad, Yugoslavia

*MSC 2000*: 65L10, 65L12, 65L70

We concerned singularly perturbed boundary value problem of convection-diffusion type and its discretization via quadratic spline on the piecewise uniform mesh of Shishkin type. The family of schemes is derived by using the collocation method. By means of detailed analysis of consistency error, the almost second order of convergence is proved for some of the schemes from the family. We present numerical results in support of this results.

# TWO-SCALE CONVERGENCE WITH RESPECT TO MEASURES IN CONTINUUM MECHANICS

*Jiří Vala*, Brno, Czech Republic

*MSC 2000*: 74Q05

In most problems of continuum mechanics two length scales can be distinguished —a macroscopic scale (e.g., the size of strengthening particles in a composite structure) and a microscopic one (e.g., the total size of a deformable body). One general homogenization technique (for any medium with a periodic microstructure) makes use of the idea of the two-scale convergence, developed by Nguetseng (1989), Allaire (1992) and Holmbom (1997).

If the periodic medium contains pores, like Allaire & al. (1995), or other discontinuities, or if thin structures are considered (the reduction of dimension is required), the proof technique mostly depends on the special shape characteristics. Following Bouchité & Fragalà (2000), this can be avoided, using the concept of the two-scale convergence in measures, under certain connectedness assumptions. The paper demonstrates how this approach can be applied to the construction of sequences of approximate solutions for some strongly nonlinear elliptic problems and to the study of their convergence properties.

# ATTRACTORS OF NONAUTONOMOUS MULTIVALUED DYNAMICAL SYSTEMS

T. Caraballo, J. A. Langa, Sevilla, Spain, V. S. Melnik, Kiev, Ukraine,
*J. Valero*, Alicante, Spain

*MSC 2000*: 35B40, 35B41, 35K55, 35K57, 35K90

In this work we study the existence of pullback global attractors for multivalued processes. The theory of pullback attractors has been developed for nonautonomous systems in which the trajectories can be unbounded when times rises to infinite. In such systems the classical theory of global attractors is not applicable. Hence, a different approach has been considered. The global attractor is defined as a parameterized family of sets $\mathcal{A}(\sigma)$, which attracts the solutions of the system "from $-\infty$." This means that the initial moment of time goes to $-\infty$ and the final time remains fixed. A new difficulty appears if the solution corresponding to each initial state can be non-unique. The system is then nonautonomous and multivalued. We prove an abstract result on the existence of a global attractor for multivalued dynamical processes and apply it to nonautonomous differential inclusions of the reaction-diffusion type.

## MIXED-HYBRID MODEL OF THE FRACTURE FLOW

Jiří Maryška, Otto Severýn, Liberec, *Martin Vohralík*, Praha

*MSC 2000*: 76M10, 65C20, 65N50, 65N15, 35J25

The paper deals with mathematical models describing percolation of groundwater in the fractured matrix of a solid rock, a medium supposed as a possible repository of dangerous nuclear waste. The flow in a single fracture is supposed as governed by Darcy's law and we statistically generate a discrete 2-D fracture network in a 3-D domain. After the construction of a triangular mesh, resultant locally second-order elliptic boundary value problem is discretized by the mixed-hybrid finite element method (due to the geometrical situation, the mixed method is not generally applicable; thus already mixed-hybrid weak formulation). Existence and uniqueness of weak and discrete solutions, as well as error estimates are given. First model problems are tested and the computational results are compared to the theoretical ones. Prospectively, finite-volume contaminant transport models based on presented flow models will be build and used for simulations of real situations.

## ON MEASURE SOLUTIONS TO THE ZERO PRESSURE GAS MODEL AND THEIR UNIQUENESS

*Gerald Warnecke*, Magdeburg, Germany

The talk describes a uniqueness theorem for the 2 by 2 system of conservation laws describing conservation of mass and momentum in a gas with zero pressure. Smooth solutions may not only develop shocks but also become singular measures. The model is related to the scalar Burgers equation in some respects. The existence of solution has been shown a few years ago. The question of uniqueness had remained open. The classical Oleinik entropy is not sufficient to guarantee uniqueness. An additional cohesion condition is introduced which leads to uniqueness.

The method of proof involves an extension of the theory of generalized characteristics to the measure solutions and the use of a flow map. The uniqueness result was achieved jointly with Jiequan Li.

## ON NONLINEAR RIEMANN-HILBERT PROBLEMS

M. Efendiev and *W. L. Wendland*, Stuttgart, Germany

We consider a class of holomorphic solutions to nonlinear Riemann-Hilbert problems for arbitrary, $p$-connected domains. The boundary condition bundle here violates the so-called "zero" conditions for the boundary curves. We present sufficient conditions for these boundary curves which provide existence of a countable number of holomorphic solutions depending on their number of zeros and having additional surprising properties. In particular, we prove the existence of holomorphic solutions of the nonlinear Riemann-Hilbert problems whose "distribution of zeros" is defined by an arbitrarily given absolute positive measure. In contrast to topological methods we present here a constructive proof which might also be relevant for the numerical computation of solutions. Moreover, we introduce the quasicylindrical structure in a Banach space related to the violation of the so-called "zero" conditions.

## A CRITICAL EXPONENT IN A DEGENERATE PARABOLIC EQUATION

*Michael Winkler*, Aachen, Germany

*MSC 2000*: 35K55, 35K65

We consider positive solutions of the Cauchy problem in $\mathbb{R}^n$ for the equation

$$u_t = u^p \Delta u + u^q, \qquad p \geqslant 1,\ q \geqslant 1,$$

and show that concerning global solvability, the number $q = p + 1$ appears as a critical growth exponent in the following sense:

- For $1 \leqslant q < p + 1$ (resp. $1 \leqslant q < \frac{3}{2}$ if $p = 1$), all positive solutions are global but unbounded, provided that $u_0$ decreases sufficiently fast in space.
- For $q = p + 1$, all positive solutions blow up in finite time.
- For $q > p+1$, there are both global and non-global positive solutions, depending on the size of $u_0$.

## ON SOME FURTHER QUADRATIC INTEGRAL INEQUALITIES OF THE SECOND ORDER

*Katarzyna Wojteczek*, Opole, Poland

*MSC 2000*: 26D10

Integral inequalities of the form

$$\int_I uh'^2 dt \leqslant \int_I (sh^2 + rh''^2)\,\mathrm{d}t, \quad h \in H, \tag{1}$$

where $I = (\alpha, \beta), -\infty \leqslant \alpha < \beta \leqslant \infty$, $r$, $s$ and $u$ are real functions of the variable $t$, $H$ is a class of functions absolutely continuous on $I$ have been derived by Florkiewicz and Wojteczek (see [1]) using the uniform method of obtaining integral inequalities. Now using the modification of this method we derive inequalities of the form (1) in a class $\hat{H}$, which doesn't cover with $H$. It allows us to derive some new integral inequalities with Czebyshev weight functions, which cannot be obtained in [1].

*References*

[1] *B. Florkiewicz and K. Wojteczek*: Some second order integral inequalities.. Journal of Nonlinear Analysis: Series A Theory and Methods, in print.

## A NEW CRITERION FOR HETEROCLINIC CONNECTIONS IN SCALAR PARABOLIC PDE

*Matthias Wolfrum*, Berlin, Germany

*MSC 2000*: 35K57, 37L30, 35B41, 34C37

We address the problem of heteroclinic connections in the attractor of dissipative scalar semilinear parabolic equations

$$u_t = u_{xx} + f(x, u, u_x), \qquad 0 < x < 1$$

on a bounded interval with Neumann conditions. Introducing a sequence of order relations, we prove a new and simple criterion for the existence of heteroclinic connections, using only information about nodal properties of solutions to the stationary ODE problem. This result allows also for a complete classification of possible attractors in terms of the permutation of the equilibria, given by their order at the two boundaries of the interval.

## IRREGULAR BOUNDARY VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

YAKOV YAKUBOV, Tel-Aviv

*MSC 2000*: 34L10, 34B05, 35K15

Birkhoff-irregular boundary value problems for quadratic ordinary differential pencils of the second order have been considered. The spectral parameter may appear in a boundary condition, the equation contains an abstract linear operator while the boundary conditions contain internal points of an interval and a linear functional. Isomorphism and coerciveness with a defect are proved for such problems. Two-fold completeness of root functions of corresponding spectral problems is also established. As an application of the obtained results, an initial boundary value problem for second order parabolic equations is considered, and the well-posedness and completeness of the elementary solutions are proved.

## EFFECTIVE STABILITY OF GENERALIZED HAMILTONIAN SYSTEMS

YINGFEI YI, Atlanta and Singapore

The lecture will present an effective (or Nekhoroshev) stability result for generalized Hamiltonian systems in which dimensions of action variables and angle variables can be distinct and odd. Estimate of Arnold's diffusion rate will be given, and a KAM type of perturbation result will also be introduced.

## IMPLICIT INITIAL VALUE PROBLEMS: SOLVABILITY, ASYMPTOTICS, NUMBER OF SOLUTIONS

Oleksandr E. Zernov, Odessa, Ukraine

*MSC 2000*: 34A08, 34C11

The following initial value problems are under consideration:

$$\begin{aligned} x'(t) &= f_1\big(t, x(t), x'(t)\big), \quad x(0)=0,\\ \alpha(t)x'(t) &= f_1\big(t, x(t), x'(t)\big), \quad x(0)=0,\\ x'(t) &= f_2\big(t, x(t), x(g(t)), x'(t), x'(h(t))\big), \quad x(0)=0,\\ \alpha(t)x'(t) &= f_2\big(t, x(t), x(g(t)), x'(t), x'(h(t))\big), \quad x(0)=0, \end{aligned}$$

where $x\colon (0,\tau) \to \mathbb{R}^n$ is an unknown function, $\alpha(t) = diag(\alpha_1(t), \dots, \alpha_n(t))$ is an $n \times n$-matrix, $\alpha_i\colon (0,\tau) \to (0,+\infty)$ are continuous functions, $\lim\limits_{t\to+0} \alpha_i(t) = 0$, $i \in \{1,\dots,n\}$, $g\colon (0,\tau) \to (0,+\infty)$, $h\colon (0,\tau) \to (0,+\infty)$ are continuous functions, $0 < g(t) \leqslant t$, $0 < h(t) \leqslant t$, $t \in (0,\tau)$, $f_1$, $f_2$ are continuous functions. Existence of continuously differentiable solutions with needed asymptotic properties is being proved. The number of such solutions is being determinated too.

## ON POSITIVE SOLUTIONS OF SECOND ORDER BOUNDARY VALUE PROBLEMS ON THE HALF-LINE

Mirosława Zima, Rzeszów, Poland

*MSC 2000*: 34B15

We discuss the existence of positive solutions of the following boundary value problems:

$$\begin{aligned} &x''(t) - k^2 x(t) + f(t, x(t)) = 0,\\ &x(0) = 0, \lim_{t\to\infty} x(t) = 0 \end{aligned} \tag{1}$$

and

$$\begin{aligned} &x''(t) - k^2 x(t) + f(t, x(t), x'(t)) = 0,\\ &x(0) = 0, \lim_{t\to\infty} x(t) = 0, \end{aligned} \tag{2}$$

where $t \in [0,\infty)$, $k > 0$ and $f$ is continuous and non-negative function. We present existence theorems for (1) and (2) in the appropriate function spaces equipped with Bielecki's norm. The proofs of these theorems are based on the Krasnoselskii fixed point theorem on cone expansion and compression of norm type.

# POSTERS

## A PARTIAL GENERALIZATION OF DILIBERTO'S THEOREM FOR CERTAIN DIFFERENTIAL EQUATIONS OF HIGHER DIMENSION

LADISLAV ADAMEC, Brno

*MSC 2000*: 37E99, 34C05, 34C30, 34D10

In the theory of autonomous perturbations of periodic solutions of ordinary differential equations is the method of Poincaré mapping widely used. For analysis of properties of this mapping is, in the case of two-dimensional system, sometimes used a result first obtained probably by Diliberto in 1950. This result is here (partially) extended to certain class of autonomous ordinary differential equations of higher dimension.

## THE ASYMPTOTIC PROPERTIES OF THE SOLUTIONS OF THE $N$-TH ORDER NEUTRAL DIFFERENTIAL EQUATIONS

DÁŠA LACKOVÁ, Košice, Slovak Republic

*MSC 2000*: 34C10, 34K11

The aim of this paper is to deduce oscillatory and asymptotic behavior of the solutions of the $n-$th order neutral differential equations

$$(x(t) - qx(t-\tau))^{(n)} - q(t)x(\sigma(t)) = 0,$$

where $\sigma(t)$ is delayed or advanced argument.

## REDUCTION OF DIMENSION VIA TWO-SCALE CONVERGENCE

*Sanja Marušić*, Zagreb, Croatia

*MSC 2000*: 35B25, 35B40, 76D05, 76D08

The notion of two scale convergence was introduced for periodic homogenization by Nguetseng and fully developed by Allaire. It is a powerful tool that avoids formal two-scale expansions and generalising the idea of the energy method enables an easy proof of convergence of homogenization process. Two-scale asymptotic expansions are also the most common tools for study of processes in thin domains and derivation of lower-dimensional models for their description. In analogy with the homogenization theory we develop the notion of two-scale convergence as a tool for deriving lower-dimensional approximations for problems in thin domains. We illustrate our method by several examples.

## OSCILLATORY PROPERTIES OF ITERATIVE FUNCTIONAL EQUATIONS

*W. Nowakowska* and J. Werbowski, Poznań

*MSC 2000*: 39B20

We give some oscillatory criteria for solutions of iterative functional equations.

## DARBOUX TRANSFORMATION OF LAMÉ-INCE POTENTIALS AND ISO-MONODROMIC DEFORMATION ON THE TORUS

Mayumi Ohmiya, Kyotanabe, Japan

*MSC 2000*: 34M55

Let us consider the $n$-th Lamé-Ince potential $u_n(x,\tau) = n(n+1)\wp(x,\tau)$ of the 2-nd order ordinary differential operator

$$H = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + u_n(x,\tau),$$

where $\wp(x,\tau)$ is Weierstrass elliptic function with the primitive periods 1 and $\tau$, $\operatorname{Im}\tau > 0$, and $n$ is a natural number. If $\tau$ satisfies the degenerate condition established by the present author, one can construct the double Darboux transformation $\widehat{u}_n(x,\tau,t)$, which is the 1-parameter family of the algebro geometric elliptic potentials, by using some specified eigenfunction of the operator $H$. In this paper, it is shown that the 1-parameter family of ordinary differential equations

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \widehat{u}_n(x,\tau,t)y$$

is the isomonodromic family of Fuchsian equations on the torus $\mathbb{C}/L$, where $L = \mathbb{Z} \oplus \mathbb{Z}\,\tau$ is the lattice.

## SIMULATIONS ON THE CHEMOSTAT FOOD CHAIN MODEL WITH DELAY

Sael Romero
Departamento de Matemáticas, UDO-Sucre, Cumaná, Venezuela, e-mail: sromero@sucre.udo.edu.ve

In this talk we consider a food chain model, the competition take place in the Chemostat, a basic piece of laboratory apparatus that occupies a central place in mathematical ecology. We consider three trophic levels, with input nutrient $S(t)$ and the organism $P(t)$ growing on the nutrient, and one predator $Q(t)$ that has $P(t)$ as a prey. Additionally we suppose that the dynamic of the predator at the end of the chain depend on the past history of the prey by mean a distributed delay that take an average of the Michaelis-Menten functional response of the prey $P(t)$. We show the existence of the global attractor for the solutions and some simulations of the solutions under particular constellations of the parameters are obtained. These simulations predict the existence of periodic solutions for the model.

## INVERSE PROBLEMS OF CELESTIAL MECHANIC

*Natalia Sadovskaia*, Barcelona, and Rafael Ramirez, Tarragona

We proposed for the mechanical system with configuration space $X$ and kinetic energy $T$ a completely solution to the inverse problem in celestial mechanics of constructing the force field capable of generating the given orbits.

## FIELDLESS METHODS FOR THE SIMULATION OF STATIONARY AND NONSTATIONARY INDUCTION HEATING

Ivo Doležel, *Pavel Šolín*, Prague, Bohuš Ulrych, Pilsen

*MSC 2000*: 78A25, 78A55, 78M10, 46N20, 46N40, 65J10

The study presents a class of numerical methods for the simulation of stationary and nonstationary induction heating of non-ferromagnetic metallic objects in two and three spatial dimensions by an external harmonic electromagnetic field. The model consists of a system of Fredholm integral equations of the second kind with a weakly-singular kernel function for the eddy currents in the material, and a heat transfer equation. Nonlinear temperature-dependent material parameters are respected. A first-order collocation scheme for the solution of the integral equations is used. The operator related to the system of the integral equations is analyzed and the solvability and uniqueness of the system of the integral equations is shown. Convergence of the collocation scheme for the solution of the integral equations is shown. Two- and three-dimensional numerical examples are presented.

## SOME OSCILLATION CRITERIA FOR DIFFERENTIAL EQUATIONS WITH DEVIATED ARGUMENTS

*A. Szawiola* and J. Werbowski, Poznań

*MSC 2000*: 34K15

In this paper the solutions of differential equations with deviating arguments are considered. We formulate some conditions to obtaine the oscillatory behaviour of solutions of the mentioned equations.

## OSCILLATORY PROPERTIES OF SOLUTIONS OF DIFFERENCE EQUATIONS

G. Grzegorczyk and *A. Wyrwińska*, Poznań

*MSC 2000*: 39A10

We consider oscillatory behavior of solutions of some difference equations of higher order.

# EXTENDED ABSTRACTS

# PHASE PORTRAITS OF QUADRATIC ODE'S AND POLYNOMIAL EQUATIONS IN NON-ASSOCIATIVE COMMUTATIVE ALGEBRAS

*Zalman Balanov*, Netanya and Yakov Krasnov, Tel Aviv

*MSC 2000*: Primary 34C11, 34C14, 34C41; Secondary 17A01

**1. Quadratic homogeneous systems.** Let $f\colon \mathbb{R}^n \to \mathbb{R}^n$ be a homogeneous quadratic map meaning that its coordinate functions are homogeneous quadratic forms. We are interested in the study of autonomous homogeneous quadratic systems of ODE's of type

$$\dot{x} = f(x) \qquad (x \in \mathbb{R}^n). \tag{1}$$

The following result is well-known in the case $n = 2$ (see, for instance, [1], [2], [4]; we also refer to [1] for the ODE's terminology used in what follows).

**Theorem A.** *Let $n = 2$. Two systems of type* (1) *are topologically equivalent if their phase spaces have the same number of:*

(i) *elliptic, parabolic and hyperbolic sectors, and*

(ii) *orbitally stable/unstable ray solutions and equilibria ray solutions.*

Observe that this "geometric" result is essentially two-dimensional with no meaning to be extended to $n$-dimensional systems in any respect. In particular, it does not allow to look for bounded solutions to system (2) for $n > 2$.

**2. Algebraic approach.** In contrast to the "geometric" techniques developed in [1], [4]), M. Markus [5] was the first to consider an algebraic approach to study systems (1) (see also [3], [6] for a detailed exposition). To be more specific, let $f$ be as in (1) and define a bilinear map $\beta\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ by $\beta(x,y) = \frac{1}{2}(f(x+y) - f(x) - f(y))$. Setting $x * y = \beta(x,y)$ one equips $\mathbb{R}^n$ with the structure of a real commutative algebra (in general, non-associative), denoted $(A,*)$. Then system (1) can be rewritten as

$$\dot{x} = x * x = x^2 \qquad (x \in (A,*)) \tag{2}$$

and one can study the behavior of the solutions to (1) bearing in mind the structure of the algebra $(A,*)$. For instance, idempotents correspond to ray solutions; 2-nilpotents are equilibria, etc. Clearly, idempotents and 2-nilpotents are solutions to a (real) polynomial equation: $x^2 = \lambda x$ ($\lambda \in \mathbb{R}$) for $\lambda = 1$ or 0, respectively. We arrive at the following question: how is related the solubility of (real) polynomial equations in $(A,*)$ to the behavior of solutions to (1)? In particular (in view of Theorem A), which equations are enough for the topological classification of systems (1)? Which equations are "responsible" for the existence of bounded solutions?

**3. Basic equations.** In this subsection $A$ stands for a real commutative finite-dimensional algebra and $x \in A$ denotes a *non-zero* element.

**Definition.** *We refer to the following equations in $A$ as the basic ones:*

(i) $x^2 = \lambda x$; (ii) $x^2 x^2 = \lambda x^2$;

(iii) $x^3 = \lambda x$; (iv) *equations determining ideals in* $A$.

*For* $\lambda = -1$ *the solutions to* (ii) (*respectively, to* (iii)) *are called negative square idempotents* (*respectively, negative* 3*-idempotents*).

**4. Main results.** The following statement clarifies the meaning of the word "basic" in the above definition.

**Theorem B.** *Let* $f, g\colon \mathbb{R}^2 \to \mathbb{R}^2$ *be two quadratic maps. Let* $(A, *)$ *and* $(A, \circ)$ *be algebras related to* $f$ *and* $g$*, respectively. Then the systems* $\dot{x} = f(x)$ *and* $\dot{y} = g(y)$ *are topologically equivalent iff the solution sets of the basic equations in* $(A, *)$ *and* $(A, \circ)$ *have the same cardinalities.*

Assume now that the origin is an isolated equilibrium for system (1). Then the topological index $\mathrm{ind}(0, \mathrm{f})$ is correctly defined. As is well-known, if $n = 2$ then $\mathrm{ind}(0, \mathrm{f})$ takes three values: $-2$, 0, 2.

**Theorem C.** *Let* $n = 2$ *and the origin be an isolated equilibrium for system* (1) *or* (2)*. Then:*

(i) $\mathrm{ind}(0, \mathrm{f}) \neq 0$ *iff* $(A, *)$ *contains a negative square idempotent;*

(ii) $\mathrm{ind}(0, \mathrm{f}) = 2$ *iff* $(A, *)$ *contains both negative square idempotent and negative 3-idempotent.*

As is well-known, under the assumptions of Theorem C, system (1) has a bounded solution iff $\mathrm{ind}(0, \mathrm{f}) = 2$. Combining this fact with Theorem C yields

**Theorem D.** *Given system* (2) *assume* $(A, *)$ *has both negative square idempotent* $a$ *and negative* 3*-idempotent* $b$*. Assume* $a$ *and* $b$ *generate a two-dimensional subalgebra of* $(A, *)$*. Then system* (1) *has a bounded solution.*

**Remark.** In a standard way Theorems B, C and D can be applied to study solutions to (1) when the associated algebra $(A, *)$ is of rank three (i.e. each one-generated subalgebra of $(A, *)$ has dimension $\leqslant 2$ (see [7] for details)).

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## A NOTE ON CLOSED GEODESICS

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*MSC 2000*: 58E10, 53C22, 53C20

In [1] we studied the existence of a closed geodesic on Riemannian manifolds $(M, \langle\cdot,\cdot\rangle_R)$ which are convex close to their boundaries according to the following definition (by $\overline{M}^c = M \cup \partial_c M$ we shall denote the canonical Cauchy completion of $M$ as a metric space, by using Cauchy sequences).

**Definition 1.** $M$ is *convex close to its boundary (CCB)* if there exist $\delta, b > 0$ and $\varphi \in C^0(\overline{M}^c, \mathbb{R}_+) \cap C^2(M, \mathbb{R}_+)$ s.t.: $\varphi^{-1}(0) = \partial_c M$; $0 < \langle\nabla\varphi(x), \nabla\varphi(x)\rangle_R \leqslant b$ if $0 < \varphi(x) < \delta$; $H_R^{\varphi}(x)[v,v] \leqslant 0$ if $0 < \varphi(x) < \delta$ and $v \in T_x M$ is s.t. $\varphi'(x)[v] = 0$.

The authors obtained the following result in [1] which extends to non-complete manifolds a result in [3].

**Theorem 2.** *Let $M$ be CCB. Assume:*
(1) *for some $x_0 \in M$ $\limsup_{d(x,x_0)\to\infty} K(x) \leqslant 0$, where $K(x) = \sup\{K(\pi) \mid \pi \subset T_x M\}$, $x \in M$, and* (2) *there exist $q \in \mathbb{N}$, $q > 2\dim M$, such that $H_q(\Lambda(M), \mathcal{F}) \neq \{0\}$. Then there exists a non-trivial closed geodesic on $M$.*

Aim of this note is to stress the convenience of Def. 1 by comparing Th. 2 with previous results. First of all, remark that in Th. 2 we do not assume the compactness of $\partial_c M$, as previous results [4], [6] do. Now, let us assume that $\partial_c M$ is compact in order to make this comparison. By Def. 1, we can state the result in [4] as follows.

**Theorem 3.** *Let $M$ be CCB with compact $\partial_c M$. Assume:* (1) *there exist $U \in C^2(M, \mathbb{R})$ and $r, \nu > 0$ s.t. $H_R^U(x)[v,v] \geqslant \nu\langle v, v\rangle_R \quad \forall x \in M \setminus B_r(x_0),\ v \in T_x M$ and* (2) *$M$ is not contractible in itself and either $\pi_1(M)$ is finite or it has infinitely many conjugacy classes. Then there exists a non-trivial closed geodesic on $M$.*

For the relation between Th. 3 and 2, notice first that the assumptions of Th. 3 are fulfilled also if all the sectional curvatures outside a compact subset are non-positive (indeed in this case the distance function plays the same role of $U$). Nevertheless, in Th. 2 we only require that the *limsup* of the sectional curvatures is non-negative. Now, recall the following result in [2].

**Theorem 4.** *Let $M$ be CCB with compact $\partial_c M$. Assume that there exist $U \in C^2(M, \mathbb{R})$ and $r, \nu > 0$ s.t.* (2) *from Theorem 3 holds. Then there exists $\{D_m\}$ of open subsets of $M$ s.t.: $D_m \subset D_{m+1}$, $M = \bigcup_{m\in\mathbb{N}} D_m$; for any $m \in \mathbb{N}$ $D_m$ is a bounded, CCB with compact $\partial_c D_m$; there exists $\overline{m} \in \mathbb{N}$ such that $D_m \cup \partial_c D_m$ is a deformation retract of $M$ for any $m \geqslant \overline{m}$.*

By using Th. 4, Th. 3 is reduced now to search closed geodesics on a *bounded* CCB manifold $D \equiv D_m$ having the same topological properties of the whole manifold, thus (1) of Th. 2 is satisfied. So, essentially Th. 3 turns out to be contained in Th. 2; the difference between their topological assumptions (2) comes from the different tools of the proof (Morse index for Th. 2 and Ljusternik-Schnirelmann theory for Th. 3). Even more, the hypotheses on $U$ in Th. 4 can be sharpened; this implies to improve Th. 3, [2]. Next, the results in [6] will be compared. The main result in [6] can be stated as follows.

**Theorem 5.** *Let $M$ be complete and let $K \subset M$ be compact and locally convex. If there exists an $i > 0$ s.t. the $i$-th homotopy group $\pi_i(K)$ is not trivial, then there exists a non-trivial closed geodesic on $M$.*

In order to relate this result with Th. 2 notice that by [5, Th. 5.5], *there exists a (necessarily bounded) totally geodesic submanifold $N$ of $M$ such that $K = \overline{N}$.* Moreover, $K$ is also a topological manifold with boundary $\partial K (\equiv \partial_c K) = \partial N$ but this boundary is not necessarily differentiable (for example: consider a convex polygon in $\mathbb{R}^n$). Nevertheless, Th. 5 cannot be compared with Th. 2 yet: the hypotheses of this last theorem can be imposed on $N$ intrinsically but Th. 5 may find a closed geodesic even in $\partial N$. In order to simplify, we can assume that $\partial K$ is differentiable (and, thus, *convex*). Then, we can obtain [2]:

**Theorem 6.** *Let $\overline{N} = N \cup \partial N$ be compact with (smooth) convex boundary. If one of the topological assumptions (2) in Th. 2 or 3 hold then there exists a closed geodesic in $\overline{N}$.*

For the comparison between the topological assumptions in Th. 6 and 5, recall that if $\pi_1(K) \neq 0$ then one can find a closed geodesic easily in each non-trivial free conjugacy or homotopy class. When $\pi_1(K) = 0$, topological assumptions in Th. 6 would extend in principle those in Th. 5, see [2]. Summing up Th. 2 and related results in [2] solves with reasonable generality the problem of the existence of a closed geodesic because: (a) for bounded manifolds, they extend or complement previous results in [4] and [6], (b) they are applicable when the manifold is unbounded and, in this case, they cannot be reduced to a result in the bounded case (as explained with those in [4] and [6]), and (c) they are also applicable when the boundary is not compact.

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## SOME REMARKS ON SUPERLINEAR INDEFINITE EQUATIONS

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The aim of this note is to present recent results and further developments on the equation

$$\ddot{x} + q(t)g(x) = 0, \tag{E}$$

where $q\colon [0,\omega] \longrightarrow \mathbb{R}$ changes sign and $g\colon \mathbb{R} \longrightarrow \mathbb{R}$ is superlinear. In order to simplify the exposition, assume that $q \in \mathrm{BV}([0,\omega])$, $g \in C^1(\mathbb{R})$, $g(x) \sim x|x|^{p-1}$ for $|x| \to +\infty$ and for some $p > 1$, and $g(x)x > 0$ for every $x \neq 0$. Moreover, suppose that $q$ has exactly one change of sign in $[0,\omega]$ and that $q(t) < 0$ in $(0,\tau)$, $q(t) \geqslant 0$, $q \not\equiv 0$, in $[\tau,\omega]$; the case when $q$ has a greater number of changes of sign is similar. Finally, assume that $q$ is monotone in a right neighbourhood of $\tau$ and in a left neighbourhood of $\omega$. For every $p \in \mathbb{R}^2$, we denote by $x(\cdot;t_0,p)$ the solution of (E) such that $(x(t_0),\dot{x}(t_0)) = p$; we also recall that (E) is equivalent to a planar system and set $z(\cdot;t_0,p) = (x(\cdot;t_0,p),\dot{x}(\cdot;t_0,p))$. Finally, if $p \neq 0$, we denote by $\mathrm{rot}(p)$ the number of rotations of $z(\cdot;\tau,p)$ (in the phase-plane $(x,\dot{x})$) in the time $[\tau,\omega]$.

Equations like (E) have been intensively studied in the last years; very recent results [1]-[3]-[4]-[5] are related to the study of the oscillatory properties of the solutions.

Since $q$ is changing sign, in the study of (E) it is crucial to relate the features of the solutions of (E) in the intervals where $q$ is positive or negative, respectively. Roughly speaking, we can say that, in the interval of positivity of $q$, the difference between the number of rotations (in the phase-plane) of solutions with large initial conditions and the number of rotations (in the phase-plane) of solutions with small initial conditions is arbitrarily large. The behaviour of solutions to (E) in the interval of negativity of $q$ is very different: first of all, we cannot ensure the continuability of $x(\cdot;0,p)$ up to $[0,\tau]$ for every $p \in \mathbb{R}^2$, but only for initial data with small norm. Moreover, (continuable) solutions having initial data of small norm can become very large at the time $\tau$.

In view of these remarks, some problems seem of interest: for instance, the existence of solutions to (E) defined on $(0,\tau)$ which blow up exactly at the time $t = 0$ and $t = \tau$ or the existence of solutions satisfying periodic (or Dirichlet) boundary conditions in $[0,\omega]$.

On these lines, the authors, together with A. Capietto, J. Mawhin and F. Zanolin, proved the following results:

**Theorem 1** [3]. *Under the previous assumptions, there exists at least one positive solution to* (E) *such that* $x(t) \to +\infty$ *for* $t \to 0^+$ *and* $t \to \tau^-$.

**Theorem 2** [4]. *Under the previous assumptions, there exists* $n^* \in \mathbb{N}$ *such that for every integer* $n \geqslant n^*$ *and for* $\delta \in \{0,1\}$ *there exist two solutions* $u$ *and* $v$ *of the*

*Dirichlet (or periodic) problem, in* $[0, \omega]$, *associated to* (E) *such that* $\dot{v}(0) < 0 < \dot{u}(0)$; *moreover,* $u$ *and* $v$ *have exactly* $\delta$ *changes of sign in* $(0, \tau)$ *and* $n$ *zeros in* $(\tau, \omega)$.

Assume now that $q\colon \mathbb{R} \longrightarrow \mathbb{R}$ and that there exists $\{t_k\}$, $k \in \mathbb{Z}$, with $t_k \to \pm\infty$ for $k \to \pm\infty$, such that $q(t) \leqslant 0$, $q \not\equiv 0$, in $[t_{2k}, t_{2k+1}]$, $q(t) \geqslant 0$, $q \not\equiv 0$, in $[t_{2k+1}, t_{2k+2}]$, for every $k \in \mathbb{Z}$. Then, we have the following:

**Theorem 3** [1]. *Under the previous assumptions, there exists* $\mathbf{n}^* = (n_k^*)$, $k \in \mathbb{Z}$, *with* $n_k^* \in \mathbb{N}$, *for every* $k \in \mathbb{Z}$, *such that for every sequence* $\mathbf{n} = (n_k)$, $k \in \mathbb{Z}$, *with* $n_k \in \mathbb{N}$ *and* $n_k > n_k^*$, *and for every sequence* $\delta = (\delta_k)$, $k \in \mathbb{Z}$, *with* $\delta_k \in \{0, 1\}$, *there exist two solutions* $u_{\mathbf{n},\delta}\colon \mathbb{R} \longrightarrow \mathbb{R}$ *and* $v_{\mathbf{n},\delta}\colon \mathbb{R} \longrightarrow \mathbb{R}$ *of* (E) *such that* $v_{\mathbf{n},\delta}(t_0) < 0 < u_{\mathbf{n},\delta}(t_0)$ *and* $\dot{u}_{\mathbf{n},\delta}(t_0) < 0 < \dot{v}_{\mathbf{n},\delta}(t_0)$. *Moreover, for every* $k \in \mathbb{Z}$, $u_{\mathbf{n},\delta}$ *and* $v_{\mathbf{n},\delta}$ *have exactly* $\delta_k$ *changes of sign in* $(t_{2k}, t_{2k+1})$ *and* $n_k$ *zeros in* $(t_{2k+1}, t_{2k+2})$.

In this situation, when $q$ is $\omega$-periodic, it is possible to prove [1] some chaotic features of the set of solutions to (E); indeed, it is possible to show the existence of an uncountable number of bounded non-periodic solutions (see also [5]). Moreover, the Poincaré map associated to (E) is topologically semiconjugate to a Bernoulli shift and has positive topological entropy.

The proofs of these results are based on a detailed phase-plane analysis combined with several topological arguments.

We end this note pointing out some open problems and further developments; first of all, it is well-known that a result analogous to Theorem 1 holds true when $g$ is sublinear near zero. As for different asymptotic behaviours of $g$? In a forthcoming paper [2] the authors study the asymptotically linear case, by adapting some of the previous techniques; the analysis of the situation when $g$ has an asymmetric behaviour seems of some interest. Finally, as far as the chaotic features of the problem on the real line are concerned, it would be important to study the set of initial conditions leading to solutions globally defined and with the prescribed oscillatory properties; in particular, we are interested in proving if it is a Cantor set. Moreover, we are looking for some classes of equations for which the Poincaré map is topologically conjugate (and not only semiconjugate) to a Bernoulli shift.

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# $\mathcal{L}^{2,\Phi}$ REGULARITY FOR NONLINEAR ELLIPTIC SYSTEMS

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In this paper we consider the problem of the regularity of the first derivatives of weak solutions to the nonlinear elliptic system

$$-D_\alpha a_i^\alpha(x,u,Du) = a_i(x,u,Du), \quad \text{in } \Omega,\ i=1,\dots,N, \alpha=1,\dots,n, \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geqslant 3)$ is bounded open set, $N > 1$ and $a_i^\alpha$, $a_i$ are Caratheodorian mappings from $(x,u,z) \in \Omega \times \mathbb{R}^N \times \mathbb{R}^{nN}$ into $\mathbb{R}$.

As it is known, in case of a general system (1) only partial regularity can be expected for $n > 2$ (see e.g. [Ca]). Under the assumptions below we will prove $\mathcal{L}^{2,\Phi}$-regularity of gradient of weak solutions for the system (1) whose coefficients $a_i^\alpha$ have the form

$$a_i^\alpha(x,u,Du) = A_{ij}^{\alpha\beta}(x) D_\beta u^j + g_i^\alpha(x,u,Du), \quad i,j=1,\dots,N, \alpha,\beta=1,\dots,n, \tag{2}$$

where $A_{ij}^{\alpha\beta}$ is a matrix of functions, the following condition of strong ellipticity

$$A_{ij}^{\alpha\beta}(x)\xi_\alpha^i \xi_\beta^j \geqslant \nu|\xi|^2, \quad \text{a.e. } x \in \Omega,\ \forall\,\xi \in \mathbb{R}^{nN};\ \nu > 0 \tag{3}$$

holds and the functions $g_i^\alpha$, $a_i$ satisfy of the following conditions

$$|a_i(x,u,z)| \leqslant f_i(x) + L|z|^{\gamma_0}, \quad g_i^\alpha(x,u,z)z_\alpha^i \geqslant \nu_1|z|^{1+\gamma} - f^2(x), \tag{4}$$

$$|g_i^\alpha(x,u,z_1) - g_i^\alpha(y,v,z_2)| \leqslant L\left(|f_i^\alpha(x) - f_i^\alpha(y)| + |z_1 - z_2|^\gamma\right) \tag{5}$$

for a.e. $x \in \Omega$ and all $u$, $v \in \mathbb{R}^N$, $z_1$, $z_2 \in \mathbb{R}^{nN}$. Here $L$, $\nu_1$ are positive constants, $1 \leqslant \gamma_0 < (n+2)/n$, $0 \leqslant \gamma < 1$, $f$, $f_i^\alpha \in \mathcal{L}^{2,n}(\Omega)$, $f_i \in L^{\sigma q_0, n q_0}(\Omega)$, $\sigma > 2$, $q_0 = n/(n+2)$.

Such result may open a way to prove BMO-regularity of gradient. In [Da] the first author has proved $L^{2,\lambda}$-regularity of gradient of weak solutions to (1) in the situation when the coefficients $A_{ij}^{\alpha\beta}$ are continuous and BMO-regularity of gradient in the case when coefficients $A_{ij}^{\alpha\beta}$ are Hölder continuous. In [DV] the coefficients $A_{ij}^{\alpha\beta}$ are discontinuous in general and the $L^{2,\lambda}$-regularity is proved.

If we want to sketch our method of proof, we have to say that its crucial point is the assumption on $A_{ij}^{\alpha\beta}$: $A_{ij}^{\alpha\beta} \in L^\infty(\Omega) \cap \mathcal{L}^{2,\Psi}(\Omega)$ (for the definition see below). Taking into account higher integrability of gradient $Du$ we obtain $\mathcal{L}^{2,\Phi}$-regularity of gradient.

Beside the usually used Morrey $L^{q,\lambda}$ and Campanato spaces $\mathcal{L}^{q,\lambda}$ (for more details see [Ca]) we use the following generalization of Campanato spaces introduced by Spanne [Sp].

**Definition.** Let $\Psi$ be a positive function on $(0, \operatorname{diam}\Omega]$. A function $u \in L^2(\Omega, \mathbb{R}^N)$ is said to belong to $\mathcal{L}^{2,\Psi}(\Omega, \mathbb{R}^N)$ if $[u]_{2,\Psi,\Omega} = \sup\{\Psi^{-1}(r)(\int_{\Omega(x,r)} |u(y) - u_{x,r}|^2 \, \mathrm{d}y)^{1/2}; x \in \Omega,\ r \in (0, \operatorname{diam}\Omega]\} < \infty$ and by $l^{2,\Psi}(\Omega, \mathbb{R}^N)$ we denote subspace of all $u \in \mathcal{L}^{2,\Psi}(\Omega, \mathbb{R}^N)$ such that $[u]_{2,\Psi,\Omega,r_0} = \sup\{\Psi^{-1}(r)(\int_{\Omega(x,r)} |u(y) - u_{x,r}|^2 \, \mathrm{d}y)^{1/2}; x \in \Omega, r \in (0, r_0]\}$ as $r_0 \searrow 0$.

The $\mathcal{L}^{2,\Psi}$, $l^{2,\Psi}(\Omega, \mathbb{R}^N)$ are Banach spaces and the sets $C^0(\overline{\Omega}, \mathbb{R}^N) \setminus \mathcal{L}^{2,\Psi}(\Omega, \mathbb{R}^N)$, $(L^\infty(\Omega, \mathbb{R}^N) \cap l^{2,\Psi}(\Omega, \mathbb{R}^N)) \setminus C^0(\overline{\Omega}, \mathbb{R}^N)$ are not empty for $\Psi(r) = r^{n/2}/(1 + |\ln r|)$ (for the proofs see [Ac] and [Sp]). In the next we assume that $\Psi\colon (0, d] \to (0, \infty)$ is the form

$$\Psi(r) = r^{\zeta/2}\psi(r), \quad 0 \leqslant \zeta \leqslant n + 2, \tag{6}$$

where $\psi$ is a continuous, non-decreasing function such that $\lim_{r\to 0} \psi(r) = 0$ and $r \to \Psi(r)/r^\xi$ for some $\xi > 0$ is almost decreasing, i.e. there exists $k_\psi \geqslant 1$ and $d_0 \leqslant d$ such that $k_\psi \psi(r)/r^\xi \geqslant \psi(R)/R^\xi$ for all $0 < r < R \leqslant d_0$. The function $\psi(r) = 1/(1+|\ln r|)$ satisfy the last inequality with the arbitrary $\xi > 0$.

The next Theorem is generalizing the main result from the paper [DV].

**Theorem.** *Let $u \in W^{1,2}_{\mathrm{loc}}(\Omega, \mathbb{R}^N)$ be a weak solution to the system* (1) *and suppose that the conditions* (2)–(5) *hold. Let further $A^{\alpha\beta}_{ij} \in L^\infty(\Omega) \cap \mathcal{L}^{2,\Psi}(\Omega)$, for each $i, j = 1, \ldots, N$, $\alpha, \beta = 1, \ldots, n$ and $\Psi$ be a function satisfying the condition* (6) *with $\zeta = n$ and $0 \leqslant \xi \leqslant 2$. Then $Du \in \mathcal{L}^{2,\Phi}_{loc}(\Omega, \mathbb{R}^{nN})$ with $\Phi(R) = R^\lambda \psi^{(r-2)/r}(R)$ for some $r > 2$ and arbitrary $\lambda < n$.*

**Remark**. For example, the condition (6) in Theorem is satisfied with the function $\psi(r) = 1/(1 + |\ln r|)$.

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## ON A BOUNDARY PROBLEM CONCERNED WITH THE NONHOMOGENEOUS NAVIER-STOKES EQUATIONS

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Belonosov and El-Sirafy solved the homogeneous equations of Navier-Stokes in the half-plane for which the velocities are given on the boundary. Belonosov solved the same boundary problem of the nonhomogeneous Navier-Stokes equations. When the stresses are given on the boundary and the volumetrical forces are absent, the solution was obtained by El-Sirafy in a closed form.

In this work we need to solve the linearized nonhomogeneous NS equations

$$\mu\nabla^2 w - \gamma\frac{\partial w}{\partial t} - 2\frac{\partial p}{\partial\bar z} + f = 0, \quad \mathrm{Re}\Big\{\frac{\partial w}{\partial z}\Big\} = 0,$$

in the half-plane $D(-\infty < x < \infty,\, y > 0\big|t > 0)$ with the initial condition $w(x, y, 0) = 0$ and boundary conditions

$$2\mu\frac{\partial w}{\partial\bar z} + p\Big|_{y=0} = -N_2(x,t) - \iota N_1(x,t), \qquad \lim_{|z|\to\infty}(x,y,t) = 0,$$

where $z = x + \iota y$, $w = u + \iota v$ is the complex velocity, $p(x, y, t)$ is the pressure, $\mu$ is the coefficient of viscosity, $\gamma$ is the density, $N_1$, $N_2$ are the tangential and normal components of the stresses at the boundary and $f(x, y, t) = f_1(x, y, t) + \iota f_2(x, y, t)$ is a given function satisfies the condition $f(x, y, t) = O(1/|z|^{1+\beta})$ at $|z| \to \infty, \beta > 0$. The above system can be transformed to

$$w = -2\iota\frac{\partial}{\partial\bar z}(\varphi_2 + \iota\varphi_1 + \varphi_0), \qquad p = \mu F_1 - \gamma\frac{\partial\varphi_1}{\partial t},$$

where $\varphi_k(x, y, t)$ $(k = 1, 2)$ are real functions satisfy the equations

$$\nabla^2\varphi_1 = 0; \quad \mu\nabla^2\varphi_2 - \gamma\frac{\partial\varphi_2}{\partial t} = 0,$$

$$\text{while} \quad \varphi_0(x,y,t) = \frac{\gamma}{4\pi\mu}\int_0^t\iint_D \frac{F_2(\xi,\eta,\tau)}{t-\tau} e^{-\frac{\gamma|z-\zeta|^2}{4\mu(t-\tau)}}\,\mathrm{d}\xi\,\mathrm{d}\eta\,\mathrm{d}\tau$$

is a particular solution of the heat equation $\nabla^2\varphi_0 - \gamma/\mu\cdot\partial\varphi_0/\partial t = F_2$; $F_1(x, y, t)$, $F_2(x, y, t)$ are given real functions satisfying the equation

$$F_1 + \iota F_2 = \frac{1}{2\pi\mu}\iint_D \frac{f(\xi,\eta,t)}{z-\zeta}\,\mathrm{d}\xi\,\mathrm{d}\eta,$$

which is a particular solution of $\partial/\partial\bar z(F_1 + \iota F_2) = f/2\mu$ and the equation of stresses on the boundary become

$$\Big(4\frac{\partial^2}{\partial x\partial\bar z} - \gamma_0\frac{\partial}{\partial t}\Big)(\varphi_2 + \iota\varphi_1)|_{y=0} = \frac{1}{\mu}[F_1^*(x,t) - \iota F_2^*(x,t)],$$

where

$$F_1^*(x,t)-\iota F_2^*(x,t) = N_1-\iota N_2+\left[\mu(F_2-\iota F_1)+\gamma\frac{\partial\varphi_0}{\partial t}-2\mu\frac{\partial^2\varphi_0}{\partial x^2}-2\mu\iota\frac{\partial^2\varphi_0}{\partial x\partial y}\right]_{y=0};\ \gamma_0=\frac{\gamma}{\mu}$$

and assuming that

$$\lim_{|z|\to\infty}\varphi_k(x,y,t)=0,\quad \varphi_k(x,y,0)=0\quad (k=0,1,2).$$

Using the technique of Laplace-Fourier transform,

$$\tilde{\varphi}_k = \frac{1}{2\pi}\int_0^\infty \mathrm{e}^{-\lambda t}\int_{-\infty}^\infty \mathrm{e}^{-\iota qx}\varphi_k\,\mathrm{d}x\,\mathrm{d}t;$$

$$\varphi_k = \frac{1}{2\pi\iota}\int_{\sigma-\iota\infty}^{\sigma+\iota\infty}\mathrm{e}^{\lambda t}\int_{-\infty}^\infty \mathrm{e}^{\iota qx}\tilde{\varphi}_k dq\,\mathrm{d}\lambda,\qquad \sigma=\operatorname{Re}\lambda>0$$

we find after some calculations the solution by quadratures, the kernels of which contain some elementary functions and integrals of probability with complex arguments. For example

$$\mu\varphi_1(x,y,t) = \int_{-\infty}^\infty [(x-\xi)\alpha_{13}^*(\xi,t)-y\alpha_{21}^*(\xi,t)]\frac{\mathrm{d}\xi}{(x-\xi)^2+y^2}$$

$$-\int_0^t\int_{-\infty}^\infty [\alpha_{22}(\xi,\tau)\operatorname{Re}\{g(x-\xi,y,t-\tau)\}+\alpha_{14}(\xi,\tau)\operatorname{Im}\{g(x-\xi,y,t-\tau)\}]\,\mathrm{d}\xi\,\mathrm{d}\tau,$$

where

$$g(x-\xi,y,t-\tau) = -\frac{\Gamma^2}{\pi\sqrt{\gamma_0(t-\tau)}}\left[\frac{2}{\sqrt{\pi}Z^2}+\frac{\iota}{Z}\left(1+\frac{1}{Z^2}\right)\mathrm{e}^{-Z^2}\operatorname{erfc}(-\iota Z)\right],$$

$$Z=\Gamma(z-\xi),\quad \Gamma=\frac{1}{2}\sqrt{\frac{\gamma_0}{t-\tau}},\quad \operatorname{erfc}(Z)=1-\Phi(z)=\frac{2}{\sqrt{\pi}}\int_0^\infty \mathrm{e}^{-(\lambda+Z)^2}\,\mathrm{d}\lambda,$$

$$\pi\alpha_{13}^*(\xi,t)=\alpha_{13}(\xi,t)-\frac{1}{2\gamma_0}\int_0^t F_1^*(\xi,\tau)\,\mathrm{d}\tau,\ \ \pi\alpha_{21}^*(\xi,t)=\alpha_{21}(\xi,t)-\frac{1}{2\gamma_0}\int_0^t F_2^*(\xi,\tau)\,\mathrm{d}\tau$$

$$\alpha_{ji}(\xi,\tau)=\sum_{k=1}^3\delta_k^{(i)}F_{jk}(\xi,\tau)\qquad (j=1,i=3,4;j=2,i=1,2),$$

$\delta_k^{(i)}$ are some known constants, for example, $\delta_k^{(1)}=2(4-6s_k+s_k^2)/[s_kB'(-s_k)]$;

$$B(s)=s^3+8s^2+24s+16=\prod_{k=1}^3(s+s_k)\quad (s_1=0.9126, s_2=\bar{s}_3=3.5437-2.2303\iota)$$

and

$$F_{jk}(x,t) = -\frac{1}{2\sqrt{\pi\gamma_0 s_k}}\int_0^t\int_{-\infty}^\infty \frac{F_j^*(\xi,\tau)}{\sqrt{t-\tau}}\mathrm{e}^{-\frac{\gamma_0(x-\xi)^2}{4s_k(t-\tau)}}\,\mathrm{d}\xi\,\mathrm{d}\tau.$$

When $f=0$, the impulsive problem of a rigid body acts in the direction of the normal to the surface of the fluid is considered. The velocities of the corresponding axisymmetrical impulsive problem in the upper half-space are also obtained by quadratures, the kernels of which contain the hypergeometric series and the Legendre function of degree $-\frac{1}{2}$ of the 2nd kind.

# GRAPHICAL AND NUMERICAL SOLUTIONS OF LINEAR AND NON-LINEAR BOUNDARY VALUE PROBLEMS IN DIFFERENTIAL EQUATIONS

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*MSC 2000*: 65S05 (65L05)

The different methods for solution linear and non-linear boundary value problems in differential equations, which cannot be expressed in analytical form, we can find in reference [1].

As known, two-point boundary value problems occur in all branches of engineering and science. In these problems the boundary conditions are specified at two points. To complicate the matter, the governing differential equations for a majority of such problems are nonlinear; since analytic solutions do not in general exist, solutions have to be such problems can be separated, or can be solved graphically.

The combinations of the graphical and numerical methods for solution of three problems: by the method of superposition, by the method of transformation and by the method of chasing will be considered.

On particular examples are graphically solved differential equations:

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} - k^2 \frac{\mathrm{d}y}{\mathrm{d}x} + a = 0, \qquad 1.$$

where $k^2$ and $a$ are physical constants which depend on the elastic properties of the lamina. For the free ends, the condition of zero shear bimoment at both ends leads to the boundary conditions

$$\frac{\mathrm{d}y(0)}{\mathrm{d}x} = \frac{\mathrm{d}y(1)}{\mathrm{d}x} = 0, \quad y(1/2) = 0. \frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}\left(x\frac{\mathrm{d}y}{\mathrm{d}x}\right) + 4N\frac{1}{x^4} = 0 \qquad 2.$$

subject to the boundary conditions

$$y(k) = 0, \quad y(1) = 1.$$

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} + \frac{q_S}{k} = 0, \quad \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 1 = 03.$$

subject to the boundary conditions

$$\frac{\mathrm{d}y(0)}{\mathrm{d}x} = 0, \quad -\frac{\mathrm{d}y(1)}{\mathrm{d}x} = N_{\mathrm{bi}} y(1),$$

where $N_{\mathrm{bi}}$ is the Biot number.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \beta y \qquad 4.$$

subject to the boundary conditions

$$y(0) = 1, \quad -\mathrm{d}y(1)/\mathrm{d}x = N_{\mathrm{bi}} y(1),$$

where $\beta = m^2L^2$, $N_{\rm bi} = hL/k$.

Our object is to extend the possibilities of these methods to solutions of problems comprising some physical parameter either in a differential equation or in boundary conditions and, what is most important, to establish a complex of solutions at a required interval of parameter values that may be represented graphically, or presented in the form of tables, according to the investigation purposes.

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# FLOQUET THEORY FOR ELLIPTIC PROBLEMS

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*MSC 2000*: 35P10 (47N20)

The periodic partial differential equations are both of theoretical and of practical importance and recently they attracted the attention of a number of researchers. The main goal of our work is to study periodic elliptic equations by technic and tools similar to classical Floquet theory for periodic ordinary differential equations.

We consider the following elliptic boundary problem of second order:

$$\begin{cases} Lu = L(x,D)u = \sum_{j,k=1}^{3} \frac{\partial}{\partial x_j}\left(a_{jk}(x)\frac{\partial u}{\partial x_k}\right) + a_0(x)u = 0 & (\Omega) \\ Tu = 0 & (\partial\Omega) \end{cases} \tag{1}$$

where

(i) $\Omega \subset \mathbb{R}^3$ is an infinite and 1-periodic with respect to $x_1$ ($\mathbb{R}^3 \ni x = (x_1, x_2, x_3)$) domain with infinitely smooth boundary, such that any section of $\Omega$ by plane orthogonal to $x_1$ is bounded,

(ii) All coefficients of $L$ and $T$ are infinitely smooth 1-periodic with respect to $x_1$ functions,

(iii) $L$ is elliptic operator with formal selfadjoint differential expression, that is $a_{jk}(x) = \overline{a_{kj}(x)}$ for all $x$ and $j,k = 1,2,3$ and $a_0(x)$ is a real valued function,

(iv) $T$ is boundary differential operator of the first order or the order zero, $T$ satisfies so-called normality condition, $T$ is "formal selfadjoint" with respect to Green's formula.

Similarly to o.d.e. case the *Floquet solution* of problem (1) is said to be a solution of the form

$$u(x_1,x_2,x_3) = e^{\mu x_1} \sum x_1^k v_k(x_1,x_2,x_3), \quad v_k(x_1+1,x_2,x_3) = v_k(x_1,x_2,x_3)\ \forall x,k$$

Functions of such a type play the same role in studying periodic partial differential problems as Floquet solutions for periodic o.d.e. and Bloch waves for Schrödinger equation. There are lot of properties of elliptic periodic problem which can be obtained by applying these functions. This is the reason for studying completeness and basisness properties of these functions and a distribution of non-zero complex numbers $\lambda = e^{\mu}$ (so-called *Floquet multipliers* of problem (1)).

We define the monodromy operator for general elliptic selfadjoint periodic problem as a shift operator on the space of solutions of this problem, similarly to the ordinary case. It is very difficult to deal with this operator since its definition is connected with solving of Cauchy problem for elliptic equation but we overcome this difficulty considering the elliptic boundary value problem instead of the Cauchy problem. In this way we reduce the spectral problem for the monodromy operator to the spectral problem for the quadratic operator pencil. Then we study this pencil by indefinite inner product technique. The most part of results obtained here concerns the symmetric case, that is the case when the domain of the problem has a plane of symmetry and all coefficients of the problem are even w.r.t. this plane.

The main goal of the next part of our work is the detail studying of the monodromy operator in the general case (without symmetry). First we prove that the monodromy operator is a closed operator with trivial kernel and a dense domain of definition. Then we establish the quasi-isometric property for the monodromy operator and obtain some useful corollaries concerning Floquet multipliers and Floquet solutions of the problem. Next, we study properties of Floquet multipliers (that is, spectral properties of the monodromy operator) more precisely. For this purpose we classify the multipliers. Such a classification is usual in the indefinite scalar product theory. This approach has been applied by M.G. Krein for periodic systems of linear differential equations (so, in a finite-dimensional case).

In the next (and last) part we deal with elliptic selfadjoint periodic problems with a parameter. We are interested in a motion of the multipliers of such problems. Behavior of multipliers under small perturbation is very important in various theoretical and applied problems, generally speaking it is important in all problems that lead to periodic partial differential equations. To describe the motion of the multipliers we use the results obtained in the previous part of our work.

# THE FILIPPOV'S TYPE THEOREM FOR INTEGRO-DIFFERENTIAL INCLUSIONS WITH THE HUKUHARA'S DERIVATIVE

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*MSC 2000*: 34C29, 34A60, 34K15

The aim of this paper is to present the existence theorem for integro-differential inclusions of the form:

$$D_hX(t) \in F\left(t, X_t, \int_0^t \Phi(t,s,X_s)\,\mathrm{d}s\right) \tag{1}$$

where $D_hX$ denotes the Hukuhara's derivative ([2]) of a multivalued mapping $X$, $X_t\colon \Theta \to X_t(\Theta) = X(t+\Theta)$ for $\Theta \in [-r,0]$ $(r>0)$; $F$ is a map from $[0,T]\times C_0\times \mathrm{conv}(\mathbb{R}^n)$ into $CC(\mathbb{R}^n)$, $\Phi$ is a map from $[0,T]\times[0,T]\times C_0$ into $\mathrm{conv}(\mathbb{R}^n)$, where $CC(\mathbb{R}^n)$ denotes the collection of all nonempty compact subset of the compact, convex subsets of Euclidean space $\mathbb{R}^n$ i.e. with $\mathrm{conv}\,\mathbb{R}^n$ and $C_0$ is a metric space of all continuous mapping from $[-r,0]$ into $\mathrm{conv}\,\mathbb{R}^n$.

We study the Filippov's type theorem for inclusion (1) with the initial conditions $X(t) = \Psi(t)$, where $\Psi(t)\colon [-r,0] \to \mathrm{conv}(\mathbb{R}^n)$ is a given absolutely continuous multifunction, where the multivalued mappings $F$ and $\Phi$ satisfy the following conditions:

$1^0$ $F$ is measurable for $t\in[0,T]$ and for fixed $(U,W)\in C_0\times\mathrm{conv}(\mathbb{R}^n)$
$2^0$ $F$ is Lipschitzean with respect to $(U,W)$
$3^0$ there exists a $M>0$ such that $d(F(t,U,W);\{0\}) \leqslant M$ for $(t,U,W)\in[0,T]\times C_0\times\mathrm{conv}\,\mathbb{R}^n$
$4^0$ $\Phi$ is continuous for $(t,s)\in[0,T]\times[0,T]$ and for fixed $U\in C_0$
$5^0$ $\Phi$ is Lipschitzean with respect to $U\in C_0$
$6^0$ there exists a $N>0$ such that $H(\Phi(t,s,U);\{0\}) \leqslant N$ for $(t,s,U)\in[0,T]\times[0,T]\times C_0$

By $d$ we will denote the distance between two collections $A,B\in CC(\mathbb{R}^n)$ and $H$ is the Hausdorff metric.

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# A POSTPROCESSING OF BIFURCATION POINTS

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The current state-of-the-art of the numerical bifurcation analysis of steady states, see [7], could be briefly described as follows: Going Down the hierarchy of bifurcation points, find an *organizing center.* This is a bifurcation point with the highest *codimension* locally available.

Going Up the hierarchy starts with a *postprocessing* of the already computed organizing center. It consists, for example, in finding tangents to the curves of codim$-1$ bifurcation points passing through the organizing center, see [7], Section 7.8.

In papers [10], [2], [3] and [9] a much more complex postprocessing analysis is proposed: The aim is to supply analytical predictors to *all* bifurcation points in a neighborhood of the processed organizing center. The technique is based on

- generalized version of Liapunov-Schmidt reduction algorithm (see [7])
- numerical treatment of the relevant *bifurcation equation*

The immediate motivation of the proposed analysis was The Theory for Imperfect Bifurcation, see [5] and [6]. This theory yields a *qualitative information* concerning the behavior of the bifurcation when the problem is subject to an arbitrary sufficiently small perturbation. In particular, there are analyzed *unfolded normal forms* of the bifurcation scenarios. These are considered to be models of the actual bifurcation problems under perturbations.

It can be shown that there exists a kind of an *unfolded contact diffeomorphism* between the actual bifurcation problem and a relevant unfolded normal form. Therefore, the actual solution set and the roots of the unfolded normal form are linked via a diffeomorphism $\Phi$. The differential of $\Phi$ computed at the organizing center yields a natural first-order predictors of all the imperfect bifurcation phenomena.

In [3] we gave algorithms for computing the differential of $\Phi$ for all organizing centers with codim $\leqslant 3$, corank $= 1$. In the case studies [10] and [2] we had verified that the idea really worked numerically. In [9], we attempted to apply our postprocessing analysis to symmetry-breaking bifurcation problems (so far, as a case study).

As far as the symmetry is concerned, computation of symmetry-breaking bifurcation points was proposed in [4] and [11]. Numerical treatment of higher codimension phenomena was suggested in [1]. In particular, the symmetry-breaking bifurcation points with nonlinear degeneracies were considered. A classification was also proposed. In fact, nonlinear degeneracies are characterized via a classification of a scalar bifurcation equation with $\mathbf{t}$-symmetry or $\mathbb{Z}_2$-symmetry, see [1]. We conclude that the postprocessing can be also restricted to the just mentioned symmetry classes.

As the overhead of the proposed analysis is concerned, the main cost represents a computation of the required partial derivatives of the Liapunov-Schmidt reduction.

But the majority of these derivatives comes out as a by-product of the computation of the particular organizing center via a Newton-like method, see [7], Section 7.4; see also the case study [2].

For another related topic namely computing parameter dependent center manifolds, see [12] and [8].

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# A SIMPLE EXAMPLE OF LATTICE DYNAMICAL SYSTEM WITH SPATIAL CHAOS

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*MSC 2000*: 37L60

Let $\mathbb{Z}$ be the set of all integers, the so called *lattice space* and $\mathbb{R}$ be the standard 1-dimensional real space. Let us consider the set

$$\mathbb{B} = \{\boldsymbol{u};\, \boldsymbol{u} = \{u_j\},\ j \in \mathbb{Z}\},$$

where $u_j \in \mathbb{R}$ for all $j \in \mathbb{Z}$, i.e. $\mathbb{B}$ is the set of all bounded two-sided sequences of real numbers. For $\boldsymbol{u} = \{u_j\}_{j\in\mathbb{Z}}$ we denote $(\boldsymbol{u})_j = u_j$.

Now, we define the mapping $\mathcal{F}\colon \mathbb{B} \to \mathbb{B}$ by the relation

$$(\mathcal{F}\boldsymbol{u})_j = f(u_{j-1}, u_j, u_{j+1}) = u_j + u_{j-1}^2 + u_j^2 + u_{j+1}^2 - 1, \qquad j \in \mathbb{Z}.$$

**Definition 1.** The fixed point $\boldsymbol{u}^*$ of the mapping $\mathcal{F}$, i.e. $\mathcal{F}\boldsymbol{u} = \boldsymbol{u}$, is called a *steady-state solution* of the LDS.

The $\boldsymbol{u} = \{u_j^*\}_{j\in\mathbb{Z}}$ is fixed point of $\mathcal{F}$ *iff* the following relation holds

$$u_j^* = f(u_{j-1}, u_j, u_{j+1}), \quad \text{i. e.} \quad u_{j-1}^2 + u_j^2 + u_{j+1}^2 - 1 = 0.$$

for all $j \in \mathbb{Z}$.

Let as denote $\mathbb{D}_0 = \{(x, y);\, x^2 + y^2 < 1\}$ and

$$\Sigma_2 = \{0,1\}^{\mathbb{Z}} = \{\boldsymbol{\omega} = (\ldots, \omega_{-1}, \omega_0, \omega_1, \ldots); \omega_j \in \{0,1\}, \quad \text{for all} \quad j \in \mathbb{Z}\}.$$

For every $(x, y) \in \mathbb{D}_0$ and $\boldsymbol{\omega} \in \Sigma_2$ we can construct inductively a steady-state solution $U(x, y, \boldsymbol{\omega}) = \{u_j\}_{j\in\mathbb{Z}}$ as follows:

a. Set $u_0 = (-1)^{\omega_0} x$ and $u_1 = (-1)^{\omega_1} y$.

b. For $j = 2, 3, \ldots$ set $u_j = (-1)^{\omega_j} \sqrt{1 - u_{j-1}^2 - u_{j-2}^2}$.

c. For $j = -1, -2, \ldots$ set $u_j = (-1)^{\omega_j} \sqrt{1 - u_{j+1}^2 - u_{j+2}^2}$.

Let us fix $(x_0, y_0) \in \mathbb{D}_0$, $x_0 > 0$, $y_0 > 0$. We denote

$$\tilde{\Lambda} = \tilde{\Lambda}(x_0, y_0) = \{U(x_0, y_0, \boldsymbol{\omega});\, \boldsymbol{\omega} \in \Sigma_2\}.$$

Let us fix $j_0 \in \mathbb{Z}$ and define the mapping $\mathcal{S}_{j_0} : \mathbb{B} \to \mathbb{B}$ by the relation

$$(\mathcal{S}_{j_0}\boldsymbol{u})_j = u_{j+j_0}, \quad \text{for all } j \in \mathbb{Z} \text{ and put } \Lambda = \bigcup_{n\in\mathbb{Z}} \mathcal{S}_1^n \tilde{\Lambda}.$$

Clearly, the set $\Lambda$ consists entirely of steady-state solutions and is closed invariant set of the TDS $(\mathcal{S}_1^n, \mathbb{B})$. We shall show, that the $\mathcal{S}_1|_\Lambda$ behaves stochastically, that means the $\mathcal{S}_1|_\Lambda$ has *positive topological entropy.* To this aim we shall use some results from theory of topological Markov chains (TMC), see e.g. [2].

Let us denote

$$z_0 = \sqrt{1 - x_0^2 - y_0^2}.$$

Then every element $\boldsymbol{u}^* \in \Lambda$, $\boldsymbol{u}^* = \{u_j^*\}_{j\in\mathbb{Z}}$, is two-sided sequence of *six* elements (or symbols) $x_0$, $y_0$, $z_0$, $-x_0$, $-y_0$, $-z_0$, numbered by turn 1, 2, 3, 4, 5, 6.

All admissible transitions between these symbols are determined by the following *transition matrix*:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

We remark that $\mathcal{S}_1|_\Lambda = \sigma_A$-*the subshift of finite type.* The matrix $\mathbf{A}$ has the following *eigenvalues*:

$$\lambda_{1,2,3} = 0; \;\; \lambda_4 = 2, \quad \lambda_{5,6} = -1 \pm i\sqrt{3}.$$

Since the *greatest positive* eigenvalue $\lambda(\mathbf{A})$ of the matrix $\mathbf{A}$ is equal to 2, the topological entropy of the mapping $\mathcal{S}_1|_\Lambda$ is (see [2])

$$h(\mathcal{S}_1|_\Lambda) = \ln \lambda(\mathbf{A}) = \ln 2 > 0.$$

Topological entropy of the mapping $\mathcal{S}_1|_\Lambda$ is *positive*, hence the *spatial chaos* in our LDS is realized, see [1].

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## NOTES TO THE FÖPPL-KÁRMÁN-MARGUERRE EQUATIONS

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*MSC 2000*: 73K10, 73K15

Let $H_i \subset H^2(\Omega)$, $\Omega \subset \mathbb{R}^2$, $i = 1, 2$. Let scalar products $(\cdot,\cdot)_i$ generate the norms, equivalent with the norm $H^2$. We consider about the following boundary value problems:

$(w, F) \in H_1 \times H_2$ is her weak solution, if

$$\begin{aligned} \forall\varphi \in H_1\colon \qquad & (w,\varphi)_1 = ([w,F],\varphi)_{L_2} + \lambda([F_0,w],\varphi)_{L_2} + \langle g_1,\varphi\rangle_1, \\ \forall\psi \in H_2\colon \qquad & (F,\psi)_2 = -\frac{1}{2}([w,w],\psi)_{L_2}, \end{aligned} \tag{1}$$

where $F_0 \in H^2$, $g_1 \in H_1'$, $[\xi,\eta] = \operatorname{div}(\xi_{x_1}\eta_{x_2x_2} - \xi_{x_2}\eta_{x_1x_2}, \xi_{x_2}\eta_{x_1x_1} - \xi_{x_1}\eta_{x_1x_2},)$.

Equations in form (1) describe geometrically nonlinear plates, webs and shells near different types of boundary conditions.

The system (1) is a certain generalisation of the equations of one step of the Kachanov method for system elasto-plastic plates with large deflections [1]. It is also a generalisation of the system proposed by Jevstratov [2] as iteration steps of the solution to an elasto-plastic problem. Also represent a generalisation of that system which under certain assumptions is employed by Jershow [3] for an investigation into the elasto-plastic flexure of plates and for materials from composites.

With the advances e.g. [4] the equations (1) maybe expressed in the form

$$\begin{aligned} w &= B_2(w,F) + \lambda L_1 w + g, \\ F &= -\frac{1}{2}B_1(w,w), \end{aligned} \tag{2}$$

where $L_1 \in L(H_1,H_1)$, $B_2 \in L(H_1 \times H_2, H_1)$, $B_1 \in L(H_1 \times H_1, H_2)$ are linear, continuous and compact mappings, $g \in H_1$ and $\lambda \in \mathbb{R}^1$.

We now formulate an iteration processes:

$$\begin{aligned} a)\ \widetilde{w}_0 = 0,\ w_{n+1} &= -\frac{1}{2}B_2(\widetilde{w}_n, B_1(\widetilde{w}_n,\widetilde{w}_n)) + \lambda L_1\widetilde{w}_n + g, \\ \widetilde{w}_{n+1} &= \widetilde{w}_n + \alpha_{n+1}(w_{n+1} - \widetilde{w}_n), \\ n &= 0,1,2,\dots \end{aligned} \tag{3}$$

$$\begin{aligned} & b)\ \widetilde{F}_0 = 0,\ \widetilde{w}_0 = g, \\ & F_{n+1} = -\frac{1}{2}B_1(\widetilde{w}_n,\widetilde{w}_n),\ w_{n+1} = B_2(\widetilde{w}_n,\widetilde{F}_n) + \lambda L_1\widetilde{w}_n + g, \\ & \widetilde{F}_{n+1} = \widetilde{F}_n + \beta_{n+1}(F_{n+1} - \widetilde{F}_n),\ \widetilde{w}_{n+1} = \widetilde{w}_n + \alpha_{n+1}(w_{n+1} - \widetilde{w}_n), \\ & n = 0,1,2,\dots \end{aligned} \tag{4}$$

**Theorem** [5]. *There exists constants* $\alpha_n$, $\beta_n \in (0,1)$, $C > 0$,

$$\max[(\alpha_{n+1}, \beta_{n+1}) / \min(\alpha_n, \beta_n)] \leqslant 1,$$

$\|g\|_1 \leqslant C$, *such that procedures a), b) converge in the norm* $H^2$ *to solution of the system* (1).

**Remark** 1. The linear boundary value problems in (3), (4) have the solutions. The solutions can be realized with variational methods and exist the aposteriori estimates for errors.

**Remark** 2. For circular plates with arbitrary axisymetric edge conditions and subjected to arbitrary axisymetric transverse londs, can be formulated convergence iterative procedures with replaced boundary value problems on integral equations [5].

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# ON A RELATION BETWEEN CARRIERS OF TWO LINEAR SECOND-ORDER DIFFERENTIAL EQUATIONS OF JACOBIAN FORM

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*MSC 2000*: 34A30

Let $q = q(t)$, $q(t) \neq 0$, $q \in C_2(\mathcal{J})$, and $Q = Q(t)$, $t \in \mathcal{J}$, $\mathcal{J} = (a, b)$, be carriers of linear second-order differential equations of Jacobian form

$$y'' = q(t)y, \tag{q}$$

$$Y'' = Q(t)Y, \tag{Q}$$

where $(Q)$ is the first associated equation to the differential equation $(q)$ [1].

It holds

$$Q = q(t) + \sqrt{|q(t)|}\Big(\frac{1}{\sqrt{|q(t)\|}}\Big)''. \tag{1}$$

Let us define a function $f = f(t)$ by the identity

$$Q(t) = \sqrt{|q(t)|}f\Big(\frac{1}{\sqrt{|q(t)|}}\Big) \tag{2}$$

and investigate if there are other pairs of carriers $(\hat{q}, \hat{Q})$ of the differential equations $(q)$ and $(Q)$ related by (2) with the function $f$ such that $(\hat{Q})$ is the first associated equation to the differential equation $(\hat{q})$. We will find out that there are such pairs as stated in the theorem mentioned below.

If we set $\sqrt{|q(t)|} = z(t)$ in (1) we get a differential equation for $z$

$$z'' = Q(t)z - \frac{1}{z} \tag{4}$$

and the condition (2) is of the shape

$$Q(t)z = f(t) \tag{5}.$$

Thus the function defined by (3) is a partial solution of the differential equation (4). Excluding the function $Q$ from (4) and (5) we obtain a differential equation for $z = z(t)$

$$z'' = f(z) - \frac{1}{z} \tag{6}.$$

Thus if $z = z(t)$ is a solution of the differential equation (4) which satisfies (5) then $z = z(t)$ is a solution of the differential equation (6) and on the contrary, if $z = z(t)$ is a solution of the differential equation (6) satisfying (5) then $z = z(t)$ is a solution of the differential equation (4).

The differential equation (6) we multiply by $2z'(\neq 0)$ and integrate and the obtained equation leads to a first-order separable differential equation

$$h'(z)\,\mathrm{d}z = \pm\,\mathrm{d}t, \tag{7}$$

where $h'(z) = 1/\sqrt{2F(z) - 2\ln|z| + c_0}$ and $F$ is an antiderivative of $f$. The differential equation (7) yields $h[z(t)] = \pm t + c$ and from here

$$z(t) = h^*(\pm t + c),$$

where $h^*$ denotes an inverse function to the function $h$.

**Theorem.** *Let the differential equation*

$$y'' = \hat{q}(t)y \tag{$\hat{q}$}$$

*have the carrier $\hat{q}(t)$ given by the equation*

$$\hat{q}(t) = [h^*(\pm t + c)]^{-2}. \tag{8}$$

*Then the carrier of its associated equation is*

$$\hat{Q}(t) = \frac{1}{h^*(\pm t + c)} f[h^*(\pm t + c)]. \tag{9}$$

By a calculation we can verify that the carrier $\hat{Q}(t)$ given by (9) satisfies

$$\hat{Q} = \hat{q}(t) + \sqrt{|\hat{q}(t)|}\Big(\frac{1}{\sqrt{|\hat{q}(t)|}}\Big)''.$$

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# CONTINUATION OF ATOMIC OPERATORS

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*MSC 2000*: 47B38, 47A67, 34K05

Let $(\Omega_1, \Sigma_1, \mu_1)$ and $(\Omega_2, \Sigma_2, \mu_2)$ be two measure spaces, and $\Sigma_1^0 \subset \Sigma_1$, $\Sigma_2^0 \subset \Sigma_2$ be the $\sigma$-ideals of $\mu_1$- and $\mu_2$-nullsets respectrively. We denote by $\tilde{\Sigma}_i := \Sigma_i/\Sigma_i^0$, $i = 1, 2$ the respective *measure algebrae*. Further on we will frequently abuse the notation and identify the elements of the measure algebrae $\tilde{\Sigma}_i$ (i.e. the equivalence classes of sets) with the elements of the respective original $\sigma$-algebrae of sets $\Sigma_i$. A map $F : \tilde{\Sigma}_1 \to \tilde{\Sigma}_2$ is called a *$\sigma$-homomorphism*, if $F(\Omega_1) = \Omega_2$, $F(\Omega_1 \setminus e) = \Omega_2 \setminus F(e)$ whenever $e \in \tilde{\Sigma}_1$ and

$$F\left(\bigsqcup_{i=1}^{\infty} e_i\right) = \bigsqcup_{i=1}^{\infty} F(e_i),$$

for any pairwise disjoint collection of $\{e_i\}_{i=1}^{\infty} \subset \tilde{\Sigma}_1$, where $\sqcup$ stands for the disjoint union. Every $(\Sigma_2, \Sigma_1)$-measurable map $g : \Omega_2 \to \Omega_1$ satisfying

$$\mu_2(g^{-1}(e_1)) = 0 \quad \text{when } \mu_1(e_1) = 0 \tag{1}$$

generates a $\sigma$-homomorphism according to the formula $F(e_1) := g^{-1}(e_1)$.

All the measure spaces we will be dealing with in the sequel are assumed to be complete.

Let $X_i := L^0(\Omega_i, \Sigma_i; \mathcal{X}_i)$, $i = 1, 2$.

**Definition 1.** An operator $T : X_1 \to X_2$ is called **atomic** with respect to the $\sigma$-homomorphism $F : \tilde{\Sigma}_1 \to \tilde{\Sigma}_2$, if $T(x)\big|_{F(e_1)} = T(y)\big|_{F(e_1)}$ whenever $x|_{e_1} = y|_{e_1}$ for $x, y \in X_1$.

For instance, the local operator is just the operator atomic with respect to the identity $\sigma$-homomorphism. We will omit the reference to the $\sigma$-homomorphism, if it is unnecessary.

Assume now that $\Sigma_1 \subset \Sigma_1'$, where $\Sigma_1'$ is a larger $\sigma$-algebra of subsets of $\Omega$. We would like to know under which condition every atomic operator $T : X_1 \to X_2$ can be extended to an atomic operator $T' : X_1' \to X_2$, where $X_1' := L^0(\Omega_1, \Sigma_1'; \mathcal{X}_1)$.

**Definition 2.** Let $\Sigma_1 \subset \Sigma_1'$ be $\sigma$-algebrae of subsets of $\Omega_1$. Then $\Sigma_1$ is said to satisfy $\Omega$-condition with respect to $\Sigma_1'$ (written $\Sigma_1 \in \Omega(\Sigma_1')$), if there is an at most countable cover of $\Omega_1$ by pairwise disjoint sets $\Omega_1 = \sqcup_j \Omega_1^j$, $\Omega_1^j \in \Sigma_1'$, such that for each $j \in \mathbb{N}$ one has $\Sigma_1 \cap \Omega_1^j = \Sigma_1' \cap \Omega_1^j$.

One has then the following simple result.

**Theorem 1.** *Let $T : X_1 \to X_2$ be atomic with respect to the $\sigma$-homomorphism $F : \tilde{\Sigma}_1 \to \tilde{\Sigma}_2$, while $\Sigma_1 \in \Omega(\Sigma_1')$ and $F$ admits an extension to a $\sigma$-homomorphism*

*$F'\colon \tilde{\Sigma}_1' \to \tilde{\Sigma}_2$. Then the operator $T$ admits the unique extension to the operator $T'\colon X_1' \to X_2$, which is atomic with respect to $F'$. Moreover, the extended operator $T'$ is continuous (in measure) whenever so is $T$.*

# ON A CONTROL PROBLEM FOR A CIRCULAR PLATE WITH A FINITE NUMBER OF RIGID OBSTACLES

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*MSC 2000*: 49J20, 49J40, 74K20

We deal with an elastic circular plate of a variable thickness fixed on an elastic unilateral foundation and axisymmetrically supported by finite number of rigid obstacles. The plate is subjected to an axially symmetrical load. The thickness of the plate and the stiffness characteristics of the foundation play the role of control variables. The state problem is modelled by a variational inequality. The convex set of admissible states is a subset of a weighted Sobolev space (see [3] for details).

We assume that the plate occupies the region

$$\begin{aligned} Q &= \{[\bar{x}, z] \in \mathbb{R}^3 : \bar{x} \in \Omega,\ -\tfrac{1}{2}H(|x|) < z < \tfrac{1}{2}H(|x|)\},\\ \Omega &= \{\bar{x} \in \mathbb{R}^2 : |\bar{x}| < a\}. \end{aligned}$$

It is simply supported along its boundary. Assuming the axial symmetry the deflections belong to the weighted space

$$V := \Big\{v \in L^2_r(0,a) \colon \frac{1}{r}v'_r,\ v''_{rr} \in L^2_r(0,a),\ v(a) = 0\Big\}.$$

Considering the radially symmetric inner obstacles, the closed convex set of admissible deflections has the form

$$K(H) = \{v \in V \colon v(r) \geqslant \tfrac{1}{2}H(r) - \Theta_i,\ 0 < a_i < r < b_i < a,\ i = 1, \ldots, N\}.$$

The unilateral elastic foundation of Winkler type (see [2]) is considered on the rest of the middle surface. We introduce the form

$$\begin{aligned} a(e; u, v) := {}& \frac{E}{12(1-\nu^2)} \int_0^a rH^3 \Big(w''_{rr} + \frac{1}{r}u'_r\Big)\Big(v''_{rr} + \frac{1}{r}v'_r\Big)\,\mathrm{d}r\\ &+ \sum_{i=0}^{N} \int_{b_i}^{a_{i+1}} rZ[u]^- v\,\mathrm{d}r,\ b_0 = 0,\ a_N = a. \end{aligned}$$

We assume further the following sets of admissible thicknesses and the reaction forces

$$\begin{aligned} U^1_{ad} &= \{H \in C^{(0),1}[0,a] \colon 0 < H_{\min} \leqslant H(r) \leqslant H_{\max},\ |H'_r| \leqslant C_1\},\\ U^2_{ad} &= \{Z \in C^{(0),1}([0,a] \setminus I) \colon 0 \leqslant Z(r) \leqslant Z_{\max},\ |H'_r| \leqslant C_2\},\ I = \bigcup_{i=1}^{N} (a_i, b_i). \end{aligned}$$

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Next we define a linear functional by

$$\langle L(H), v\rangle = \sum_{i=1}^{M} F_i R_i v(R_i) + \int_0^a r(p - 2\omega H)v \,\mathrm{d}r,\ 0 < R_i < a,\ i = 1, \ldots, M,$$

where $F_i \in \mathbb{R}$ describe the forces concentrated on the circles with centers at origin and radius $R_i$, $p \in L_1(0,a)$ is the distributed load and $\omega$ is the constant material density. We set $U_{\text{ad}} = U^1_{\text{ad}} \times U^2_{ad}$. The deflection $u(e)$ is a solution of a variational inequality

$$u(e) \in K(H)\colon a(e; u(e), v - u(e) \geqslant \langle L(H), v - u(e)\rangle\ \forall v \in K(H). \tag{1}$$

The existence and the uniqueness of a solution ca be verified using standard methods. Let us introduce following cost functionals

$$\mathcal{L}_1(v) = \int_0^a r(v - z_{ad})^2,\ z_{ad} \in L^2_r(0,a),$$
$$\mathcal{L}_2(e,v) = \int_0^a r[a(e; v, \Theta) + (Z_*[v]^- p - 2\omega H)\Theta]\,\mathrm{d}r.$$

The last functional represents a resultant of transverse reactive forces on the inner obstacles, $\Theta \in H^1_r(0,a)$ is such that $\Theta = 1$ on $I$ and $Z_*$ is the extension of $Z \in U^2_{\text{ad}}$ by zero on $I$.

**Optimal Control Problems.** To find $e_i \in U_{ad}$, $i = 1, 2$ such that

$$e_1^* = \operatorname{Arg} \min_{e \in U_{ad}} \mathcal{L}_1(u(e)), \tag{2}$$
$$e_2^* = \operatorname{Arg} \min_{e \in U_{ad}} \mathcal{L}_2(e, u(e)), \tag{3}$$

where $u(e)$ denotes the solution of the state problem (1).

Applying the approach similar to [1] there can be verified

**Theorem 1.** *There exist solutions $e_1^*$ and $e_2^*$ of the Optimal Control Problems* (2) *and* (3) *respectively.*

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# METHOD OF AVERAGING FOR INTEGRAL-DIFFERENTIAL INCLUSIONS WITH THE HUKUHARA'S DERIVATIVE

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*MSC 2000*: 34C29, 34A60, 34K15

In this paper presents the theorem of middling for integral-differential equation with Hukuhara's derivative of the form

$$D_h X(t) \in \varepsilon F\left(t, X_t, \int_0^t \Phi(t, s, X_s)\,\mathrm{d}s\right) \tag{1}$$

with the initial conditions $X(t) = \Psi(t)$ for $t \in [-r, 0]$ where $\varepsilon > 0$ is a small parameter $\Psi\colon [-r, 0] \to \operatorname{conv}(\mathbb{R}^n)$ is given absolutely continuous function.

Let us denote by $(\operatorname{conv}(\mathbb{R}^n), H)$ and $(CC(\mathbb{R}^n), d)$ the metric space all nonempty compact, convex subsets of $n$-dimensional Euclidean space $\mathbb{R}^n$ with the Hausdorff metric $H$ and space all nonempty compact subsets of $\operatorname{conv}(\mathbb{R}^n)$ with the metric $d$, respectively.

Let's assume that the multivalued mappings $F\colon [0,\infty) \times C_0 \times \operatorname{conv}(\mathbb{R}^n) \to CC(\mathbb{R}^n)$ and $\Phi\colon [0,\infty) \times [0,\infty) \times C_0 \to \operatorname{conv}(\mathbb{R}^n)$ satisfy the following conditions:

$1^0$ $F$ is measurable for $t \in [0,\infty)$ and for fixed $(U, W) \in C_0 \times \operatorname{conv}(\mathbb{R}^n)$

$2^0$ $F$ is Lipschitzean with respect to $(U, W)$

$3^0$ there exists a $M > 0$ such that $d(F(t, U, W); \{0\}) \leqslant M$ for $(t, U, W) \times [0,\infty) \times C_0 \times \operatorname{conv}\mathbb{R}^n$

$4^0$ $\Phi$ is continuous for $(t, s) \in [0,\infty) \times [0,\infty)$ and for fixed $U \in C_0$

$5^0$ $\Phi$ is Lipschitzean with respect to $U \in C_0$

$6^0$ there exists a $N > 0$ such that $H(\Phi(t, s, U); \{0\}) \leqslant N$ for $(t, s, U) \in [0,\infty) \times [0,\infty) \times C_0$

$7^0$ there exists a limit

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \mathrm{F}\left(t, X_t, \Phi_1(t, X_t)\right) dt = \overline{F}(X_t)$$

where

$$\Phi_1(t, X_t) = \int_0^T \Phi_1(t, s, X_s)\,\mathrm{d}s.$$

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# ON SOME GENERALIZATION OF DIFFERENTIAL EQUATIONS FOR HERMITE FUNCTIONS

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*MSC 2000*: 33C25

As it is well-known the classical Hermite polynomials $\{H_n(x)\}_{n=0}^{\infty}$ are the polynomials orthogonal in the interval $(-\infty,\infty)$ with respect to the weight function $H(x)=\mathrm{e}^{-x^2}$ (cf. [1–3]). The Hermite polynomial $H_n(x)$ of the $n$th degree satisfies the differential equation

$$y''-2xy'+2ny=0.$$

The Hermite function

$$h_n(x)=H_n(x)\mathrm{e}^{-x^2/2}$$

satisfies the differential equation

$$y''+(2n+1-x^2)y=0.$$

In this paper we consider the differential equations

$$y''+[2n+1-x^2+a_n(x)]y=0, \tag{1}$$

where $\{a_n(x)\}_{n=0}^{\infty}$ is the sequence of functions which have the first derivatives $\{a_n'(x)\}_{n=0}^{\infty}$ in $(-\infty,\infty)$ and the following properties:

i) $|a_n(x)|<c_1[(1+x^2)^{-k_1}n^{\frac{1}{2}}+(1+x^2)^{-1}n^{k_2}]$, where $k_1>1$, $k_2<\frac{1}{2}$ and $c_1$ is a constant,

ii) $|a_n'(x)|<c_2[(1+x^2)^{-k_3}n^{\frac{1}{2}}+(1+x^2)^{-k_4}n^{k_2}]$, where $k_1<k_3\leqslant k_1+1$, $1<k_4\leqslant 2$ and $c_2$ is a constant.

Let $\mathcal{A}$ be the set of all sequences $\{a_n(x)\}_{n=0}^{\infty}$ fulfiling the above properties in the interval $-(\infty,\infty)$. Let $\{Q_n(x)\}_{n=0}^{\infty}$be the system of polynomials orthogonal in the interval $(-\infty,\infty)$ with respect to the function $Q(x)$ such that there exists $\{a_n(x)\}_{n=0}^{\infty}\in\mathcal{A}$so that for the functions

$$q_n(x)=Q_n(x)\sqrt{Q(x)},\quad n=0,1,2,\dots$$

the differential equations (1) hold.

Further, we introduce the set $\mathcal{M}$ of those systems of orthogonal polynomials $\{Q_n(x)\}_{n=0}^{\infty}$ for which $\{a_n(x)\}_{n=0}^{\infty}\in\mathcal{A}$ exist such that corresponding functions $\{q_n(x)\}_{n=0}^{\infty}$ satisfy the differential equations (1).

The set $\mathcal{M}$ is not empty, e.g. Korous polynomials investigated in [2, 4] belong to $\mathcal{M}$. Our polynomials $\{Q_n(x)\}_{n=0}^{\infty}$ generalize the Hermite polynomials and the Korous polynomials. We give the Theorem on their zeros.

**Lemma.** *Every function* $b_n(x) = 2n + 1 - x^2 + a_n(x)$, $n = 0, 1, 2, \ldots$, *where* $a_n(x)$ *fulfils the properties* i) *and* ii), *has two and only two real zeros* $b_1^{(n)}$ *and* $b_2^{(n)}$. *If* $b_1^{(n)} < b_2^{(n)}$, *then*

$$b_n(x) > 0 \ for \ x \in (b_1^{(n)}, b_2^{(n)})$$

*and*

$$b_n(x) < 0 \ for \ x \in [(-\infty, \infty) - (b_1^{(n)}, b_2^{(n)})].$$

*Further,*

$$b_1^{(n)} = -\sqrt{2n+1} + O(n^{-\frac{1}{2}})$$

*and*

$$b_2^{(n)} = \sqrt{2n+1} + O(n^{-\frac{1}{2}}).$$

**Theorem.** *Let* $\{x_k^{(n)}\}_{k=1}^{n}$ *be an increasing sequence of all real zeros of the polynomial* $Q_n(x)$. *Then*

$$(x_1^{(n)}, x_n^{(n)}) \subset (b_1^{(n)}, b_2^{(n)}).$$

**Acknowledgment.** This work is devoted to professor Josef Korous whose 95th anniversary of his birthday was in february 2001.

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## ASYMPTOTICS OF EIGENFUNCTIONS FOR A WEAKLY COUPLED SCHRÖDINGER OPERATOR

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It is well-known that the Schrödinger equation $(-\Delta + U)\psi = E\psi$ in the case when $U$ describes a shallow potential well (i.e., $U = \varepsilon V(x)$, $V(x) \in C_0^\infty(\mathbb{R}^n)$, $\varepsilon \to 0$) has exactly one eigenvalue $E_0 = -\beta^2$, $\beta \in \mathbb{R}$, below the essential spectrum $[0, \infty)$ in the case when $\int_{\mathbb{R}^n} V(x)\,\mathrm{d}x \leqslant 0$ and the dimension $n$ of the configuration space is 1 or 2. This was established for $n = 1$ and in the radially symmetric case for $n = 2$ already in the famous textbook of Landau & Lifshitz [1] and later was demonstrated in the general case in dimension 2 by Simon [2]. The methods used by those authors are quite different and consist, in brief, in the following. Landau & Lifshitz construct the asymptotics of the eigenfunction in the domains where $V \equiv 0$ and $V \not\equiv 0$ separately and then glue then together; thus, the asymptotics of the eigenfunction is nonuniform and the method *per se* is applicable only in the radially symmetric case for $n = 2$. The asymptotics of the eigenvalues is obtained from the gluing conditions. On the other hand, Simon reduces the problem to an equation for the eigenvalues (secular equation) which he solves by means of a Taylor expansion using the implicit function theorem; thus in his approach the asymptotics of the eigenfunction does not appear at all. Moreover, Simon's method is by no means trivial because it uses, for example, the theory of nuclear operators.

Our goal here is to construct a uniform asymptotics of the eigenfunction in this situation assuming that $\|\psi\| = O(1)$ as $\varepsilon \to 0$ (the norm is that of $L_2(\mathbb{R})$). It turns out that this construction is completely elementary when one passes to the momentum representation. More exactly, we prove the following theorem. Denote $\tilde{V}(p) = (2\pi)^{-n/2} \int_{\mathbb{R}^n} \mathrm{e}^{-ipx} V(x)\,\mathrm{d}x$.

**Theorem.** (i) $n = 1$. 1) *Let* $\int_{\mathbb{R}} V(x)\,\mathrm{d}x < 0$. *Then*

$$\psi_0 = \beta_0^{3/2} \int_{\mathbb{R}} \mathrm{e}^{ipx} \frac{A_0(p)}{p^2 + \beta_0^2}\,\mathrm{d}p,$$

*where* $\beta_0 = -\varepsilon\sqrt{\pi/2}\tilde{V}(0)$, $A_0(p) = \tilde{V}(p)$, *is the asymptotics of the eigenfunction belonging to the eigenvalue* $E = -\beta_0^2(1 + O(\varepsilon))$, *i.e.* $\|\psi - \psi_0\| = o(1), \|\psi_0\| = O(1)$ *as* $\varepsilon \to 0$.

2) *Let* $\int_{\mathbb{R}} V(x)\,\mathrm{d}x = 0$. *Then*

$$\psi_0 = \mu_0 \int_{\mathbb{R}} \mathrm{e}^{ipx} \frac{A_0(p) + \varepsilon A_1(p)}{p^2 + (\beta_0 + \varepsilon\beta_1)^2}\,\mathrm{d}p,$$

*where* $\beta_0 = \varepsilon^2/2 \int_{\mathbb{R}} |V(t)|^2/t^2\,\mathrm{d}t$, $\beta_1 = \varepsilon^2 B$, $B = -\frac{1}{2\sqrt{2\pi}} \int_{\mathbb{R}^2} \frac{\tilde{V}(t-s)\tilde{V}(s)\tilde{V}(-t)}{t^2 s^2}\,\mathrm{d}t\,\mathrm{d}s$, $A_0(p) = \tilde{V}(p)$, $A_1(p) = -B\tilde{V}(p) - i\sqrt{\pi/2}\tilde{V}(p)\tilde{V}'(0) - \sqrt{2/\pi}\int_{\mathbb{R}+i} \tilde{V}(p-t)\tilde{V}(t)/t^2\,\mathrm{d}t$,

* Supported in part by CONACYT, Mexico

*is the asymptotics of the eigenfunction belonging to the eigenvalue* $E = -\beta_0^2(1+O(\varepsilon))$ *in the above sense.*

(ii) $n = 2$. 1) *Let* $\int_{\mathbb{R}^2} V(x)\,\mathrm{d}x < 0$. *Then*

$$\psi_0 = \beta_0 \int_{\mathbb{R}^2} \mathrm{e}^{ipx} \frac{A_0(p)}{p^2+\beta_0^2}\,\mathrm{d}p,$$

*where* $\beta_0 = C\exp(1/(\varepsilon\tilde{V}(0)))$, $A_0(p) = \tilde{V}(p)$, *and* $C = \exp\left\{\frac{1}{2\pi\tilde{V}(0)} \fint \frac{\tilde{V}(p)\tilde{V}(-p)}{p^2}\,\mathrm{d}p\right\}$, *is the asymptotics of the eigenfunction belonging to the eigenvalue* $E = -\beta_0^2(1+O(\varepsilon))$ *in the above sense. Here* $\fint f(p)/p^2 \equiv \int_{|p|<1} f(p) - f(0)/p^2\,\mathrm{d}p + \int_{|p|>1} f(p)/p^2\,\mathrm{d}p$.

2) *Let* $\int_{\mathbb{R}^2} V(x)\,\mathrm{d}x = 0$. *Then*

$$\psi_0 = \frac{\beta_0}{\varepsilon} \int_{\mathbb{R}^2} \mathrm{e}^{ipx} \frac{A_0(p)+\varepsilon A_1(p)}{p^2+\beta_0^2}\,\mathrm{d}p,$$

*where*

$$\beta_0 = C\exp\left\{\frac{1}{\alpha_1\varepsilon^2} - \frac{\alpha_2}{\varepsilon\alpha_1^2}\right\}, \quad \alpha_1 = -\frac{1}{2\pi}\int_{\mathbb{R}^2} \frac{|\tilde{V}(p)|^2}{p^2}\,\mathrm{d}p,$$

$$\alpha_2 = \frac{\alpha_1}{2\pi}\int_{\mathbb{R}^4} \frac{\tilde{V}(-p)\tilde{V}(p-t)\tilde{V}(t)}{p^2t^2}\,\mathrm{d}p\,\mathrm{d}t, \quad A_0(p) = \tilde{V}(p),$$

$$A_1(p) = -\alpha_2 A_0(p) - \frac{\alpha_1}{2\pi}\fint_{\mathbb{R}^2} \frac{\tilde{V}(p-t)A_0(t)}{p^2}\,\mathrm{d}t, \quad C = \exp\left\{\frac{\alpha_2^2}{\alpha_1^3}\right\},$$

*is the asymptotics of the eigenfunction belonging to the eigenvalue* $E = -\beta_0^2(1+O(\varepsilon))$ *in the above sense.*

**Remarks**. 1. It is possible to construct corrections of any order to the asymptotic eigenfunction $\psi_0$. In fact, the proof of the theorem consists exactly in an explicit construction of these corrections using the same representation for $\psi_0$ with $A_0$ and $\beta_0$ changed to the corresponding expansions. The theorem on closeness of formal asymptotics to the exact solution [3] provides the final step of the proof.

2. The (strange at the first sight) presence of the correction $\varepsilon A_1$ in the formula for $\psi_0$ in the cases (i) 2) and (ii) 2) is due to the fact that $A_0(0) = 0$, $A_1(0) = 1$; and thus the $L_2$-norm of the "correction" turns out to be even greater than that of the leading term.

3. We note that in Simon's paper [2] the asymptotics of the eigenvalue is calculated in the case $n = 2$ only up to the constant $C$ (case (ii) 1) and up to $C$ and $\alpha_{1,2}$ (case (ii) 2).

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# ASYMPTOTIC BEHAVIOUR OF THE SOLUTIONS OF SOME DISCRETE VOLTERRA EQUATIONS

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We consider the perturbed system of Volterra equations

$$x(n) = f(n) + \sum_{s=0}^{n-1} K(n,s)[x(s) + y(s, x(s))] \tag{1}$$

where $f$, $g$ are given vectors and $K$ is a given $k \times k$ matrix; the unperturbed system is the linear system

$$y(n) = f(n) + \sum_{s=0}^{n-1} K(n,s) y(s). \tag{2}$$

The purpose of this note is to give sufficient conditions for the asymptotic equivalence of (1) and (2) on $N$.

## ON NON-LOCAL BITSADZE PROBLEM FOR SINGULAR ELLIPTIC EQUATIONS

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The following problem is considered:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{k}{y}\frac{\partial u}{\partial y} = 0; \quad x^2 + y^2 < 1,\ y > 0 \tag{1}$$

$$u\Big|_{\varrho=1} - u\Big|_{\varrho=\delta} = f(\varphi); \quad \varphi \in [0, \pi] \tag{2}$$

$$\frac{\partial u}{\partial y}\Big|_{y=0} = 0, \tag{3}$$

where $\varrho$, $\varphi$—are the polar co-ordinates, parameters $k$ and $\delta$ belong to $[1, +\infty), (0, 1)$ correspondingly.

The regular prototype of the above problem is the well-known non-local Bitsadze problem for Laplace equation in the disk (see e.g. [1]). Singularities of the considered type (i.e. Bessel operator with respect to a selected variable) arise in models of mathematical physics with degenerative space heterogeneities (see e.g. [3]).

We establish the necessary and sufficient condition of (classical) solvability for (1)–(3):

$$\int_0^{\pi} f_0(\varphi)\, \mathrm{d}\varphi = 0.$$

We also find the espansion of the solution of (1)–(3) with respect to Jacobi polynomials.

Note that the maximum principle for the considered kind of equations is already proved (see e.g. [2]); this immediately yields (as in the regular case) the uniqueness of the solution.

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# ON TWO-SCALE CONVERGENCE

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*MSC 2000*: 35B27

We shall deal with two-scale convergence. The notion is motivated by the following problems. Let $f_\varepsilon(x)$ be a sequence of periodic functions $f_\varepsilon(x) = f(x/\varepsilon)$. As $\varepsilon \to 0$, the sequence $\{f_\varepsilon\}$ converges weakly in $L^2(\Omega)$ to a constant, e.g. $f_\varepsilon(x) = \sin(x/\varepsilon) \rightharpoonup 0$ in $L^2(I)$. We see that the information about periodic behaviour of original functions is lost. Further, weak convergence lacks some useful properties: $u_\varepsilon \rightharpoonup u$, $v_\varepsilon \rightharpoonup v$ does not imply $u_\varepsilon v_\varepsilon \rightharpoonup uv$. Similarly, $u_\varepsilon \rightharpoonup u \Rightarrow 1/u_\varepsilon \rightharpoonup 1/u$ does not hold etc. However, such situations occur in proofs. A new convergence called *two-scale* solves this problem. Particularly it is very useful tool in homogenization theory.

There are two ways to two-scale convergence. The first was introduced by G. Nguetseng [4] and then expanded by G. Allaire [1]. Another approach based on a two-scale transform is worked out J. Casado-Díaz [3], who followed T. Arbogast's idea [2].

Let $\Omega$ be a bounded domain in $\mathbb{R}^N$ $(N \geqslant 1)$ and $Y = \langle 0;1\rangle^N$ be the unit closed cube. The symbol # denotes functions periodic in variable $y$ with period $Y$. Let us begin with Allaire's definition.

**Definition 1.** We say, that a sequence of functions $u_\varepsilon(x)$ two-scale converges to a limit $u_0(x,y)$, if the relation

$$\lim_{\varepsilon\to 0}\int_\Omega u_\varepsilon(x)\,\psi\left(x,\frac{x}{\varepsilon}\right)\,\mathrm{d}x = \int_\Omega\int_Y u_0(x,y)\psi(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

holds for any function $\psi(x,y) \in D[\Omega; C^\infty_\#(Y)]$.

This new definition of convergence is useful due to the following compactness theorem.

**Theorem.** *Each bounded sequence $\{u_\varepsilon\}$ in $L^2(\Omega)$ contains a subsequence two-scale converging to a limit $u_0(x,y) \in L^2(\Omega\times Y)$.*

The two-scale limits are of double number of variables. Thanks to variable $y$ the information about periodic behaviour is kept.

The second approach consists in defining a change of variables which transforms a sequence of one variable functions $u_\varepsilon\colon \mathbb{R}^N \to \mathbb{R}$ into a sequence of two variables functions $\hat{u}_\varepsilon\colon \mathbb{R}^N\times Y \to \mathbb{R}$, where $Y = (-1/2;1/2)^N$. The change is

$$\hat{u}_\varepsilon(x,y) = u_\varepsilon\left(\varepsilon k\left(\frac{x}{\varepsilon}\right) + \varepsilon y\right),$$

where $k(x/\varepsilon)$ denotes the round of $x/\varepsilon$ for almost every $x$. This implies that in any cube $C^k_\varepsilon = (\varepsilon(k-1/2);\varepsilon(k+1/2))^N$ the function $\hat{u}_\varepsilon(x,y)$ is constant in $x$. As the function of $y$ it is a transformed function from $u_\varepsilon(x)$ by the change of variables $y = x/\varepsilon - k$. Thus, behaviour of function $\hat{u}_\varepsilon(x,y)$ is the "same" to the original function $u_\varepsilon$, but it is described by two variables.

**Definition 2.** We say that a sequence $\{u_\varepsilon(x)\}$ two-scale converges to a function $u_0(x,y)$, if

$$\lim_{\varepsilon\to 0} \int_\Omega \int_Y \hat{u}_\varepsilon(x,y)\,\psi(x,y)\,\mathrm{d}x = \int_\Omega \int_Y u_0(x,y)\psi(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

for any function $\psi(x,y) \in D[\Omega, C^\infty_\#(Y)]$.

This second definition leads to the same results as the more usual Allaire's presentation, since it can be proved

**Theorem.** *Let $\psi(x,y) \in D[\Omega; C^\infty_\#(Y)]$. Then, we have*

$$\int_\Omega u_\varepsilon(x)\psi\left(x, \frac{x}{\varepsilon}\right)\,\mathrm{d}x = \int_{\Omega\times Y} \hat{u}_\varepsilon(x,y)\psi(x,y)\,\mathrm{d}x\,\mathrm{d}y + O_\varepsilon.$$

*where $O_\varepsilon$ is the sequence which converges to zero when $\varepsilon \to 0$.*

According to this property, the compactness theorem follows by weak compactness of bounded sequences in $L^2(\Omega \times Y)$. Moreover, the second definition is more general, since we can describe more difficult problems like homogenization of porous media, where two or more periodic ratios appear [3].

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# OSCILLATORY BEHAVIOR IN LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS OF NEUTRAL TYPE

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The aim of this note is to obtain explicit oscillation criteria for the linear functional differential system of neutral type

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t)-\int_{-1}^{0}x(t-\tau(\theta))\mathrm{d}\left[\nu(\theta)\right]\right]=\int_{-1}^{0}x(t-r(\theta))\mathrm{d}\left[\eta(\theta)\right], \tag{1}$$

where $x(t)\in\mathbb{R}^n$, the $\nu(\theta)$ and $\eta(\theta)$ are real $n$-by-$n$ matrix valued functions of bounded variation on $[-1,0]$, and the $\tau(\theta)$ and $r(\theta)$ are real positive continuous functions on $[-1,0]$.

Considering the value $R=\max\{\|\tau\|,\|r\|\}$, where $\|\tau\|=\max\{\tau(\theta)\colon -1\leqslant\theta\leqslant 0\}$ and $\|r\|=\max\{r(\theta)\colon -1\leqslant\theta\leqslant 0\}$, by a solution of (1) we mean a continuous function $x\colon [-R,+\infty[\,\to\mathbb{R}$, such that $x(t)-\int_{-1}^{0}x(t-\tau(\theta))\mathrm{d}\left[\nu(\theta)\right]$ is differentiable and (1) is satisfied for every $t\geqslant 0$. A solution of (1), $x(t)=\left[x_1(t),\dots,x_n(t)\right]^T$, is said to be *oscillatory* if every component, $x_i(t)$, $i=1,\dots,n$, has arbitrary large zeros; otherwise it is called *nonoscillatory.* Whenever all solutions of the system (1) are oscillatory we will say that (1) is *totally oscillatory.* If (1) is totally oscillatory for every delay functions $\tau(\theta)$ and $r(\theta)$, it will be called *totally oscillatory globally in the delays.*

Kirchner and Stroinski, in [1], discuss the same problem for the system

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t)-\int_{-r}^{0}x(t+\theta)\mathrm{d}\left[\alpha(\theta)\right]\right]=\int_{-r}^{0}x(t+\theta))\mathrm{d}\left[\beta(\theta)\right], \tag{2}$$

where $r$ is a positive real number and $\alpha$ and $\beta$ are matrix valued functions of bounded variation on $[-r,0]$, $\alpha$ atomic at zero. If we allowed in (1), $\tau(\theta)$ and $r(\theta)$ to be nonnegative, then with $r(\theta)=\tau(\theta)=-r\theta$ and $\alpha(\theta)=\nu(\theta/r)$ atomic at zero (notice that in (1), the restriction on $\tau(\theta)$ to be positive makes unnecessary any atomicity assumption on $\nu$) and $\beta(\theta)=\eta(\theta/r)$, we obtain the class of systems (2). However, there will be some interest by considering (1) in order to understand the role of the delays on the oscillatory behavior of functional differential systems. Anyway, independently of the adopted formulation, the criteria obtained here are of different kind of those given in [1].

By $C^+$ we will denote the subset of $C\left([-1,0],\mathbb{R}\right)$ formed by all positive continuous functions on $[-1,0]$. With respect to a function, $r\in C^+$, it will be often considered the value $m(r)=\min\{r(\theta)\colon -1\leqslant\theta\leqslant 0\}$.

Denoting by $\mathbb{R}^{n\times n}$ the Banach space of all $n$-by-$n$ real matrices, we take the space $BV_n$ of all functions of bounded variation, $\eta\colon [-1,0]\to\mathbb{R}^{n\times n}$. For a given norm, $\|\cdot\|$, in $\mathbb{R}^{n\times n}$, with $\eta\in BV_n$ and $\theta\in[-1,0]$, by $V_\eta(\theta)$, we mean the total variation of $\eta$ on the interval $[-1,\theta]$. The total variation of $\eta$ on $[-1,0]$, $V_\eta(0)$, will also be denoted by $\int_{-1}^{0}\|\mathrm{d}\,[\eta(\theta)]\|$.

For any given norm, $\|\cdot\|$, in $\mathbb{R}^{n\times n}$, and the corresponding matrix measure, $\mu$, the function $\eta\in BV_n$, will be supposed in manner that the following assumption holds:

$$\text{(A) } \mu\circ\eta_0 \text{ is increasing and } \mu\circ\eta_1 \text{ is decreasing.}$$

**Theorem 1.** *Under* (A), *let* $\tau, r\in C^+$ *be such that* $r$ *is decreasing,* $\|r\|>\|\tau\|$ *and*

$$1+\int_{-1}^{0}\|\mathrm{d}\,[\nu\,(\theta)]\|\leqslant e\int_I\left(\|\tau\|-r\,(\theta)\right)\mathrm{d}(\mu\circ\eta_1)\,(\theta)\,,$$

*where* $I$ *is a closed interval such that* $\|\tau\|<r\,(\theta)$ *when* $\theta\in I$. *Then* (1) *is totally oscillatory, if at least one of the following assumptions is verified:*

$$1)\int_{-1}^{0}\|\mathrm{d}\,[\nu\,(\theta)]\|\leqslant 1;\qquad 2)\int_{-1}^{0}\|\mathrm{d}\,[\nu\,(\theta)]\|<\frac{em(\tau)}{\|r\|}\log\left[\|r\|\,e\,|\mu\,(\eta_0\,(-1))|\right];$$

$$3)\int_{-1}^{0}\frac{1}{e\tau\,(\theta)}\,\mathrm{d}V_\nu\,(\theta)<\int_{-1}^{0}\frac{\log\left(|\mu\,(\eta_0\,(-1))|\,r\,(\theta)\,e\right)}{|\mu\,(\eta_0\,(-1))|\,r\,(\theta)}\mathrm{d}(\mu\circ\eta_0)\,(\theta)\,.$$

**Theorem 2.** *Under* (A), *let* $\tau, r\in C^+$ *be such that* $r$ *is increasing,* $\|r\|>\|\tau\|$ *and*

$$1+\int_{-1}^{0}\|\mathrm{d}\,[\nu\,(\theta)]\|\leqslant e\int_I\left(r\,(\theta)-\|\tau\|\right)\mathrm{d}(\mu\circ\eta_0)\,(\theta)\,,$$

*where* $I$ *is a closed interval such that* $\|\tau\|<r\,(\theta)$ *when* $\theta\in I$. *Then* (1) *is totally oscillatory, if at least one of the following assumptions is satisfied:*

$$1)\int_{-1}^{0}\|\mathrm{d}\,[\nu\,(\theta)]\|\leqslant 1;$$

$$2)\int_{-1}^{0}\|\mathrm{d}\,[\nu\,(\theta)]\|<\frac{em(\tau)}{\|r\|}\log\left[\|r\|\,e\,|\mu\,(\eta_1\,(0))|\right];$$

$$3)\int_{-1}^{0}\frac{1}{e\tau\,(\theta)}\,\mathrm{d}V_\nu\,(\theta)+\int_{-1}^{0}\frac{\log\left[|\mu\,(\eta_1\,(0))|\,r\,(\theta)\,e\right]}{|\mu\,(\eta_1\,(0))|\,r\,(\theta)}\mathrm{d}(\mu\circ\eta_1)\,(\theta)<0.$$

Analogous results are valid in the scalar case.

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# PARALLEL REALIZATION OF THE FINITE DIFFERENCE METHOD SOLUTION OF THE POISSON-BOLTZMANN EQUATION

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The paper is related to with the elaboration of effective numerical algorithms for the multigrid solution of the following nonlinear Poisson-Boltzmann equation in the complex area:

$$\nabla[\varepsilon(\mathbf{r})\nabla\varphi(\mathbf{r})] - \varepsilon(\mathbf{r})\kappa(\mathbf{r})^{\mathbf{2}}\sinh[\varphi(\mathbf{r})] + \mathbf{4}\pi\varrho(\mathbf{r})/\mathbf{kT} = \mathbf{0}.$$

This equation describes a model of the energy of the electrostatic field produced by the electrical charge placed at the center of some atoms of the protein molecule, where $\varphi(\mathbf{r})$ is the dimensionless electrostatic potential in units of $kT/q$ ($k$ is the Boltzmann constant, $T$ is the absolute temperature, and $q$ is the charge on a proton), $\varepsilon(\mathbf{r})$ is the dielectric constant, and $\varrho(\mathbf{r})$ is the fixed charge density (in proton charge units). The term $\kappa = 1/\lambda$, where $\lambda$ is the Debye length. The variables $\varphi, \varepsilon, \kappa$, and $\varrho$ are all functions of the position vector $\mathbf{r}$.

We solve this problem by the multigrid finite difference successive overrelaxation method (SOR). Iterations are stopped when the relative error in solution is less than some $\delta$. Computations are organized using the multigrid approach by using the sequence of grids. This method provides a quick convergence as well as a better control and analysis of obtained approximations. The first problem is solved on the mesh with step size h=1A by SOR iteration process. The obtained solution is interpolated on the mesh with a half grid size and a new iteration is performed. Similarly the solution on the fine grid with a mesh size of 0.25A is obtained.

The program PBSOLVE is constructed for numerical solution of the linearized Poisson-Boltzmann equation. Finite difference discretization and SOR iterations [1] on the sequence of grids (multigrid) are used to obtain the approximate electrostatic potential on the grid. The error control is provided by comparison of solutions on nested grids. A parallel version of the PBSOLVE program was written in the programming language FORTRAN77 by using MPI (Message Passing Interface) [6], and we used the multiprocessor computer system *SP2000* with 8 processors.

The improvement of the program is related to the treatment of the solute-solvent interface boundary. Two ways are possible to improve the mapping of the interface boundary onto the grid. The former is a multilevel mesh refinement technique [2]. The method is intended for accurate electrostatic calculations over domains with local mesh refinement patches. The latter is the exploitation of well-developed finite [3] and boundary element [4] techniques to complex molecular surface. To account exactly the behavior of the potential on the infinity, the coupled boundary integral finite elements formulations of the problem [5] will be elaborated.

A natural way of parallelization is to divide the three-dimensional perpendicular area into the $p$ peaces by planes (we used horizontal ones), where $p$ is the number of used processors. First, the processors work step by step. We obtain our results for the method inaccuracy 1, 0.25, and 0.0625; calculation inaccuracy—$10^{-4}$; and the best time ratio is for the 3 processors (2.82). This ratio decreases for more than 3 processors because the number of data transmits increases.

The correctness of the algorithm was tested by solving the PB equation for a single spherical charge for which the exact solution is known.

The performance of the algorithm was tested on the small peptide Metenkephalin.

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# LOGISTIC DELAY EQUATIONS WITH RICHARD'S NONLINEARITY

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*MSC 2000*: 34K15, 34K25, 92D25

## Introduction

We study the following delay differential equation which is of interest in population dynamics:

$$\frac{X'(t)}{X(t)} = a - \left( c_0 X(t) + \sum_{i=1}^{m} c_i X(t - r_i) \right)^{\gamma}, \tag{1}$$

where $X(t)$ stands for the size (biomass) of a population, or its density, $X'(t)/X(t)$ denotes the growth rate, $a > 0$, $c_i \geqslant 0$, $\gamma > 0$ are given constants. We also assume that

$$K := c_0 + \sum_{i=1}^{m} c_i > 0. \tag{2}$$

The logistic equation with time lags corresponds to the case $\gamma = 1$. On the other hand, in the absence of time lags, i.e. if $c_i = 0$ $(i \geqslant 1)$, we obtain a generalized logistic differential equation which includes Richards' function: $r(\xi) = a - \frac{1}{R}\xi^{\gamma}$ $(R = \beta^{-\gamma})$. Equation (1) is assumed to be supplied with initial conditions:

$$X(\tau) = \varphi(\tau) \quad (-\max_{1 \leqslant i \leqslant m} r_i \leqslant \tau < 0), \qquad X(0) = x_0. \tag{3}$$

It is also assumed that $\varphi$ is a bounded and positive measurable function, and $x_0 > 0$.

## Main results

**Theorem 1.** 1) *For any positive $\varphi \in L^{\infty}$ and $x_0 \in \mathbb{R}^+$ there exists a unique local positive solution of the equation* (1) *satisfying the conditions* (3). 2) *Any global positive solution of the equation* (1) *will be uniformly bounded on $\mathbb{R}^+$.*

*Proof.* As $\gamma > 0$ and the initial data (3) are positive, the right hand side of the equation (3) is locally Lipschitz. Therefore, one can apply a well-known existence result for functional differential equations stating that a local positive solution does exist.

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To prove the global solvability, we follow some ideas from [1] and [2]. Due to $X(t) \geqslant 0$, one has

$$\frac{X'(t)}{X(t)} \leqslant a - c_0^\gamma X^\gamma(t) \quad (t > 0).$$

Suppose that $X(t)$ is unbounded. Then it cannot tend to $\infty$ as $t \to \infty$, because otherwise $X'(t) < 0$ for sufficiently large $t$, which is impossible. If the function $X(t)$ is both unbounded and does not tend to $\infty$, then it must have unbounded oscillations, so that there exists a sequence $t_n$ of positive numbers for which $X'(t_n) = 0$ and $X(t_n) \to \infty$ as $n \to \infty$. This would imply the estimate $X(t_n) \leqslant {c_0}^{-1} a^{1/\gamma}$ for any natural $n$, which contradicts the assumption $X(t_n) \to \infty$.

The more technical proof of the next theorems is omitted here.

**Theorem 2.** *Assume that the initial function $\varphi$ from* (3) *satisfies the estimate*

$$0 \leqslant \varphi(\tau) \leqslant \frac{a^{1/\gamma}}{K - c_0} - \varepsilon$$

*for some positive $\varepsilon$. If the solution $X(t)$ of the equation* (1)*, satisfying the initial condition* (3)*, is bounded on $\mathbb{R}^+$, then it is also strictly positive on $\mathbb{R}^+$, i.e. $X(t) \geqslant m > 0$.*

Clearly, Equation (1) has the following equilibrium state

$$X_e = K^{-1} a^{1/\gamma},$$

which is not necessarily Lyapunov stable. We will call the equilibrium $X_e$ *long time average stable* if for any strictly positive solution $X(t)$ of Equation (1)

$$\frac{1}{t} \int_0^t X(\tau)\,\mathrm{d}\tau \to X_e \qquad \text{as} \qquad t \to \infty.$$

**Theorem 3.** *If $\gamma \geqslant 1$, then the equilibrium $X_e$ of Equation* (1) *is long time average stable.*

**Remark.** The above results can be generalized to the case of the logistic equation with a distributed delay function:

$$\frac{X'(t)}{X(t)} = f_\gamma\left(\int_{-\infty}^{t} X(\tau) d_\tau k(t-\tau)\right).$$

Here $f_\gamma(\xi) = a - \xi^\gamma$ and $k(\eta)$ is a nonincreasing and right-continuous function such that $k(\infty)$ is finite.

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# SIGN-CHANGING SOLUTIONS OF SINGULAR DIRICHLET BOUNDARY VALUE PROBLEMS

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*MSC 2000*: 34B16, 34B18

We consider the singular Dirichlet boundary value problem

$$(r(x(t))x'(t))' = \mu q(t) f(t, x(t)), \tag{1}$$

$$x(0) = x(T) = 0,\ \max\{x(t)\colon 0 \leqslant t \leqslant T\} \cdot \min\{x(t)\colon 0 \leqslant t \leqslant T\} < 0, \tag{2}$$

where $\mu$ is a positive parameter and $f$ is singular at the point $x = 0$ of the phase variable $x$ in the following sense

$$\lim_{x\to 0^-} f(t,x) = -\infty,\ \lim_{x\to 0^+} f(t,x) = \infty \quad \text{for } t \in [0,T]. \tag{3}$$

We say that a function $x \in C^1([0,T])$ is a *solution of problem* (1), (2) if $x$ has precisely one zero $t_0$ in $(0,T)$, $r(x)x' \in C^1((0,T)\setminus\{t_0\})$, $x$ fulfils (2) and there exists $\mu_0 > 0$ such that (1) is satisfied for $\mu = \mu_0$ and $t \in (0,T)\setminus\{t_0\}$.

We are interested in finding effective conditions imposed on the functions $r$, $q$ and $f$ for the existence of solutions to problem (1), (2). Any such solution goes through the singularity of $f$. As far as we know, this case has not been solved yet. Up till now, only positive (negative) solutions on $(0,T)$ of the Dirichlet problem with the singularity at the point $x = 0$ of the phase variable $x$ in nonlinearities of considered second-order differential equations have been studied.

According to our above definition, solutions of problem (1), (2) belong to the class $C^1([0,T])$. The character of smoothness of solutions is very important for the consideration of their existence. We note that if we study solutions of our problem only in the class of functions having continuous first derivatives on $[0,T]$ except of zeros of solutions in $(0,T)$, we can get the existence result as well as the exact multiplicity result easier, see [2].

Since our solutions have to go through the singularity of $f$ and they have to be smooth there, we have developed a new approach to prove their existence, see [1]. This approach is based on "gluing" of positive and negative parts of solutions and on smoothing them. The needed positive and negative parts of solutions are obtained by means of the eixstence theorems from [3]. Here we assume that

(H1) $r \in C^0(\mathbb{R})$, $r(x) \geqslant r_0 > 0$ for $x \in \mathbb{R}$,

(H2) $q \in C^0((0,T))$, $q(t) < 0$ for $t \in (0,T)$ and $Q = \sup\{|q(t)|\colon t \in [0,T]\} < \infty$,

(H3) $f \in C^0([0,T]\times D)$, where $D = (-\infty,0)\cup(0,\infty)$, $f(t,\cdot)$ is nonincreasing on $D$ for $t \in [0,T]$ and for any $M > 0$ there is a function $k_M \in C^0([0,T])$ such that

$$0 < k_M(t) \leqslant f(t,x)\operatorname{sign} x \quad \text{on } [0,T]\times[-M,0)\cup(0,M],$$

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where $g \in C^0(D)$, $\int^0 g(x)\,\mathrm{d}x < \infty$, $\int_0 g(x)\,\mathrm{d}x < \infty$.

**Theorem 1.** *Let $A \in (0,\infty)$. Then there exists a solution $x$ of problem* (1), (2). *If $t_0 \in (0,T)$ denotes the unique zero of $x$, then*

$$\max\{x(t)\colon 0 \leqslant t \leqslant T\} = \max\{x(t)\colon 0 \leqslant t \leqslant t_0\} = A \quad if \quad t_0 \in [T/2, T),$$
$$\max\{x(t)\colon 0 \leqslant t \leqslant T\} = \max\{x(t)\colon 0 \leqslant t \leqslant t_0\} \leqslant A \quad if \quad t_0 \in (0, T/2).$$

**Theorem 2.** *Let $B \in (-\infty,0)$. Then there exists a solution $x$ of problem* (1), (2). *If $t_0 \in (0,T)$ denotes the unique zero of $x$, then*

$$\min\{x(t)\colon 0 \leqslant t \leqslant T\} = \min\{x(t)\colon t_0 \leqslant t \leqslant T\} = B \quad if \quad t_0 \in (0, T/2],$$
$$\min\{x(t)\colon 0 \leqslant t \leqslant T\} = \min\{x(t)\colon t_0 \leqslant t \leqslant T\} \geqslant B \quad if \quad t_0 \in (T/2, T).$$

**Example**. Let $\alpha, \beta \in (0,1)$, $a \in (0,\infty)$, $b \in (-\infty, 0)$ and

$$p(x) = \begin{cases} \dfrac{a}{x^\alpha} & \text{for } x > 0 \\[2mm] \dfrac{b}{(-x)^\beta} & \text{for } x < 0. \end{cases}$$

Consider the differential equation

$$((1 + e^{x \cos x})^\gamma x')' = \mu\Big(\sin\frac{1}{t(T-t)} - 2\Big)p(x) \tag{4}$$

with $\gamma \in (0,\infty)$. Then the assumptions (H1)–(H3) are satisfied. Consequently, Theorems 1 and 2 can be applied to problem (4), (2).

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# ESTIMATES OF CENTRAL STABILITY ZONE FOR DISCRETE HAMILTONIAN SYSTEMS

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It is considered the discrete version of the problems of strong (robust) stability of linear Hamiltonian systems with periodic coefficients

$$y_{k+1} - y_k = \lambda(B_k y_k + D_k z_{k+1}),$$
$$z_{k+1} - z_k = -\lambda(A_k y_k + B_k^* z_{k+1})$$

with $A_k$, $B_k$, $D_k$ being $N$-periodic and $A_k$, $D_k$ Hermitian; $\lambda$ is a complex parameter. In two previous papers [1], [3] this system has been studied following the line of Krein classical paper [2]; the stability zones and Krein traffic rules for multipliers have been extended to discrete case. Nevertheless not all the results of the continuous-time case may migrate, *mutatis-mutandis* to the discrete time case. A basic reason for this is that the monodromy matrix of the Hamiltonian system $U_N(\lambda)$ is not of entire but of meromorphic type, being rational with respect to $\lambda$. This is crucial for evaluating the central (around 0) stability zone with respect to $\lambda$. An estimate of it is given in the terms of the characteristic numbers of the skew-symetric boundary value problem namely this zone is included in the interval $(\Lambda_-, \Lambda_+)$ where $\Lambda_-$ is the largest (first) negative characteristic number and $\Lambda_+$ is the smallest (first) positive one. This result is valid both in the continuous and discrete case provided the boundary value problem has characteristic numbers of opposite signs. The argument of [2], which was valid in [1] fails in the general case since, as mentioned, $U_N(\lambda)$ is not of entire type.

Some sufficient conditions may be given for the existence of $\Lambda_-$ and $\Lambda_+$:

– $U_N(\lambda)$ has at least a real pole;
– the growth order for $\lambda \to \infty$ of $det(I + U_N(\lambda))$ is at least $\lambda$.

In specific cases specific methods might be used, for instance extension, by direct proof, of Liapunov criterion.

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# POSITIVE SOLUTIONS OF A SINGULAR CAUCHY PROBLEM FOR A NONLINEAR SYSTEM OF DIFFERENTIAL EQUATIONS

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Let us consider the initial problem

$$\begin{gathered} g(x)y' = A(x)\alpha(y) - \omega(x), \\ y(0^+) = 0. \end{gathered} \tag{1}$$

Here $y = (y_1, \ldots, y_n)^T$, $g(x) = \operatorname{diag}(g_1(x), \ldots, g_n(x))$ is a diagonal matrix, $\alpha(y) = (\alpha_1(y_1), \ldots, \alpha_n\ (y_n))^T$, $A(x)$ is $n \times n$ matrix with elements $a_{ij}(x)$, $i, j = 1 \ldots, n$, $\omega(x) = (\omega_1(x), \ldots, \omega_n(x))^T$. The symbol $I_s$ indicates an interval of the form $(0, s]$ with a fixed $s > 0$. The system (1) is considered under the following main assumptions:

($C_1$) $g_i \in C(I_{x_0}, \mathbb{R}^+)$, $i = 1, \ldots, n$ with $\mathbb{R}^+ = (0, \infty)$;
($C_2$) $\alpha \in C^1(I_{y_0}, \mathbb{R}^n), \alpha(y) \gg 0$ on $I_{y_0}, \alpha'(y) \gg 0$ on $I_{y_0}$ and $\alpha(0^+) = 0$;
($C_3$) $\omega \in C^1(I_{x_0}, \mathbb{R}^n)$;
($C_4$) $a_{ij} \in C^1(I_{x_0}, \mathbb{R})$, $a_{ij}(x) \neq 0$, $i, j = 1, \ldots, n$ and $\det A(x) \neq 0$ on $I_{x_0}$;
($C_5$) $\alpha_i(y) \leqslant M\alpha_i'(y)$, $i = 1, \ldots, n$ on $I_{y_0}$ with a constant $M \in \mathbb{R}^+$;
($C_6$) $\Omega(x) \equiv A^{-1}(x)\omega(x) \gg 0$, $\Omega'(x) \gg 0$ on $I_{x_0}$ and $\Omega(0^+) = 0$.

**Theorem 1.** *Suppose that conditions* ($C_1$)–($C_6$) *are satisfied. Let*
**A)** *for* $i = 1, \ldots, p \leqslant n$:

$$\begin{gathered} \omega_i(x) < 0,\ \omega_i'(x) < 0,\ x \in I_{x_0}, \\ a_{ij}(x) \geqslant 0,\ j \neq i,\ j = 1, \ldots, n,\ \ a_{ij}'(x) \geqslant 0,\ j = 1, \ldots, n,\ x \in I_{x_0} \end{gathered}$$

*and*

$$\omega_i(\delta x) > \omega_i(x) + \delta M g_i(x) \frac{\Omega_i'(\delta x)}{\Omega_i(\delta x)},\quad x \in I_{x_0}$$

*for a constant* $\delta \in (0, 1)$;
**B)** *for* $i = p + 1, \ldots, n$:

$$\begin{gathered} \omega_i(x) > 0,\ \omega_i'(x) > 0,\ \ x \in I_{x_0}, \\ a_{ij}(x) \leqslant 0,\ j \neq i,\ j = 1, \ldots, n,\ \ a_{ij}'(x) \leqslant 0,\ j = 1, \ldots, n,\ x \in I_{x_0} \end{gathered}$$

*and*

$$\omega_i(Kx) > \omega_i(x) + KMg_i(x) \frac{\Omega_i'(Kx)}{\Omega_i(Kx)},\ x \in I_{x_0}$$

*for a constant* $K > 1$. *Then there exists* $(n - p)$*-parametric family of solutions of the problem (1), having positive coordinates, on an interval* $I_{x^*} \subseteq I_{x^{**}}$ *with* $x^{**} \leqslant \min\{x_0 K^{-1}, y_0\}$.

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# AN EXISTENCE CRITERION OF POSITIVE SOLUTIONS OF $p$-TYPE RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS

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*MSC 2000*: 34K20, 34K25

The function $p \in C[\mathbb{R} \times [-1,0], \mathbb{R}]$ is called a *p-function* if it has the following properties: $p(t,0) = t$, $p(t,-1)$ is a nondecreasing function of $t$ and there exists a $\sigma \geqslant -\infty$ such that $p(t,\vartheta)$ is an increasing function in $\vartheta$ for each $t \in (\sigma,\infty)$ (see [2]). For $t \in [t_0, t_0 + A)$ with $A > 0$ we define $y_t(\vartheta) = y(p(t,\vartheta))$, $-1 \leqslant \vartheta \leqslant 0$.

Consider the system

$$\dot{y}(t) = f(t, y_t) \tag{1}$$

where $f \in C[[t_0, t_0 + A) \times \mathcal{C}, \mathbb{R}^n]$ with $\mathcal{C} = [[-1,0], \mathbb{R}^n]$. This system is called the system of *p-type retarded functional differential equations* ([2]).

We say that the functional $g \in C(\Omega, \mathbb{R})$ is *strongly decreasing* (*increasing*) with respect to the second argument on $\Omega \subset \mathbb{R} \times \mathcal{C}$ if for each $(t,\varphi), (t,\psi) \in \Omega$ with $\varphi(p(t,\vartheta)) \ll \psi(p(t,\vartheta))$, $\vartheta \in [-1,0)$: $g(t,\varphi) - g(t,\psi) > 0$ $(< 0)$. Let $k \gg 0$ and $\mu$ be constant vectors, $\mu_i = -1$, $i = 1, \dots, p$ and $\mu_i = 1$, $i = p+1, \dots, n$. Let $\lambda(t)$ denote a real vector with continuous entries on $[p^*, \infty)$, $p^* = p(t^*, -1)$. Put

$$T(k,\lambda)(t) \equiv k e^{\mu \int_{p^*}^{t} \lambda(s)\,ds} = \left(k_1 e^{\mu_1 \int_{p^*}^{t} \lambda_1(s)\,ds}, \dots, k_n e^{\mu_n \int_{p^*}^{t} \lambda_n(s)\,ds}\right).$$

**Theorem 1.** *Suppose $\Omega = [t^*, \infty) \times \mathcal{C}$, $f \in C(\Omega, \mathbb{R}^n)$ is locally Lipschitzian with respect to the second argument and, moreover:*

(i) $f(t,0) \equiv 0$ *if* $t \geqslant t^*$.

(ii) *The functional $f_i$ is strongly decreasing if $i = 1, \dots, p$ and strongly increasing if $i = p+1, \dots, n$ with respect to the second argument on $\Omega$.*

*Then for the existence of a positive solution $y = y(t)$ on $[p^*, \infty)$ of the system* (1) *a necessary and sufficient condition is that there exists a vector $\lambda \in C([p^*, \infty), \mathbb{R}^n)$, such that $\lambda \gg 0$ on $[t^*, \infty)$, satisfying the system of integral inequalities*

$$\lambda_i(t) \geqslant \frac{\mu_i}{k_i} e^{-\mu_i \int_{p^*}^{t} \lambda_i(s)\,ds} \cdot f_i\left(t, T(k,\lambda)_t\right), \quad i = 1, \dots, n$$

*for $t \geqslant t^*$, with a positive constant vector $k$.*

Consider the equation

$$\dot{y}(t) = -\int_{\tau(t)}^{t} K(t,s) y(s)\,ds, \tag{2}$$

where $K : [t^*, \infty) \times [p^*, \infty) \to \mathbb{R}^+$ is a continuous function, and $\tau : [t^*, \infty) \to [p^*, \infty)$ is a nondecreasing function with $\tau(t) < t$.

**Theorem 2.** *The equation* (2) *has a positive solution* $y = y(t)$ *on* $[p^*, \infty)$ *if and only if there exists a function* $\lambda \in C([p^*, \infty), \mathbb{R})$, *such that* $\lambda(t) > 0$ *for* $t \geqslant t^*$ *and*

$$\lambda(t) \geqslant \int_{\tau(t)}^{t} K(t,s) \mathrm{e}^{\int_s^t \lambda(u)du} \, \mathrm{d}s$$

*on the interval* $[t^*, \infty)$.

Let us consider a partial case of Eq. (2) when $\tau(t) \equiv t - l$, $l \in \mathbb{R}^+$ and $K(t,s) \equiv c(t)$ for $t \in [t^*, \infty)$. Then Eq. (2) takes the form

$$\dot{y}(t) = -c(t) \int_{t-l}^{t} y(s) \, \mathrm{d}s. \tag{3}$$

**Theorem 3.** *For the existence of a solution of Eq.* (3), *positive on* $[t^* - l, \infty)$, *the inequality*

$$c(t) \leqslant M, \quad t \in [t^*, \infty)$$

*is sufficient for* $M = \alpha(2-\alpha)/l^2 = \text{const}$ *with a constant* $\alpha$ *being the positive root of the equation* $2 - \alpha = 2\mathrm{e}^{-\alpha}$. (*The approximate values are* $\alpha \doteq 1.5936$ *and* $M \doteq 0.6476/l^2$.)

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# OSGOOD CONDITION FOR THE EQUATION $x^{(m)} = f(t,x)$ IN BANACH SPACES

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*MSC 2000*: 34G20

Assume that $I = [0,a]$, $E$ is a Banach space, $B = \{x \in E\colon \|x\| \leqslant b\}$ and $f\colon I \times B \mapsto E$ is a bounded continuous function. Consider the Cauchy problem

$$\begin{gathered} x^{(m)} = f(t,x) \\ x(0) = 0, x'(0) = \eta_1, \ldots, x^{(m-1)}(0) = \eta_{m-1} \end{gathered} \tag{1}$$

Let $M = \sup\{\|f(t,x)\|\colon t \in I, x \in B\}$. We choose a positive number $d$ such that $d \leqslant a$ and $\sum\limits_{j=1}^{m-1} \| \eta_j \| \frac{d^j}{j!} + M\frac{d^m}{m!} \leqslant b$. Put $J = [0,d]$ and

$$F(x)(t) = p(t) + \frac{1}{(m-1)!}\int_0^t (t-s)^{m-1} f(s,x(s))\,\mathrm{d}s \quad \text{for } x \in \tilde{B}, t \in J,$$

where $\tilde{B}$ is the closed ball in the space $C(J,E)$ with center 0 and radius $b$.

**Theorem.** *Let $w\colon [0,2b] \mapsto \mathbb{R}_+$ be a continuous nondecreasing function such that $w(0) = 0$, $w(r) > 0$ for $r > 0$ and*

$$\int\limits_{0+} \frac{\mathrm{d}r}{\sqrt[m]{r^{m-1}w(r)}} = \infty.$$

*If*

$$\|f(t,x) - f(t,y)\| \leqslant w(\|x-y\|) \quad \text{for } t \in I, \quad x,y \in B, \tag{2}$$

*then the successive approximations $u_n$, defined by*

$$u_0 = 0, u_{n+1} = F(u_n) \quad \text{for } n \in \mathbb{N}, \tag{3}$$

*converge uniformly on $J$ to the unique solution $u$ of* (1).

*Proof.* From (2) it follows that

$$\alpha\left(f(t,X)\right) \leqslant w(\alpha(X)) \quad \text{for } t \in J \text{ and } X \subset B, \tag{4}$$

where $\alpha$ is the Kuratowski measure of noncompactness. We shall show that the sequence $(u_n)$ has a limit point. Let $V = \{u_n\colon n \in \mathbb{N}\}$. Then $V$ is a bounded equicontinuous subset of $\tilde{B}$. Denote by $v$ the function defined by $v(t) = \alpha(V(t))$ for $t \in J$, where $V(t) = \{u_n(t)\colon n \in \mathbb{N}\}$. It is well known that the function $v$ is continuous. As $V = F(V) \cup \{0\}$, we have

$$V(t) \subset F(V)(t) \cup \{0\}$$

and consequently $\alpha(V(t)) \leqslant \alpha(F(V)(t))$. Since

$$F(V)(t) \subset \frac{1}{(m-1)!}\left\{\int_0^t (t-s)^{m-1} f(s,u_n(s))\,\mathrm{d}s\colon n \in \mathbb{N}\right\},$$

Heinz's lemma [2] proves that

$$\begin{aligned}\alpha(F(V)(t)) &\leqslant \frac{1}{(m-1)!}\alpha\left(\left\{\int_0^t (t-s)^{m-1} f(s,u_n(s))\,\mathrm{d}s\colon n \in \mathbb{N}\right\}\right)\\ &\leqslant \frac{1}{(m-1)!}\int_0^t \alpha(\{(t-s)^{m-1} f(s,u_n(s))\colon n \in \mathbb{N}\})\,\mathrm{d}s\\ &\leqslant \frac{1}{(m-1)!}\int_0^t (t-s)^{m-1}\alpha(\{f(s,u_n(s))\colon n \in \mathbb{N}\})\,\mathrm{d}s.\end{aligned}$$

Moreover, in view of (4), we have

$$\alpha(\{f(s,u_n(s))\colon n \in \mathbb{N}\}) \leqslant w(\alpha(V(s))).$$

Hence

$$v(t) \leqslant \alpha(F(V)(t)) \leqslant \frac{1}{(m-1)!}\int_0^t (t-s)^{m-1} w(v(s))\,\mathrm{d}s \quad \text{for } t \in J.$$

Applying now Lemma 2 from [3], we deduce that $v(t) \equiv 0$ on $J$. This proves that $V(t)$ is relatively compact for $t \in J$ and, consequently, by Ascoli's theorem $V$ is relatively compact in $C(J,E)$. Hence the sequence $(u_n)$ has a limit point.

Further we argue in the same way as in the proof of Th. III. 9.1 in [1].

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# STABILITY OF SPATIALLY HOMOGENEOUS SOLUTIONS IN LATTICE DYNAMICAL SYSTEMS

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*MSC 2000*: 37L60

The investigation of stability of Spatially Homogeneous Solutions in Lattice Dynamical Systems leads to study of spectra of the following types of linear operators $\mathcal{A}$.

We consider the complex Banach space

$$\mathbb{B} = \{\boldsymbol{u} = \{u_j\}_{j\in\mathbb{Z}}\,;\ u_j \in \mathbb{C},\ \sup\{|u_j|;\ j \in \mathbb{Z}\} < \infty\}$$

with the sup-norm and a linear bounded operator $\mathcal{A}\colon \mathbb{B} \longrightarrow \mathbb{B}$ given by

$$(\mathcal{A}\boldsymbol{u})_j = \sum_{k=-s}^{s} a_k u_{j+k},$$

where $a_{-s}, \ldots, a_s$ are given numbers. We describe the spectrum $\sigma(\mathcal{A})$ of the operator $\mathcal{A}$ and in some cases we give a simple formula for the spectral radius $r_\sigma(\mathcal{A})$ of $\mathcal{A}$. (Let us recall that $r_\sigma(\mathcal{A}) = \sup\{|\lambda|;\ \lambda \in \sigma(\mathcal{A})\}$.)

**Theorem 1.** *The spectrum of the operator $\mathcal{A}$ is given by*

$$\sigma(\mathcal{A}) = \left\{\sum_{k=-s}^{s} a_k \mathrm{e}^{ik\varphi}\,;\ \varphi \in [0, 2\pi]\right\}.$$

*Moreover, the whole spectrum $\sigma(\mathcal{A})$ is formed by eigenvalues of $\mathcal{A}$.*

At first, we prove Theorem 1 for a particular case of the left-shift operator $\mathcal{S}\colon \mathbb{B} \longrightarrow \mathbb{B}$,

$$(\mathcal{S}\boldsymbol{u})_j = u_{j+1}.$$

**Lemma 2.** *The spectrum of the operator $\mathcal{S}$ is given by*

$$\sigma(\mathcal{S}) = \{\mathrm{e}^{i\varphi}\,;\ \varphi \in [0, 2\pi]\}.$$

*Proof of Theorem* 1. It is easy to see that

$$\mathcal{A} = \sum_{k=-s}^{s} a_k \mathcal{S}^k,$$

where $\mathcal{S}^{-1}$ is the inverse operator to $\mathcal{S}$. As the function $g(z) = \sum_{k=-s}^{s} a_k z^k$ is analytical on the set $\{z \in \mathbb{C};\ z \neq 0\}$, Theorem 1 now follows immediately from the following Theorem (see [2, Chapter 7, Theorem 11]):

*Let $\mathcal{T}\colon X \longrightarrow X$ be a linear bounded operator on a Banach space $X$. Let $g$ be an analytical function on a neighbourhood of $\sigma(\mathcal{S})$. Then $\sigma(g(\mathcal{T})) = \{g(\lambda);\ \lambda \in \sigma(\mathcal{T})\}$.*

□

**Theorem 3.** *Let $a_k$, $-s \leqslant k \leqslant s$, be real numbers satisfying one of the following four conditions*

a) $a_k \geqslant 0$ *for* $-s \leqslant k \leqslant s$,

b) $a_k \leqslant 0$ *for* $-s \leqslant k \leqslant s$,

c) $a_{-s+2k} \geqslant 0$ *for* $0 \leqslant k \leqslant s$ *and* $a_{-s+2k+1} \leqslant 0$ *for* $0 \leqslant k \leqslant s-1$, *or*

d) $a_{-s+2k} \leqslant 0$ *for* $0 \leqslant k \leqslant s$ *and* $a_{-s+2k+1} \geqslant 0$ *for* $0 \leqslant k \leqslant s-1$.

*Then* $r_\sigma(\mathcal{A}) = \sum\limits_{k=-s}^{s} |a_k|$.

P r o o f. Clearly

$$\|\mathcal{A}\| = \sup_{\|\boldsymbol{x}\|=1} \|\mathcal{A}(\boldsymbol{x})\| = \sup_{\|\boldsymbol{x}\|=1} (\sup_{j\in\mathbb{Z}} |(\mathcal{A}(\boldsymbol{x}))_j|)$$
$$= \sup_{\|\boldsymbol{x}\|=1} \left(\sup_{j\in\mathbb{Z}} \left|\sum_{k=-s}^{s} a_k x_{j+k}\right|\right) = \sum_{k=-s}^{s} |a_k|.$$

Hence $r_\sigma(\mathcal{A}) \leqslant \sum\limits_{k=-s}^{s} |a_k|$. On the other hand, the conditions a)–d) in fact mean that all $a_k$ have the same sign or their sign alternates. If we choose $\varphi = 0$ for cases a), b) and $\varphi = \pi$ for cases c), d) and apply Theorem 1, we get an eigenvalue $\lambda$ satisfying $|\lambda| = \sum\limits_{k=-s}^{s} |a_k|$. □

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# AN EXAMPLE OF NONTRIVIAL MARKOV PROCESS IN THE SPACE OF LIPSCHITZ FUNCTIONS

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*MSC 2000*: 60G17

Markov processes defined on spaces of Lipschitz continuous functions (with a given constant of Lipschitz continuity) have identically zero diffusion coefficients. Obviously, such processes are not diffusion processes in the classical sense. A wide class of such processes are those defined as Markov chains in equidistant moments and developing as deterministic processes in time intervals between these moments. If we consider physical, biological and other systems the real parts of such systems are changed with finite velocity, i.e. their trajectories are Lipschitz continuous. This restriction reduces applicability of diffusion processes, especially of the theory of stochastic differential equations which are driven by Wiener process. Trajectories of these processes have generally unbouded total variation and the velocity is not defined at all. From the other point of view the applications of diffusion processes are studied in [1] where it is shown that in some concrete cases the charecteristics of real processes are different from those given by Wiener process. The Markov chains described above are not suitable for examination of physical or biological systems which are developing continuously and "at every moment they behave as a stochastic process".

It is well known [2], [3] that if the diffusion coefficients are identically equal to zero and if the trend $a(t,x)$ is sufficiently smooth and defined on the whole half-space then the process is deterministic and it is a solution of a system of ordinary differential equations. We can pose a natural question how complicated Markov processes defined on the space of Lipschitz functions may be. A certain answer is given by Theorem 1 which is a slight generalization of an interesting result of G. Colombo [4]. The assumptions of the Theorem 1 (the local BV of trajectories) are substantially weaker than the Lipschitz continuity required in [1].

**Theorem 1.** *Let $(\Omega,\mathcal{F},\mathcal{F}_t,X(.),P)$ be an $\mathbb{R}^n$ valued Markov process with locally BV (bounded variation) and right-continuous trajectories for almost all $\omega$. Then there exists a subset $\Lambda$ in $\mathbb{R}$ of full Lebesgue measure such that for $t\in\Lambda$ the derivatives $\mathrm{d}X(t,\omega)/\mathrm{d}t$ exist for almost all $\omega\in\Omega$. Moreover there exists a function $\Gamma\colon\Lambda\times\mathbb{R}^n\to\mathbb{R}^n$ such that $\Gamma(t,.)$ is Borel-measurable and*

$$\mathrm{d}X(t,\omega)/\mathrm{d}t=\Gamma(t,X(t,\omega)) \tag{1}$$

*holds for almost all $\omega$.*

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This Theorem gives a certain bound for the complexity of processes defined on the space of Lipschitz continuous functions. For such processes we can define a trend whose meaning is similar as in the theory of diffusion processes.

**Definition 1.** Let $t \in [0,T]$ be given. A function

$$a(t,.)\colon \operatorname{Dom}(a(t,.)) \to \mathbb{R}^n$$

is called a trend at a moment $t$ if there exist modifications of $E[X(t+h)|\mathcal{G}_t]$ such that

$$\lim_{h\searrow 0} \frac{1}{h}\Big(\mathbb{E}\big[X(t+h,.)|\mathcal{G}_t\big](\omega) - \mathbb{E}\big[X(t,.)|\mathcal{G}_t\big](\omega)\Big) = a(t, X(t,\omega))$$

for almost all $\omega$ where $\operatorname{Dom}(a(t,.)) \subset \mathbb{R}^n$ is a domain of the function $a(t,.)$, $\mathcal{G}_t$ is the complete $\sigma$-field generated by $X(t)$ and $\mathbb{E}(|)$ denotes the conditional expectation.

Using Colombo's theorem we can prove

**Theorem 2.** *There exists a set $\Lambda \subset [0,T]$, $\lambda(\Lambda) = T$ such that*

$$\lim_{h\searrow 0} \frac{1}{h}\Big(\mathbb{E}^\star\big[X(t+h,.)|\mathcal{G}_t\big](\omega) - \mathbb{E}^\star\big[X(t,.)|\mathcal{G}_t\big](\omega)\Big) = a(t, X(t,\omega))$$

*for $t \in \Lambda$ and almost all $\omega$ where $\mathbb{E}^\star[X(t,.)|\mathcal{G}_t](\omega)$ is a modification of the conditional expectation.*

Using the notion of trend we can formulate conditions which assure that the process is just deterministic. These conditions are weaker than those stated in [2], [3].

Our goal is to construct a Markov process which is defined on the space $\mathcal{L} = \{x(.)\colon [0,1] \to \mathbb{R}^n,\ x(0) = 0,\ x(.)$ is a nondecreasing function with Lipschitz coefficient $1\}$.

This process acquires only a set consisting of countable many values for every $t \in (0,1]$ which is dense in the interval $[0,t]$. We can say that the process is "*weakly diffusive*" since it successively covers increasing regions even if the process starts from arbitrarily small region.

The process is constructed in three steps. First we construct Markov chains $X_n(.)$ which are defined only at moments $t_{k,n} = k/2^n$, $k$ is nonnegative integer. Using the weak convergence a process $Y(.)$ can be constructed which is defined for all $t_{k,n}$, $n$ arbitrary. Since $Y(.)$ is defined on the dense set of moments in $[0,1]$ and the trajectories are nondecreasing and Lipschitz continuous $Y(.)$ can be extended to the whole time interval $[0,1]$.

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## SINGULAR SETS OF SOBOLEV FUNCTIONS

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We would like to answer the question how large (in appropriate sense) can be a set of singular points for Sobolev functions taken from a given space $W_0^{1,p}(\Omega)$.

Let $u\colon \Omega \to \overline{\mathbb{R}}$ be a measurable function. We say that $u$ has singularity at least of order $\alpha > 0$ at $x_0 \in \Omega$ if there exist $r > 0$ and $C > 0$ such that $u(x) \geqslant C\cdot|x-x_0|^{-\alpha}$ for a.e. $x \in B_r(x_0)^\circ\Omega$. The set of all singular points of a given measurable function $u\colon \Omega \to \overline{\mathbb{R}}$, having order at least $\alpha > 0$, will be denoted by $\operatorname{Sing}_\alpha u$. We define the *singular set of* $u$ by $\operatorname{Sing} u = \bigcup\limits_{\alpha>0} \operatorname{Sing}_\alpha u$. It is natural to define the following nonnegative real number

$$\text{s-dim}\, W_0^{1,p}(\Omega) = \sup\{\dim_{\mathcal{H}} (\operatorname{Sing} u)\colon u \in W_0^{1,p}(\Omega)\}, \tag{1}$$

which we call **singular dimension of the Sobolev space** $W_0^{1,p}(\Omega)$. Here $\dim_{\mathcal{H}} A$ on the right-hand side is the Hausdorff dimension of $A^\circ\mathbb{R}^N$. Expression (1) measures how large can be singular sets of Sobolev functions from $W_0^{1,p}(\Omega)$.

From inequality (3) below we can easily derive the following estimate for $p = 2$ and all $N > 2$:

$$\text{s-dim}\, H_0^1(\Omega) \geqslant \begin{cases} 3/5 & \text{if } N = 3, \\ 4/3 & \text{if } N = 4, \\ 15/7 & \text{if } N = 5, \\ N-3 & \text{if } N \geqslant 6. \end{cases} \tag{2}$$

**Theorem 1.** ([5]) *Assume that* $1 < p < N$. *Then*

$$\text{s-dim}\, W_0^{1,p}(\Omega) \geqslant \max\left\{N\frac{N-p}{N+p'}, N - \lfloor p \rfloor - 1\right\}. \tag{3}$$

More generally, assume that $\Omega$ is a given domain in $\mathbb{R}^N$, and let $X$ be an arbitrary Banach space (or just a set) of functions $u\colon \Omega \to \overline{\mathbb{R}}$. We define **singular dimension** of $X$ by

$$\text{s-dim}\, X = \sup\{\dim_{\mathcal{H}}(\operatorname{Sing} u)\colon u \in X\}. \tag{4}$$

Clearly, s-dim $X \leqslant N$. We have the following surprising result:

**Theorem 2.** ([5]) *Assume that* $1 \leqslant p < \infty$, *and let* $\Omega$ *be a domain in* $\mathbb{R}^N$. *Then the singular dimension of* $L^p(\Omega)$ *is maximal, that is,* s-dim $L^p(\Omega) = N$.

**Theorem 3.** ([5]) *Let* $1 < p < N$, *and assume that* $A$ *is a compact subset of a bounded domain* $\Omega$ *in* $\mathbb{R}^N$, *such that its upper box dimension satisfies:*

$$\overline{\dim}_{\mathrm{B}} A < N \cdot \frac{N-p}{N+p'}. \tag{5}$$

*Then there exist a Sobolev function* $u \in W_0^{1,p}(\Omega) \cap C(\Omega \setminus A)$ *such that* $\operatorname{Sing}_\alpha u = A$ *for some* $\alpha > 0$. *Furthermore,* $u$ *can be chosen so that it is locally Hölder continuous on* $\Omega \setminus A$, $u \geqslant 0$ *in* $\Omega$. *If* $p = 2$, *then we can achieve that* $u \in H_0^1(\Omega) \cap C^1(\Omega \setminus A)$.

**Corollary 1.** ([5]) *Let* $1 < p < N$, *and assume that* $A$ *is a compact,* $k$*-rectifiable subset of* $\Omega$, *where* $k = \left\lceil N \cdot \frac{N-p}{N+p'} \right\rceil - 1$. *Then we have the same conclusion as in Theorem 3.*

Example 1. ([5]) Antoine's necklace $A$ in $\mathbb{R}^3$ is clearly 3-rectifiable. Using Corollary 1 we obtain that for any $N \geqslant 7$ there exist a Sobolev function $u \in H_0^1(\Omega) \cap C^1(\Omega \setminus A)$, $\Omega^\circ \mathbb{R}^N$, such that $A = \operatorname{Sing}_\alpha u$ for some positive $\alpha$, where $A \subseteq \mathbb{R}^3 \subseteq \mathbb{R}^N$. We do not know if Antoine's necklace can be equal to a singular set of any Sobolev function in $H_0^1(\Omega)$ for $N < 7$.

It is a well known fact that $|x - a|^{-\gamma}$ is integrable on any $r$-neighbourhood of $\{a\}$ in $\mathbb{R}^N$, $r \in (0, \infty)$, if and only if $\gamma < N$. Here is a generalization of this fact (the if part has been proved in [1]):

**Proposition 1.** ([5]) *Let* $0 \leqslant s < N$ *and* $\gamma > 0$. *Assume that* $A^\circ \mathbb{R}^N$ *possesses* $s$*-dimensional Minkowski content* $\mathcal{M}^s(A)$, *and* $0 < \mathcal{M}^s(A) < \infty$. *Then* $d(x, A)^{-\gamma}$ *is Lebesgue integrable on any* $r$*-neghbourhood of* $A$, $r \in (0, \infty)$, *if and only if* $\gamma < N - s$.

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