SETTING OF COVARIANCE AND STATE PARAMETERS FOR KALMAN FILTER

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Abstract: Abstract: The estimation of queue lengths via local filters is depend on initial settings of covariances and state parameters if they are unknown. In the traffic, it is possible used the old measurements for initial setting. In the traffic data, the uncertainties are and the noise is not correlated and so it is not suitable using the method of least squares. Using of Subspace methods looks as a good possibilities for initial settings. The traffic problem is so complicated that it is not used the Subspace generally algorithm for estimation.

Keywords: traffic, subspace methods, covariances

1 INTRODUCTION

The main problem in traffic are congested situation. The problem can be solved by the control of the traffic network instead of the control of the one intersection. For traffic control we need on-line information from the road. One of the possibility, it is monitoring the queue lengths in the region and than optimizing the flow of vehicles through the traffic network. The queue length gives us the good knowledge about the real situation on the roads and in the network. In these days, the queue length is unmeasured and so it must be estimated.

The queue lengths are estimated via the traffic model (**?**). The traffic model is state space model which describe the situation in the traffic networks or in the section of the networks so-called micro-regions. The model is based on physical principle and it must be valid that the vehicles entering the system must be identical to the vehicles leave or present the system. The basic problem with estimation of queue length was described in (**?**; **?**). For the better estimation of the queue lengths, it is necessary to solve another problems.

The traffic is described by a linear state space model where the state includes the unmeasured queue lengths (?). For this case, it is possible to estimate queue lengths using the Kalman filter (KF) (?).The state transition matrix includes the unknown parameters that can not be determined from physical relations and so they need to be estimated too. The parameters can be estimated by on-line or off-line methods. In case that the off-line methods was used, the KF can be used for estimation of queue lengths otherwise we need the current estimation of the state and the model parameters which is solved by the suitable nonlinear estimation methods, in our case Sigma Point Kalman Filters (SP-KF) (?).

The both ways (linear and nonlinear estimation methods, respectively) need the good initial setting of state and measurement covariance matrix which can be determined via off-line method. Methods, which can be used for initial setting, are Subspace methods.

The paper is organized as follows. In section 2, the Subspace methods for estimation the parameters are described. In section 3, the parameters of the state matrices and the covariances are estimated. In section 4, the results are summarized in conclusion.

2 SUBSPACE METHODS (SMS)

State estimation of state-space models is widely used in a variety of computer science and application problems. The famous algorithms deal with these problems, the Kalman filter (KF) is but it is applicable to linear-Gaussian models and models with finite state spaces only. For KF, it is necessary to know the prior setting of state and covariance matrices.

The subspace methods allow us to estimate the state-space matrices A, B, C, D and covariance matrices directly without first specifying any another parameters. The aim of these methods is found the state-space model with knowledge input-output data only (?)
.

SMs are useful for different type of systems. There are 3 basic types of algorithms according to the system for which they are to used. The deterministic algorithm is for the systems without noises, the stochastic algorithm is available for auto-regression systems and combined algorithm is sufficient for system with inputs and noises.

The SMs are divided into three types of algorithms by sort of system:

2.1 Deterministic algorithm (DA)

The deterministic algorithm is not sufficient for real systems under uncertainty. Although this type is not suitable for traffic model, we introduce them for completeness.

DA is sufficient for unknown deterministic system of order n:

$$
\begin{aligned}
x_{k+1}^d &= Ax_k^d + Bu_k \\
y_k &= Cx_k^d + Du_k
\end{aligned}
$$

2.2 *Stochastic algorithm (SA)*

The stochastic algorithm works with noises but it is autonomous system. The new state is influenced upon only the last state and with the noises (state and process). The traffic model is not autonomous system and so this algorithm is not sufficient.

SA is sufficient for unknown stochastic system of order n:

$$\begin{aligned} x^s_{k+1} &= Ax^s_k + w_k \\ y_k &= Cx^s_k + v_k \end{aligned}$$

where w_k and v_k are mutually independence. w_k and v_k have zero mean with covariance matrix:

$$E[\begin{pmatrix} w_k \\ v_k \end{pmatrix} \begin{pmatrix} w_k^T & v_k^T \end{pmatrix}] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}$$

2.3 *Combined Deterministic-Stochastic Algorithm (CA)*

For complete description of traffic model, we must used the combined deterministic-stochastic algorithm. The CA obtain both external inputs u_k, the process noise w_k and the measurement noise v_k and so it is suitable for traffic model.

CA is sufficient for unknown system of order n:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + Du_k + v_k \end{aligned}$$

with w_k a v_k are mutually independence. w_k and v_k have zero mean with covariance matrix:

$$E[\begin{pmatrix} w_k \\ v_k \end{pmatrix} \begin{pmatrix} w_k^T & v_k^T \end{pmatrix}] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}$$

2.4 *SMs and the traffic model*

The traffic model has some specificities. The model is based on physical model and so the variables in state and measurement matrices must be non-negative and we do not need to estimated all values in matrices because some of them are known. The main disadvantage of SMs can be summarized into two areas.

1. problem with the parameter δ. The parameter δ is depend on the queue lengths only.

2. problem with the non-negative parameters α, β, κ and sometimes λ. For solution of this problem it is need made soma changes and added some restrictions SMs algorithm.

3 EXPERIMENTS

3.1 Simulated system

The original system The notation of state-space model is:

$$
\begin{aligned}
x_{k+1} &= \underbrace{\begin{bmatrix} 0.9 & -1 & 0.7 \\ 1 & 0 & 0 \\ 0 & 1 & 0.1 \end{bmatrix}}_{A} \cdot x_k + \begin{bmatrix} 1 \\ 0.5 \\ 0.4 \end{bmatrix} \cdot u_k + w_k \\
y_k &= \underbrace{\begin{bmatrix} 0.8 & 0.3 & -0.1 \end{bmatrix}}_{C} \cdot x_k + v_k
\end{aligned}
$$

where

w_k is state noise at time k and $Q = \begin{bmatrix} 0.15 \\ 0.15 \\ 0.15 \end{bmatrix}$,

v_k is the process noise at time k and $R = 0.15$.

The estimation of matrices The estimated matrix A, B, C and covariances for system order 3

$$A = \begin{bmatrix} 0.0865 & 1.0117 & 0.1366 \\ -0.9145 & 0.0763 & 0.2224 \\ 0.0154 & -0.0969 & 0.8443 \end{bmatrix}, \; B = \begin{bmatrix} -0.7439 \\ -0.2220 \\ -0.8414 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.9591 & 0.4591 & -0.3872 \end{bmatrix}, \; D = -0.0929$$

$$K = \begin{bmatrix} -0.0252 & 0.5658 & -0.2670 \end{bmatrix}, \; R = 0.6716$$

Time demand: $t_d = 1.1250\ s$

Comparison the estimation matrices and the original model This systems obtain the state and measurement noises and the the parameters. For the estimation from corrupted data, it is necessary to have bigger amount of data. In this case, the 3000 time steps is sufficient.

We compare the data via eigenvalues, canonical form of models and output prediction which is made on models in canonical form by reason of the using the same prior state.

Eigenvalues (L)

1. the original model

$$L_O = \begin{bmatrix} 0.0809 + 0.9736i \\ 0.0809 - 0.9736i \\ 0.8382 \end{bmatrix}$$

2. the SMs model

$$L_{SM} = \begin{bmatrix} 0.0809 + 0.9736i \\ 0.0809 - 0.9736i \\ 0.8382 \end{bmatrix}$$

Canonical form

1. the original model

$$A_O^{(c)} = \begin{bmatrix} 0.08089 & 0.9736 & 0 \\ -0.9736 & 0.08089 & 0 \\ 0 & 0 & 0.8382 \end{bmatrix},\ B_O^{(c)} = \begin{bmatrix} 0.2752 \\ 0.9065 \\ -1.394 \end{bmatrix},$$

$$C_O^{(c)} = \begin{bmatrix} -0.5122 & 0.2014 & 0.444 \end{bmatrix},\ D_O^{(c)} = 0$$

2. the SMs model

$$A_{SM}^{(c)} = \begin{bmatrix} 0.08108 & 0.9722 & 0 \\ -0.9722 & 0.08108 & 0 \\ 0 & 0 & 0.8449 \end{bmatrix},\ B_{SM}^{(c)} = \begin{bmatrix} -0.7668 \\ -0.2823 \\ -0.8916 \end{bmatrix},$$

$$C_{SM}^{(c)} = \begin{bmatrix} -0.6725 & 0.3065 & -0.57 \end{bmatrix},\ D_{SM}^{(c)} = -0.0929$$

Output prediction

The initial condition of state is $x_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. The models are in canonical form and so the initial conditions can not be recalculated. The results of estimations is seen on figure 1.

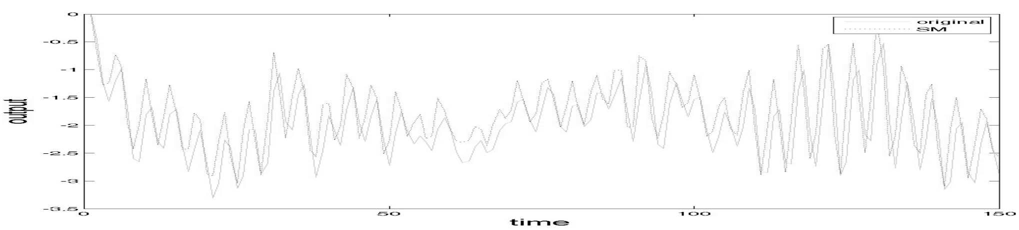

Figure 1: Output prediction

Summary: The identification of combined model via SMs is sufficient exact but for good estimation it is necessary to have enough data. For this case, it was needed circa 3000 data items.

3.2 *Traffic system*

The original system The original system is unknown because it was used the data from traffic network. Inputs are input intensities and outputs are output intensities. The intensities are measured by magnetics loops which are cover in the roads.

The estimation of matrices We have one input and output and so the order equals 1. In the traffic model, we assumed, that the state space model has minimal dimension 2 because there is queue length and occupancy. In this case, we assumed that we do not have any knowledge about the system.

1. The estimated matrix A, B, C, D and covariances:
   (a) $A = -0.6648$,
   (b) $B = -0.2232$,
   (c) $C = -1.4508$,
   (d) $D = 0.7253$,
   (e) $K = 0.2368$,
   (f) $R = 12.509$,
   (g) time demand: $t_d = 1.5625\ s$.

Comparison the estimation matrices and the original model

This system is real system and so we do not have possibility how to compare the original model with SMs model. The comparison by eigenvalues it is impossible. The main comparison is made by the output prediction. The results looks very well because we know that the traffic system is very complicated. The measurement covariance $R = 12.509$ shows that the intensities on input and output are very different. It can be cause by the time delay. Figure 2 shows the difference between outputs with the covariances in SM model.

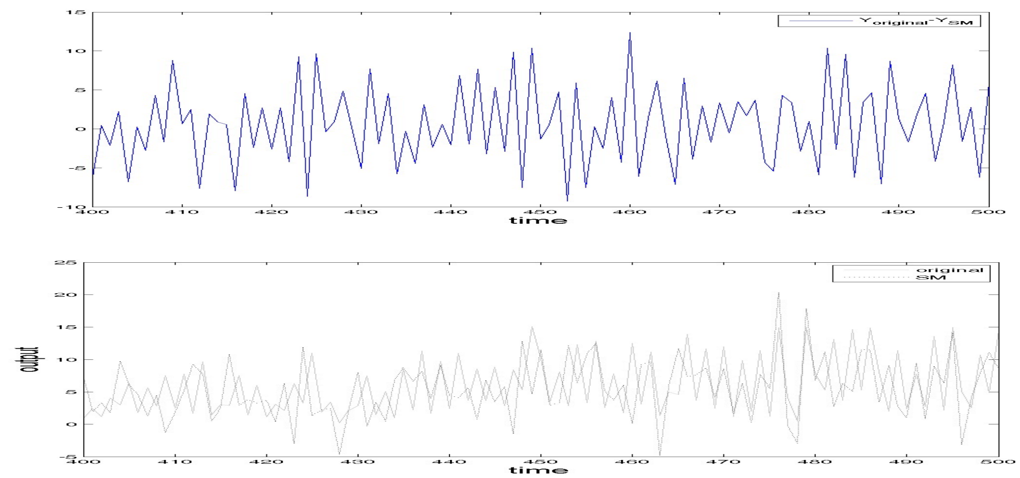

Figure 2: CA - order 1 with noise

Summary: The SMs model, which was applicated on the traffic data, give relatively good results. The noises are bigger than we assumed for this traffic system. The input covariance $R = 12,509$ looks unreliably.

4 CONCLUSION

In this paper, the SM methods and its using in the traffic model was described. The main aim of this paper was evaluation as estimator of state parameters and covariances. The SMs estimated the state parameters and covariances from knowledge of input and output. Using this method for on-line estimation and estimation of parameters β, κ etc. , it is not probable because the SMs estimate all values in matrices A, B, C and D and not only which we need. The using for off-line estimation of covariances does not looks believably because the covariances are strong depend on estimate of others matrices.

On the other hand, the prediction output for traffic system looks well. It is possible that the estimate of output intensity on roads where we do not have the output detectors or in case when the detectors break down can be made via SMs. This using will be more closely described in the next works.

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