

On the Tangential Velocity Arising in a Crystalline Approximation of Evolving Plane Curves

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Abstract: In a crystalline algorithm, a tangential velocity is used implicitly. In this short note, it is specified for the case of evolving plane curves, and is characterized by using the intrinsic heat equation.

Keywords: tangential velocity; intrinsic heat equation; crystalline algorithm; admissible polygonal curve;

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References

- [1] S. Angenent and M.E. Gurtin: Multiphase thermomechanics with interfacial structure, 2. Evolution of an isothermal interface. *Arch. Rational Mech. Anal.* 108 (1989), 323–391.
- [2] K. Deckelnick: Weak solutions of the curve shortening flow. *Calc. Var. Partial Differential Equations* 5 (1997), 489–510.
- [3] G. Dziuk: Convergence of a semi discrete scheme for the curve shortening flow. *Math. Models Methods Appl. Sci.* 4 (1994), 589–606.
- [4] Y. Giga: Anisotropic curvature effects in interface dynamics. *Sūgaku* 52 (2000), 113–127; English transl., *Sugaku Expositions* 16 (2003), 135–152.
- [5] M.-H. Giga and Y. Giga: Crystalline and level set flow – convergence of a crystalline algorithm for a general anisotropic curvature flow in the plane. *GAKUTO Internat. Ser. Math. Sci. Appl.* 13 (2000), 64–79.
- [6] M.E. Gurtin: *Thermomechanics of Evolving Phase Boundaries in the Plane*. Clarendon Press, Oxford 1993.
- [7] C. Hirota, T. Ishiwata, and S. Yazaki: Numerical study and examples on singularities of solutions to anisotropic crystalline curvature flows of nonconvex polygonal curves. In: *Advanced Studies in Pure Mathematics (ASPM); Proc. MSJ-IRI 2005 “Asymptotic Analysis and Singularity”*, Sendai 2005 (to appear).

- [8] H. Hontani, M.-H. Giga, Y. Giga, and K. Deguchi: Expanding selfsimilar solutions of a crystalline flow with applications to contour figure analysis. *Discrete Appl. Math.* 147 (2005), 265–285.
- [9] T. Ishiwata, T. K. Ushijima, H. Yagisita, and S. Yazaki: Two examples of nonconvex self-similar solution curves for a crystalline curvature flow. *Proc. Japan Academy, Ser. A* 80 (2004), 8, 151–154.
- [10] M. Kimura: Accurate numerical scheme for the flow by curvature. *Appl. Math. Letters* 7 (1994), 69–73.
- [11] K. Mikula and D. Ševčovič: Solution of nonlinearly curvature driven evolution of plane curves. *Appl. Numer. Math.* 31 (1999), 191–207.
- [12] K. Mikula and D. Ševčovič: Evolution of plane curves driven by a nonlinear function of curvature and anisotropy. *SIAM J. Appl. Math.* 61 (2001), 1473–1501.
- [13] J.E. Taylor: Constructions and conjectures in crystalline nondifferential geometry. In: *Proc. Conference on Differential Geometry, Rio de Janeiro, Pitman Monographs Surveys Pure Appl. Math.* 52 (1991), pp. 321–336, Pitman, London 1991.
- [14] J.E. Taylor: Motion of curves by crystalline curvature, including triple junctions and boundary points. *Diff. Geom.: Partial Diff. Eqs. on Manifolds (Los Angeles 1990), Proc. Sympos. Pure Math.*, 54 (1993), Part I, pp. 417–438, AMS, Providence.
- [15] J.E. Taylor, J. Cahn, and C. Handwerker: Geometric models of crystal growth. *Acta Metall.* 40 (1992), 1443–1474.
- [16] T.K. Ushijima and H. Yagisita: Approximation of the Gauss curvature flow by a three-dimensional crystalline motion. In: *Proc. Czech–Japanese Seminar in Applied Mathematics 2005; Kuju Training Center, Oita, Japan, September 15–18, 2005, COE Lecture Note 3, Faculty of Mathematics, Kyushu University (M. Beneš, M. Kimura, and T. Nakaki, eds.)*, 2006, pp. 139–145.
- [17] T.K. Ushijima and H. Yagisita: Convergence of a three-dimensional crystalline motion to Gauss curvature flow. *Japan J. Indust. Appl. Math.* 22 (2005), 443–459.
- [18] T.K. Ushijima and S. Yazaki: Convergence of a crystalline approximation for an area-preserving motion. *Journal of Comput. and Appl. Math.* 166 (2004), 427–452.
- [19] S. Yazaki: Motion of nonadmissible convex polygons by crystalline curvature. *Publ. Res. Inst. Math. Sci.* 43 (2007), 155–170.