

Computational Technique for Treating the Nonlinear Black-Scholes Equation with the Effect of Transaction Costs

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Abstract: We deal with numerical computation of the nonlinear partial differential equations (PDEs) of Black-Scholes type which incorporate the effect of transaction costs. Our proposed technique surmounts the difficulty of infinite domains and unbounded values of the solutions. Numerical implementation shows the validity of our scheme.

Keywords: transaction costs; nonlinear partial differential equation; numerical computation;

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