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DETERMINATION OF PHASE–SPACE RECONSTRUCTION PARAMETERS OF CHAOTIC TIME SERIES

Wei-Dong Cai, Yi-Qing Qin and Bing-Ru Yang

A new method called C–C–1 method is suggested, which can improve some drawbacks of the original C–C method. Based on the theory of period N, a new quantity $S(t)$ for estimating the delay time window of a chaotic time series is given via direct computing a time-series quantity $S(m, N, r, t)$, from which the delay time window can be found. The optimal delay time window is taken as the first period of the chaotic time series with a local minimum of $S(t)$. Only the first local minimum of the average of a quantity $\Delta S_2(t)$ is needed to ascertain the optimal delay time. The parameter of the C–C method – embedding dimension m – is adjusted rationally. In the new method, the estimates of the optimal delay time and the optimal delay time window are more appropriate. The robustness of the C–C–1 method reaches 40 %, whereas that of the C–C method is 30 %.

Keywords: phase-space reconstruction, embedding window, delay time, time series

AMS Subject Classification: 37D45

1. INTRODUCTION

Strange attractors embody the characteristics of chaotic systems. The evolvement of any component in a chaotic system is determined by other components which interacting each other, therefore the information of these correlated components is underlying in the evolution process of the component [2]. Analyses to chaotic time series are mostly based on the phase-space reconstruction. In [8, 10], Packard et al. suggested that the phase-space can be restructured from observing the single delay coordinate of the dynamical system, and the fundamental theorem of reconstruction, introduced by Takens [10, 11] and extended more recently in [9], gives no restriction on the time delay constant τ while for m states a sufficient (but not necessary) condition is $m \geq 2d+1$, where d is the fractal dimension of the underlying attractor, and m is the phase-space dimension. The theorem of phase-space reconstruction is as follows:

The method of delays can be used to embed a scalar time series $\{x(t_i), i = 1, 2, \ldots, N\}$ into an m-dimensional space $X(t_i) = (x(t_i), x(t_i + \tau), \ldots, x(t_i + (m-1)\tau))$, $i = 1, 2, \ldots, M$, where τ is the delay time, m is the embedding

dimension, M is the number of embedded points in the m-dimensional space, and $M = N - (m-1)\tau$. Set $\{X(t_i),\ i = 1, 2, \ldots, M\}$ shows the tracks of the strange attractor in the phase-space, and the chaotic evolvement of the dynamical system can be studied in the reconstructed m-dimensional space. Research shows that the reconstructed phase-space with appropriate m and τ has the same quality of diffeomorphism as the original dynamical system.

The selections of m and τ are rather important but difficult in the phase-space reconstruction [7]. There are two main points of selecting m and τ, see e. g. [5, 12, 13, 14].

Point 1: The selections of m are independent of the selections of τ. The selections of m and τ are based on 3 rules. The first one is the method of serial correlation, such as autocorrelation function, mutual information, high-order correlation, and so on; the second one is the method based on phase-space expansion, such as fill factor, average displacement, SVF, and so on; the last one includes the method of multiple autocorrelation function and the method of non-biased multiple autocorrelation function.

Point 2: m and τ are correlated one another for the reason that real data sets are finite and noisy. Tests show that the delay time window is $\tau_w = (m-1)\tau_d$, and it is the entire time spanned by the components of $\{X(t_i)\}$, which is independent of m instead. In this case, the delay time τ_d varies with the embedding dimension m. τ_w is comparatively steady for a certain time series, and the irrelevant partnership of m will affect the equivalence relationship between the reconstructed phase-space and the former space. Thus, some combined computing methods come into being, such as the C-C method [3], the time window length, the automated embedding and the creep phenomenon, and so on.

Many researchers agree with Point 2 above. They consider that the process of mutual information is rather cumbersome computationally, whereas the autocorrelation function only treats the linear dependence of the time series and it does not treat the nonlinearity appropriately, but it may yield an incorrect value for the delay time τ_d. The C–C method suggested is most popular, which gives the delay time τ_d and delay time window τ_w simultaneously by applying the correlation integral. Based on the statistical results, although the C–C method lacks theoretical support, it runs well in practice and it shows some advantages, such as simple operation, lower algorithm complexity, reliability for less data and better robustness, etc. It has become a regular method for analyzing the time series [4].

Aiming at improving some drawbacks of the C–C method, this paper suggests an advanced method to determine the optimal delay time τ_d and the optimal delay time window τ_w. It improves the computing process, parameter selections and the determination rules of the C–C method. The selections of the optimal delay time window τ_w are more reliable and stable, the determination of the optimal delay time τ_d is more appropriate, and the robustness is higher than that of the C–C method.

2. ANALYSIS OF THE C–C METHOD

2.1. Algorithm of the C–C method

Let the chaotic time series be $x = \{x_i, i = 1, 2, \ldots, N\}$, where m is the embedding dimension, τ_d is the delay time, and denote $X = \{X_i\}$, where X_i are the points in the m-dimensional space:

$$X_i = (x_i, x_i + \tau, \ldots, x_i + (m-1)\tau)^T, \quad i = 1, 2, \ldots, M. \tag{1}$$

Thus, the correlation integral for the embedded time series is [1]:

$$C(m, N, r, t) = \frac{2}{M(M-1)} \sum_{1 \le i < j \le M} \theta(r - d_{ij}), \quad r > 0 \tag{2}$$

where m is the embedding dimension, N is the data number of the time series, r is the search radius, t is the delay time, $M = N - (m-1)t$ is the number of embedded points in the m-dimensional space, $\theta(x)$ is the Heaviside function: $\theta(x) = 0$, if $x < 0$; $\theta(x) = 1$, if $x \ge 0$, and $d_{ij} = \| x_j - x_j \|_\infty$ denotes the sup-norm.

Correlation integral is a cumulative distribution function, which denotes the probability of distances between any pairs of points in the phase-space that are not greater than r. The distance between a pair of points is denoted by the sup-norm of the difference between the two vectors. Define the statistical quantity of the time series by

$$S(m, N, r, t) = C(m, N, r, t) - C^m(1, N, r, t). \tag{3}$$

The computing process of Eq. (3) is to subdivide the time series $x = \{x_i\}$ into t disjoint time series averagely, where t is the reconstructive delay time, i. e.

$$\begin{cases} x(1) = \{x_1, x_{t+1}, \ldots, x_{\lfloor N/t \rfloor - t + 1}\} \\ x(2) = \{x_2, x_{t+2}, \ldots, x_{\lfloor N/t \rfloor - t + 2}\} \\ \ldots \\ x(t) = \{x_t, x_{t+t}, \ldots, x_{\lfloor N/t \rfloor}\}. \end{cases} \tag{4}$$

Here, define the average of the statistical quantity given by Eq. (3) as follows:

$$S_1(m, N, r, t) = \frac{1}{t} \sum_{s=1}^{t} [C_s(m, N, r, t) - C_s^m(1, N, r, t)]. \tag{5}$$

As $N \to \infty$, we can write

$$S_1(m, r, t) = \frac{1}{t} \sum_{s=1}^{t} [C_s(m, r, t) - C_s^m(1, r, t)]. \tag{6}$$

For fixed m and t, $S_1(m, r, t)$ is identically equal to 0 for all r if $x = \{x_i\}$ is independently and identically distributed (i.i.d.) and $N \to \infty$. However, real data sets are finite, and the data may be correlated with noise; so, in general, $S_1(m, r, t) \neq$

0. Thus, the locally optimal times may be either the zero crossings of $S_1(m,r,t)$ or the times at which $S_1(m,r,t)$ shows the least variation in r, since this indicates a nearly uniform distribution of points. Hence, we select several representative values r_j, and define the quantity

$$\Delta S_1(m,t) = \max\{S_1(m,r_i,t)\} - \min\{S_1(m,r_i,t)\} \tag{7}$$

which is a measure of the variation of $S_1(m,r,t)$ in r. The locally optimal times t are then the zero crossings of $S_1(m,r,t) \sim t$ and the minima of $\Delta S_1(m,t) \sim t$. The zero crossings of $S_1(m,r,t) \sim t$ should be nearly the same for all m and r, and the minima of $S_1(m,r,t) \sim t$ should be nearly the same for all m (otherwise, the time is not locally optimal). The delay time τ_d will correspond to the first of these locally optimal times.

Appropriate choices for m, N and r may be found by examining the BDS statistic. Generally, for $N = 3000$, $m = 2,3,4,5$, $t = 1,2,\ldots,200$, $r = k \times \sigma/2$, $k = 1,2,3,4$, where $\sigma =$ std (x) is the standard deviation of the time series. We then define the following averages of the quantities given by Eqs. (6) and (7):

$$\overline{S}_1(t) = \frac{1}{16}\sum_{m=1}^{4}\sum_{i=1}^{4} S_1(m,r_i,t) \tag{8}$$

$$\Delta\bar{S}_1(t) = \frac{1}{4}\sum_{m=1}^{4} \Delta S_1(m,t) \tag{9}$$

and we look for the first zero crossing of $\overline{S}_1(t)$ or the first local minimum of $\Delta\bar{S}_1(t)$ to find the first locally optimal time for independence of the data, which gives the delay time τ_d. The optimal time is the delay time t for which $\bar{S}_1(t)$ and $\Delta\bar{S}_1(t)$ are both closest to 0. If we assign equal importance to these two quantities, then we may simply look for the minimum of the quantity

$$S_{1\_cor}(t) = \Delta\bar{S}_1(t) + \mid \bar{S}_1(t) \mid \tag{10}$$

and this optimal time gives the delay time window τ_w.

2.2. Numerical examples of the C–C method

In these examples, we observe variable x from the chaotic Lorenz system by integrating equations using function ode45 in MATLAB. The Lorenz system (11) is as follows:

$$\begin{cases} \mathrm{d}x/\mathrm{d}t = -\sigma x + \sigma y \\ \mathrm{d}y/\mathrm{d}t = -xz + rx - y \\ \mathrm{d}z/\mathrm{d}t = xy - bz \end{cases} \tag{11}$$

where σ, r and b are constants. We solve this system of equations for $[\sigma, r, b] = [16.0, 4.0, 45.92]$, with initial conditions $[x, y, z] = [-1, 0, 1]$, to generate a time series of the variable x with interval of integration from 0 to 1000, step $h = 0.01$.

The reconstruction results are the same as those given by Kim et al. in [3], tested by 3000 points selected from 53001 to 56000 (Figure 1).

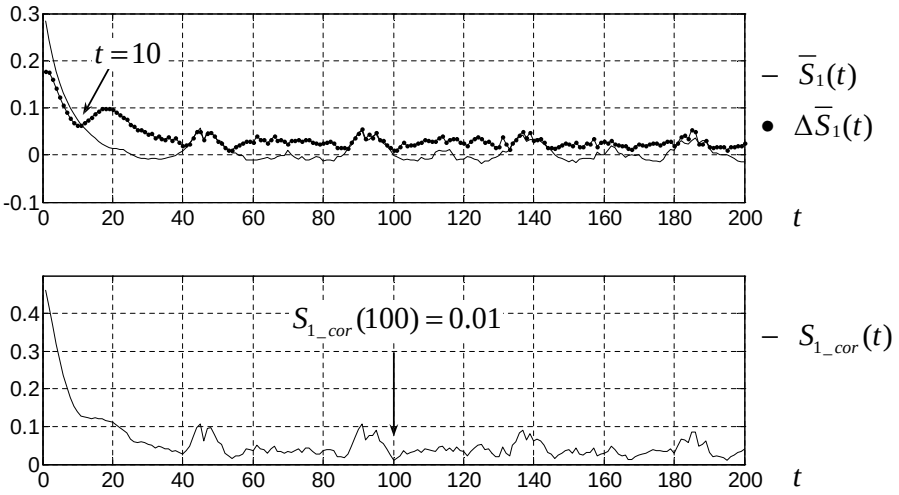

Fig. 1. C–C method: analysis on variable x from the Lorenz system.

2.3. Drawbacks of the C–C method

While the numerical examples are carrying on, we also select 3000 points from different intervals to estimate the optimal delay time τ_d and the optimal delay time window τ_w. The results are shown in Table 1.

Table 1. C–C method: results of the reconstructed variable x from the Lorenz system.

Sample Interval	m	τ_d	τ_w
10001–13000	21	10	191
20001–23000	15	10	132
30001–33000	20	10	184
40001–43000	11	11	104
50001–53000	14	11	137
60001–63000	14	11	137
70001–73000	9	11	84
80001–83000	17	10	152
90001–93000	11	11	101

There are at least 3 drawbacks in the C–C method.

(1) Ideally the minimum of $S_{1\_cor}(t)$ is the optimal delay time window τ_w, whereas in the tests there are some local minimal points whose values are much close to the minimum of $S_{1\_cor}(t)$. They disturb the estimate of the minimum of $S_{1\_cor}(t)$. And even worse, the optimal delay time window τ_w is not the exact minimum point, which may mislead the estimate of the optimal delay time window τ_w. In Figure 2, the marked points are all likely to be the optimal delay time window τ_w.

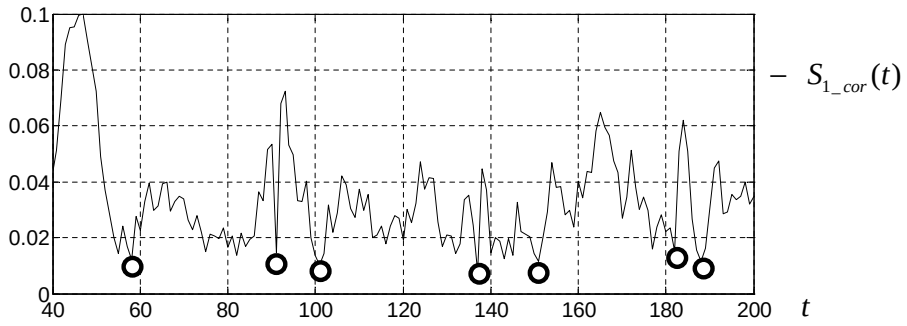

Fig. 2. C–C method: results of the local minima and the minimum of $S_{1\_cor}(t)$.

(2) In practice, the first zero crossing of $\overline{S}_1(t)$ is unequal to the first local minimum of $\Delta\overline{S}_1(t)$. But for the time series with period T, for $t = kT,\ k = 1, 2, \ldots$, one of the points is mostly not only the first zero crossing of $\bar{S}_1(t)$ but also the minimum of $S_{1\_cor}(t)$; therefore, paradoxical conclusions can be drawn. We suggest that it is not appropriate to take the first zero crossing of $\bar{S}_1(t)$ as the optimal delay time τ_d. We may consider taking the first local minimum of $\Delta\bar{S}_1(t)$ as the first locally optimal time τ_w.

(3) Strictly speaking, a chaotic system has no period. For low-dimensional chaotic systems with period N, the mean orbital period T is the mean period generated by the oscillations of the chaotic attractor in the phase-space orbits. The computing mode of Eq. (5) leads to the following result: if $t = kT,\ k = 1, 2, \ldots$, then $\Delta\bar{S}_1(t) = 0$, and $\Delta\bar{S}_1(t)$ shows high-frequency oscillations increasingly along with the increase of t. When the value of the optimal delay time τ_d is big enough, the high-frequency oscillations can even affect the estimate of the first local minimum of $\Delta\bar{S}_1(t)$.

Aiming at improving the drawbacks of the C–C method, we suggest an improved method of phase-space reconstruction, which is called C–C–1 method.

3. THE IMPROVED METHOD: C–C–1 METHOD

3.1. Algorithm of the C–C–1 method

By comparing $S(m, N, r, t)$ with $S_1(m, N, r, t)$ in Eq. (5), for fixed m, and $n \to \infty$, $S_1(m, N, r, t) \sim t$ shows high-frequency oscillations increasingly along with the increase of t. In Eq. (3), on the same conditions, generally $S(m, N, r, t) \sim t$ has the same oscillations characteristics as $S_1(m, N, r, t) \sim t$, whereas the high-frequency oscillations of $S_1(m, N, r, t) \sim t$ disappear.

Therefore, instead of subdividing the time series $x = \{x_i\}$ into t disjoint time series, the C–C–1 method computes $S(m, N, r, t)$ directly. Since chaotic time series has intrinsic determinacy, and the direct algorithm is rather cumbersome computationally, in order to reduce the time complexity, the statistical quantity $S(m, N, r, t)$ given by Eq. (3) is computed with another average method. Being different from the C–C method, a positive integer p is selected as a constant, which is independent of the delay time t, to subdivide the reconstructed phase-space $X = \{X(t_i)\}$; and according to the calculation of $\overline{S}_1(t)$ and $\Delta\bar{S}_1(t)$ in the C–C method, $\bar{S}_2(t)$ and $\Delta\bar{S}_2(t)$ are calculated.

Numbers of tests show that $S_{2\_cor}(t)$ have some clear peak values with qualitatively chaotic period N, and all of the points that bring these clear peak values are the local minima of $S_{1\_cor}(t)$. Thus, a new determinative rule of the optimal delay time window τ_w is given: to estimate the optimal delay time window τ_w, the C–C method looks for the minimum of $S_{1\_cor}(t)$, whereas the C–C–1 method combines the clear peak values of $S_{2\_cor}(t)$ with the chaotic period N and with the local minima of $S_{1\_cor}(t)$. By looking for the first local minimum peak value of $S_{1\_cor}(t) - S_{2\_cor}(t)$ with the clear quality of chaotic period N, we estimate the optimal delay time window τ_w; and aiming at the results with no clear quality of chaotic period N, we select the minimum of $S_{1\_cor}(t) - S_{2\_cor}(t)$ to estimate the optimal delay time window τ_w.

Furthermore, the C–C method looks for the first zero crossing of $\bar{S}_1(t)$ or the first local minimum of $\Delta\overline{S}_1(t)$ as the first optimal delay time τ_d, while the C–C–1 method just looks for the first local minimum of $\Delta\bar{S}_2(t)$ as the first optimal delay time τ_d.

The algorithm of the C–C–1 method is summarized as follows:

The phase-space reconstruction is the first step. Then, a positive integer p is selected as a constant, which is independent of the delay time t, to subdivide the reconstructed phase-space $X = \{X(t_i)\}$:

$$\begin{cases} X(1) = \{X_1, X_{p+1}, \ldots, X_{\lfloor N/p \rfloor - p+1}\} \\ X(2) = \{X_2, X_{p+2}, \ldots, X_{\lfloor N/p \rfloor - p+2}\} \\ \ldots \\ X(p) = \{X_p, X_{p+p}, \ldots, X_{\lfloor N/p \rfloor}\} \end{cases} \tag{12}$$

$$\begin{cases} x(1) = \{X_1(1), X_1(2), \ldots, X_1(m), X_{p+1}(1), X_{p+1}(2), \ldots, X_{p+1}(m), \ldots\} \\ x(2) = \{X_2(1), X_2(2), \ldots, X_2(m), X_{p+2}(1), X_{p+2}(2), \ldots, X_{p+2}(m), \ldots\} \\ \ldots \\ x(p) = \{X_p(1), X_p(2), \ldots, X_p(m), X_{p+p}(1), X_{p+p}(2), \ldots, X_{p+p}(m), \ldots\}. \end{cases} \tag{13}$$

We define the average of the statistical quantity given by Eq. (3) as follows:

$$S_2(m, r, t) = \frac{1}{p} \sum_{s=1}^{p} C_s(m, r, t) - \left[\frac{1}{p} C_s(1, r, t)\right]^m \tag{14}$$

where p is an adjustable parameter to balance the precision and speed of calculation. The definitions of $\Delta S_2(m, t)$, $\bar{S}_2(t)$, $\Delta \bar{S}_2(t)$ and $S_{2\_cor}(t)$ are all the same as Eqs. (7), (8), (9) and (10).

For $p = 1$, Eq. (14) is equal to Eq. (3), so the results of $S_2(m, r, t)$ have the highest precision, but the algorithm has the highest time complexity. For $p > 1$, the algorithm has the lower time complexity. Tests show that although the new algorithm still has a few errors (the same situation as that in the C–C method), these errors do not disturb the estimates of the local minima.

Furthermore, we just look for the first local minimum of $\Delta \bar{S}_2(t)$ as the first optimal delay time τ_d. Considering $S_{1\_cor}(t)$ and $S_{2\_\mathrm{cor}}(t)$ roundly, if we assign equal importance to these two quantities, and define a new quantity

$$S_{cor}(t) = S_{1\_cor}(t) - S_{2\_cor}(t) \tag{15}$$

then we may simply look for the first local minimum peak value of the quantity $S_{cor}(t)$ with the clear quality of chaotic period N. This optimal time gives the optimal delay time window τ_w; but if the results are not with clear quality of chaotic period N, the minimum of $S_{cor}(t)$ gives the optimal delay time window τ_w.

3.2. Numerical examples of the C–C–1 method

As tests, we apply the C–C–1 method to the Lorenz system. Large numbers of simulations prove that the adjustments to the ranges of the embedding dimension m can help to obtain a more appropriate optimal delay time τ_d. Let $m = 2, 3, \ldots, 7$, $p = 60$, and other conditions be the same as the C–C method.

Figures 3, 4, and 5 show some contrastive results between the C–C method and the C–C–1 method (where 3000 points are selected from 60001 to 63000).

Analyzing Figures 3, 4, and 5 shows that high-frequency oscillations are still increasing along with the increase of t but they are improved significently, and more importantly, the local maxima with the chaotic period N can be found from the graph of the phase-space reconstruction (Figure 4).

In Figure 4, when $t = 46, 92, 138, 184$, the clear maximum peak values of $S_{2\_cor}(t)$ with the quality of chaotic period N are given. Comparing with Figure 3, these values are all local minima of $S_{1\_cor}(t)$. Thus, an important conclusion is drawn: in the C–C–1 method, the optimal delay time window τ_w is estimated by the first local minimum peak value of $S_{cor}(t)$, with a clear quality of chaotic period N.

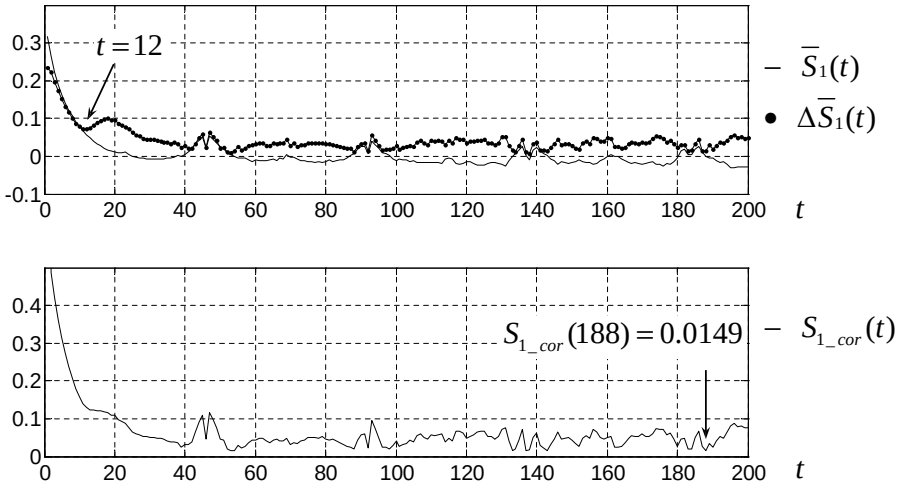

Fig. 3. C–C method: analysis on variable x from the Lorenz system ($m = 2, 3, \ldots, 7$).

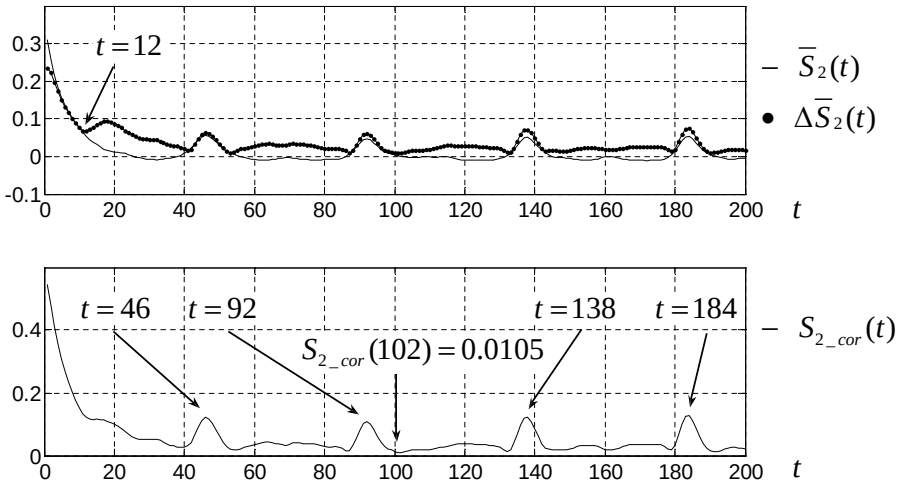

Fig. 4. C–C–1 method: analysis on variable x from the Lorenz system ($m = 2, 3, \ldots, 7,\ p = 60$).

The results of different test intervals of the Lorenz system in the C–C–1 method are shown in Table 2.

Table 2 shows that the estimates of the optimal delay time τ_d in the C–C–1 method are the same as that in the C–C method, whereas the estimates of the optimal delay time window τ_w in the C–C–1 method – stable values – are different from that in the C–C method. Therefore, by the formula $\tau_w = (m-1)\tau_d$, the results

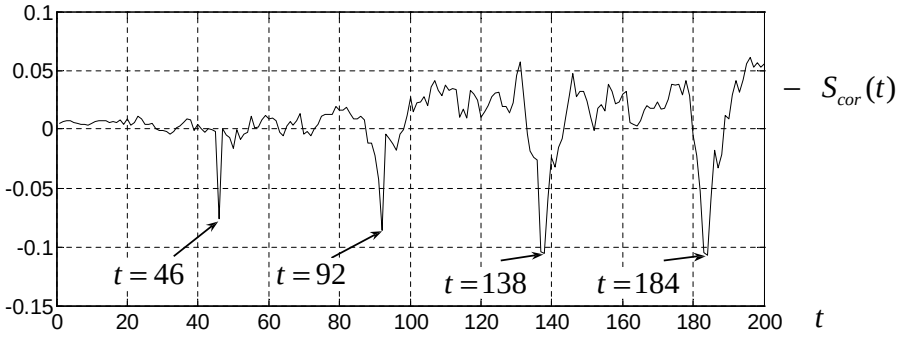

Fig. 5. C–C–1 method: results of local minimal peak values of $S_{cor}(t)$ $(m = 2, 3, \ldots, 7,\ p = 60)$.

of the optimal embedding dimension m become more robust, and are most close to the well-known optimal embedding dimension m calculated by the theoretical value of the fractal dimension, $d = 2.06$ of the Lorenz system [6]. The complexity of the prediction and the analysis on the chaotic time series based on the phase-space reconstruction is lower than before.

Table 2. Results of the reconstructed variable x from the Lorenz system $(m = 2, 3, \ldots, 7,\ p = 60)$.

Sample Interval	C–C method			C–C–1 method		
	m	τ_d	τ_w	m	τ_d	τ_w
10001–13000	14	12	148	5	12	46
20001–23000	10	12	99	6	11	46
30001–33000	17	12	184	6	11	46
40001–43000	12	12	122	5	12	46
50001–53000	14	12	151	5	12	46
60001–63000	17	12	188	5	12	46
70001–73000	15	12	167	5	12	46
80001–83000	14	12	152	5	12	46
90001–93000	10	12	101	5	12	46

In order to test the universality of the C–C–1 method, we also applied it to the Duffing chaotic system:

$$\begin{cases} \mathrm{d}x/\mathrm{d}t = y \\ \mathrm{d}y/\mathrm{d}t = -\delta y - \alpha x(1 + x^2) + f\cos z \\ \mathrm{d}z/\mathrm{d}t = \omega \end{cases} \tag{16}$$

where $\delta,\ \alpha,\ f$ and ω are constants. We solve this system of equations for $[\delta, \alpha, f, \omega] = [0.05, 0.5, 7.5, 1]$, with initial conditions $[x, y, z] = [-1, 0, 1]$, and integrate equations

by function ode45 in MATLAB, to generate a time series of the variable x with interval of integration from 0 to 5000, step $h = 0.05$.

In testing the Duffing system, we select 3000 points from 50001 to 53000. For $m = 2, 3, \ldots, 7,\ p = 60$, and in order to show the characteristic of the chaotic period N, we let $t = 1, 2, \ldots, 300$.

The reconstructed results of x from the Duffing system in the C–C method and the C–C–1 method are shown in Figures 6, 7, and 8. The local minimum peak values with the chaotic period N are also given in the graph of the phase-space reconstruction.

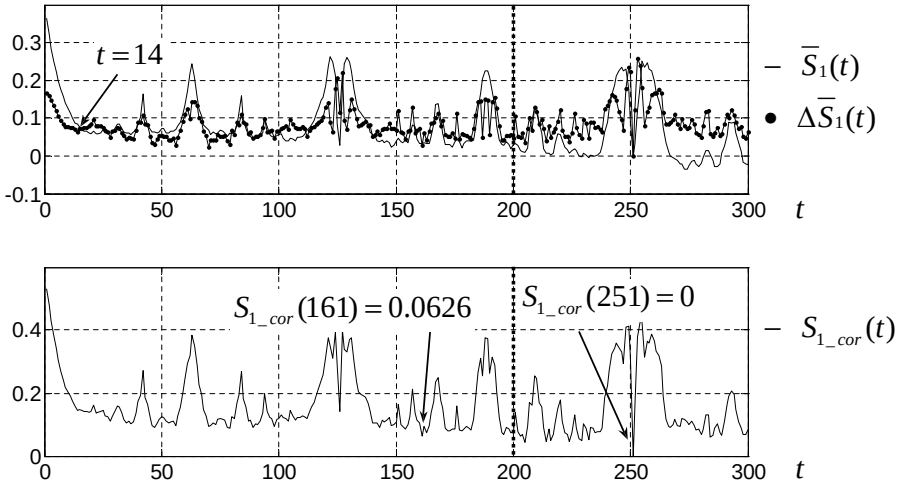

Fig. 6. C–C method: analysis on variable x from the Duffing system ($m = 2, 3, \ldots, 7$).

Analyses show that the C–C–1 method has broader universality and a more appropriate optimal delay time window τ_w, and the optimal delay time τ_d can be obtained.

4. NOISE EFFECTS

To study the effects of noise on the C–C–1 method, we add Gaussian noise to the Lorenz time series. Specifically, we examine the time series $y_i = x_i + \eta\sigma\varepsilon_i$, where x_i is the noise-free Lorenz time series, σ is its standard deviation, ε_i is a Gaussian i.i.d. random variable with zero mean and a standard deviation of 1, and η is the strength of the noise. Noise levels of 10 %, 20 %, ..., and 60 % (with $\eta = 0.1, 0.2, \ldots, 0.6$) are added to the Lorenz time series, and the C–C–1 method is performed for each of these noise levels. According to the error standard given in [3], we observe that the estimates of the optimal delay time τ_d and the optimal delay time window τ_w remain unchanged for $\eta = 0.1, 0.2, 0.3, 0.4$, but not for $\eta = 0.5, 0.6$. The results show that the robustness of the C–C–1 method reaches 40 %, whereas that of the C–C method is 30 %.

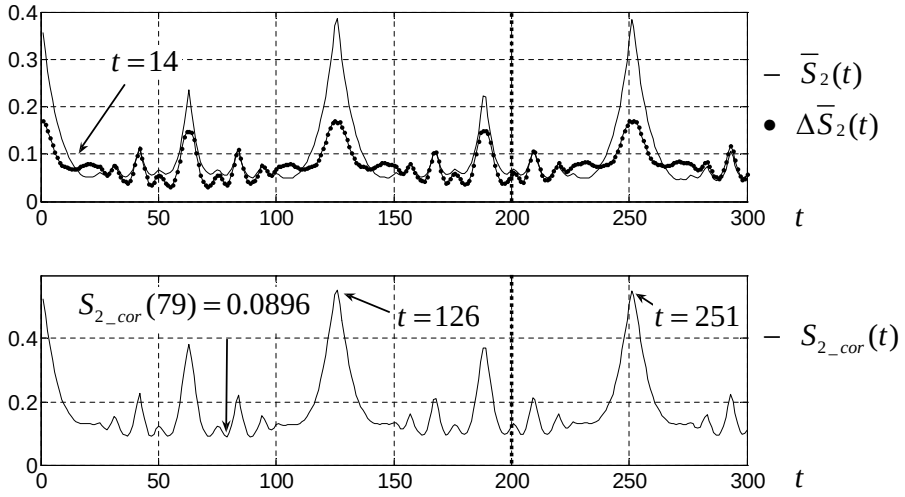

Fig. 7. C–C–1 method: analysis on variable x from the Duffing system ($m = 2, 3, \ldots, 7,\ p = 60$).

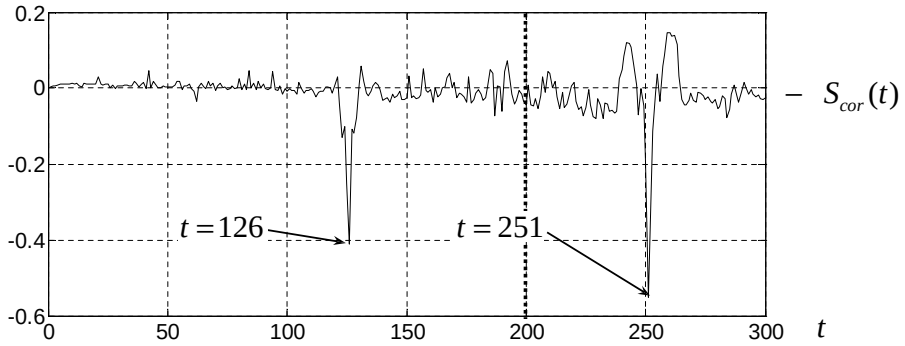

Fig. 8. C–C–1 method: results of local minimal peak values of $S_{cor}(t)$ ($m = 2, 3, \ldots, 7,\ p = 60$).

5. CONCLUSIONS

The C–C–1 method is an improvement of the C–C method. It designs one mode to subdivide the time series by a parameter p, which is independent of the embedding time t. The results of the process clearly show the quality of chaotic period N of the chaotic time series. Furthermore, a new $S_{cor}(t)$ quantity to estimate the optimal delay time window τ_w of the chaotic time series is given. As a new rule of estimating the optimal delay time window τ_w, the C–C–1 method looks for the first local minimum of the quantity $S_{cor}(t)$ and this optimal time should be the

first period of the chaotic time series. On the other hand, we point out that in the C–C method the rule of determining the optimal delay time τ_d by choosing the zero crossings of $\bar{S}_1(t)$ may lead to an incorrect value. The C–C–1 method looks for the first local minimum of $\Delta\bar{S}_2(t)$, which gives the optimal delay time τ_d. In the computing process, the parameter of the C–C method – embedding dimension m – is adjusted rationally in order to obtain more appropriate estimates of the optimal delay time τ_d. Tests show that the C–C–1 method can give more reliable and stable estimates of the optimal delay time window τ_w and the optimal delay time τ_d. We also demonstrate the robustness of this method in the presence of noise. The noise levels reach 40 %, which is about 10 % higher than that of the C–C method.

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Wei-Dong Cai, School of Information Science and Engineering, University of Ji'nan, Ji'nan 250022 and School of Information Engineering, University of Science and Technology Beijing, Beijing 100083. China.
e-mail: caiwd.jn@gmail.com

Yi-Qing Qin, Computer School, Beijing Information Science & Technology University, Beijing 100085, China and School of Information Engineering, University of Science and Technology Beijing, Beijing 100083. China.
e-mail: qyq_email@sina.com

Bing-Ru Yang, School of Information Engineering, University of Science and Technology Beijing, Beijing 100083. China.
e-mail: bryang_kd@yahoo.com.cn