Continuous Extension of Order-preserving Homogeneous Maps.

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Abstract: Maps f defined on the interior of the standard non-negative cone K in $\mathbb{R}^N$ which are both homogeneous of degree 1 and order-preserving arise naturally in the study of certain classes of Discrete Event Systems. Such maps are non-expanding in Thompson's part metric and continuous on the interior of the cone. It follows from more general results presented here that all such maps have a homogeneous order-preserving continuous extension to the whole cone. It follows that the extension must have at least one eigenvector in $K - \{0\}$. In the case where the cycle time $\chi(f)$ of the original map does not exist, such eigenvectors must lie in $\partial K - \{0\}$.