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Probabilistic Analysis of Belief Functions

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The adequate processing of indeterminism in the modelled phenomena belongs to the most urgent topics of the (not only) applied mathematics of the second half of the 20th century, and it will be probably also one of the most inspiring tasks of the mathematics in the beginning of a new century. Good management of indeterminism of any kind (randomness, vagueness, uncertainty and others) is a necessary condition for further development of some applications of mathematical tools in such branches like optimization, decision-making, economic models, sociological models and many others, in which the indeterminism represents a significant component of the modelled and investigated reality.

The referred book contributes to this field in a very important way. It is focused on various aspects of the Dempster–Shafer theory and namely of its fundamental concept of the belief function. As far as the author of this review knows, the Kramosil's book is the first general monograph summarizing the probabilistic views on the concept of the belief function, its properties, and its wider consequences for the modelling of uncertainty. Of course, the Dempster–Shafer theory and belief functions can be treated even without using the probabilistic concepts. On the other hand, including these concept in the original model means some advantages. It supports the realism of the description of the optimization problems and their solutions. Even more, it opens the way to qualified utilization of the classical and well developed probabilistic apparatus for an inspiring approach to qualitatively new types of problems.

The book is written in a lucid and mathematically perfect way. The explanation of the topic is divided into 14 relatively short parts. It simplifies the orientation of the reader in the presented text and in the relation between its particular parts. These 14 parts can be, with certain degree of simplification, divided into three groups, which are not strictly separated but which fluently pass over the each other.

The first group contains chapters of rather introductory character (*Introduction, Preliminaries on Axiomatic Probability Theory, Probabilistic Model of Decision Making Under Uncertainty* and in certain degree also *Basic Elements of Dempster–Shafer Theory*). These chapters introduce and discuss the background of the investigated problems and of the mathematical tools prepared for their analysis.

In the second group, we can include a few chapters oriented to the classical model of belief functions and to its stochastic version based on the standard probabilistic concepts. The last chapter mentioned in the previous paragraph is a sort of transition between the previous and this part. Its other chapters are *Elementary Properties of Belief Functions, Probabilistic Analysis of Dempster Combination Rule* and also *Nonspecificity Degrees of Basic Probability Assignments*. The last group contains specific modifications of the standard model, using some advanced or generalized counterparts of the classical probabilistic and generally mathematical concepts used in the chapters from the previous part. It contains chapters devoted to *Belief Functions Induced by Partial Compatibility Relations, Belief Functions Over Infinite State Spaces, Boolean Combinations of Set–Valued Random Variables, Belief Functions With Signed and Nonstandard Values, Jordan Decomposition of Signed Belief Functions* and also rather special modifications of the investigated con-

cepts and their processing in specific models, namely *Monte–Carlo Estimations for Belief Functions*, and *Boolean–Valued and Boolean–Like Processed Belief Functions*.

The monograph is completed by a list of references (113 items) which representatively covers the topics mentioned in the book, and by an Index.

The referred book offers a complete survey of the methods and results regarding the relevant branches. Moreover, the exact mathematical presentations of the theoretical concepts is completed by a heuristic and methodological comments and discussions of their mutual relations, as well as of their wider consequences for the human understanding of the real world. Even this feature turns the book to something more than "only" perfect mathematical monograph. Some of the readers may, perhaps, miss some numerical examples illustrating the theoretical explanations. Nevertheless, it is to be stated that such numerical illustrations could break the homogeneity of the referred work, and that they can be found, if necessary, in the referred literature.

The Kramosil's book can be recommended to readers who have managed the basic mathematical culture and who are interested in a representative explanation of the topic mentioned in its heading.

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