On the existence and regularity of weak solutions to the Navier-Stokes Fourier systems under Dirichlet boundary conditions

Institute of Mathematics, Prag, April, 16- April, 26 2009

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Titles and contents of the lectures:

1. On the pressure of the Navier-Stokes equations from different points of view

The Navier-Stokes equations and related system describe an incompressible viscous fluid, where the volume is preserved. Therefore the pressure p has to be introduced as an additional unknown quantity. From the theory of the non-stationary Stokes system using the well-known properties of the Stokes operator under suitable conditions the pressure for the Navier-Stokes equation can be introduced. However, in general this method does not work, for example if the viscosity ν is non-constant. In this first lecture will introduce different methods for introducing a global and local pressure in general situations.

2. Existence of suitable weak solutions to the Navier-Stokes-Fourier system and partial regularity - Part I

A heat conducting incompressible fluid will be governed by the Navier-Stokes-Fourier system, where the unknowns are the velocity u the pressure p and the temperature θ. We consider the constitutive law $S(Du) = \nu(\theta)Du$, where θ satisfies

$$(1) \qquad \partial_t\theta + (u\cdot\nabla)\theta - \mathrm{div}(\kappa(\theta)\nabla\theta) = \nu(\theta)Du : Du.$$

The first part deals with the existence of weak solutions under Dirichlet-boundary conditions involving a defect measure μ. At the same time we provide a generalized local energy inequality together with a generalized local energy equality for the total energy.

3. Existence of suitable weak solutions to the Navier-Stokes-Fourier system and partial regularity - Part II

On the basis of the local energy inequality one obtains a partial regularity result, where the singular set Σ has zero three-dimensional Hausdorff measure. As a consequence one deduces that the defect measure μ is concentrated on Σ.

4. Existence of renormalized solutions to the Navier-Stokes-Fourier system and partial regularity

Setting $s := \log(1+\theta)$ by an elementary calculus the equation (1) turns into

$$(2) \qquad \partial_t s + (u \cdot \nabla) s - \operatorname{div}(\kappa(\theta)\nabla s) \,=\, \nu(\theta)\frac{Du : Du}{1+\theta} + |\nabla s|^2.$$

Taking into account the partial regularity result from above one gets a suitable weak solution (u, p, θ, μ), simultaneously s satisfying (2) in sense of distributions.