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Bound-Based Decision Rules in Multistage Stochastic Programming

Daniel Kuhn; Panos Parpas; Berç Rustem

Abstract: We study bounding approximations for a multistage stochastic program with expected value constraints. Two simpler approximate stochastic programs, which provide upper and lower bounds on the original problem, are obtained by replacing the original stochastic data process by finitely supported approximate processes. We model the original and approximate processes as dependent random vectors on a joint probability space. This probabilistic coupling allows us to transform the optimal solution of the upper bounding problem to a near-optimal decision rule for the original problem. Unlike the scenario tree based solutions of the bounding problems, the resulting decision rule is implementable in all decision stages, i.e., there is no need for dynamic reoptimization during the planning period. Our approach is illustrated with a mean-risk portfolio optimization model.

Keywords: stochastic programming; bounds; decision rules; expected value constraints; portfolio optimization;

AMS Subject Classification: 90C15;

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