

Risk Objectives in Two-Stage Stochastic Programming Models

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Abstract: In applications of stochastic programming, optimization of the expected outcome need not be an acceptable goal. This has been the reason for recent proposals aiming at construction and optimization of more complicated nonlinear risk objectives.

We will survey various approaches to risk quantification and optimization mainly in the framework of static and two-stage stochastic programs and comment on their properties. It turns out that polyhedral risk functionals introduced in Eichorn and Römisch [17] have many convenient features. We shall complement the existing results by an application of contamination technique to stress testing or robustness analysis of stochastic programs with polyhedral risk objectives with respect to the underlying probability distribution. The ideas will be illuminated by numerical results for a bond portfolio management problem.

Keywords: two-stage stochastic programs; polyhedral risk objectives; robustness; contamination; bond portfolio management problem;

AMS Subject Classification: 90C15; 91B28;

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