

Stability of Stochastic Optimization Problems - Nonmeasurable Case

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Abstract: This paper deals with stability of stochastic optimization problems in a general setting. Objective function is defined on a metric space and depends on a probability measure which is unknown, but, estimated from empirical observations. We try to derive stability results without precise knowledge of problem structure and without measurability assumption. Moreover, ε -optimal solutions are considered.

The setup is illustrated on consistency of a ε - M -estimator in linear regression model.

Keywords: stability of stochastic optimization problem; weak convergence of probability measures; estimator consistency; metric spaces;

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