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Sixty Years of Cybernetics: A Comparison of Approaches to Solving the $\mathbf{H}_2$ Control Problem

Vladimír Kučera

Abstract: The ¡span class="tex"¿H¡sub¿2¡/sub¿ ¡/span¿ control problem consists of stabilizing a control system while minimizing the ¡span class="tex"¿H¡sub¿2¡/sub¿ ¡/span¿ norm of its transfer function. Several solutions to this problem are available. For systems in ¡em¿state space¡/em¿ form, an optimal regulator can be obtained by solving two algebraic Riccati equations. For systems described by ¡em¿transfer functions¡/em¿, either Wiener-Hopf optimization or projection results can be applied. The optimal regulator is then obtained using operations with proper stable rational matrices: inner-outer factorizations and stable projections.

The aim of this paper is to compare the two approaches. It is well understood that the inner-outer factorization is equivalent to solving an algebraic Riccati equation. However, why are the stable projections not needed in the state-space approach?

The difference between the two approaches derives from a different construction of doubly coprime, proper stable matrix fractions used to represent the plant. The transfer-function approach takes any ¡em¿fixed¡/em¿ doubly coprime fractions, while the state-space approach parameterizes ¡em¿all¡/em¿ such representations and those selected then obviate the need for stable projections.

Keywords: linear systems; feedback control; stability; norm minimization;

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