

Kybernetika

VOLUME 40 (2004), NUMBER 2

The Journal of the Czech Society for
Cybernetics and Information Sciences

Published by:

Institute of Information Theory
and Automation of the Academy
of Sciences of the Czech Republic

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The Journal has been monitored in the Science Citation Index since 1977 and it is abstracted/indexed in databases of Mathematical Reviews, Current Mathematical Publications, Current Contents ISI Engineering and Computing Technology.

Kybernetika. Volume 40 (2004)

ISSN 0023-5954, MK ČR E 4902.

Published bi-monthly by the Institute of Information Theory and Automation of the Academy of Sciences of the Czech Republic, Pod Vodárenskou věží 4, 182 08 Praha 8. — Address of the Editor: P.O. Box 18, 182 08 Prague 8, e-mail: kybernetika@utia.cas.cz. — Printed by PV Press, Pod vrstevnicí 5, 140 00 Prague 4. — Orders and subscriptions should be placed with: MYRIS TRADE Ltd., P.O. Box 2, V Štíhlách 1311, 142 01 Prague 4, Czech Republic, e-mail: myris@myris.cz. — Sole agent for all “western” countries: Kubon & Sagner, P.O. Box 34 01 08, D-8 000 München 34, F.R.G.

Published in April 2004.

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DESIGN OF ROBUST OUTPUT AFFINE QUADRATIC CONTROLLER¹

VOJTECH VESELÝ

The paper addresses the problem robust output feedback controller design with guaranteed cost and affine quadratic stability for linear continuous time affine systems. The proposed design method leads to a non-iterative LMI based algorithm. A numerical example is given to illustrate the design procedure.

Keywords: robust control, parameter dependent Lyapunov function, affine quadratic stability, LMI approach

AMS Subject Classification: 93D15

1. INTRODUCTION

Robustness has been recognized as a key issue in the analysis and design of control systems for the last two decades. During the last decades numerous papers dealing with the design of static robust output feedback control schemes to stabilize uncertain systems have been published, Benton and Smith [1], Crusius and Trofino [3], Ghaoui and Balakrishnan [4], Geromel, De Souza, and Skelton [7], Gyurkovics and Takacs [9], Hejdiš and et al [10], Henrion and et al [11], Kose and Jabbari [12], Kozáková [13], Li Yu and Jian Chu [15], Mehdi, Al Hamid and Perrin [16], Pakshin [18], Pogyeon and et al [19], Tuan and et al [21], Xu and Darouch [24], Yong Yan Cao and You Xian Sun [25]. Various approaches have been used to study the two aspects of the robust stabilization problem, namely conditions under which the linear system described in state space can be stabilized via output feedback and the respective procedure to obtain a stabilizing or robustly stabilizing control law.

The necessary and sufficient conditions to stabilize the linear continuous time invariant system via static output feedback can be found in Kučera and De Souza [14] and in Veselý [22]. In the above and other papers, the authors basically conclude that despite the availability of many approaches and numerical algorithms the static output feedback problem is still open.

Recently, it has been shown that an extremely wide array of robust controller design problems can be reduced to the problem of finding a feasible point under a

¹The first version of this paper has been presented at IFAC 15th World Congress, Barcelona 2002.

Biaffine Matrix Inequality (BMI) constraint. The BMI has been introduced in Goh, Safonov and Papavassilopoulos [8]. In this paper, the BMI problem of robust controller design with output feedback is reduced to a LMI problem, Boyd, El Ghaoui, Feron and Balakrishnan [2]. The theory of Linear Matrix Inequalities has been used to design robust output feedback controllers in Benton and Smith [1], Crusius and Trofino [3], El Ghaoui and Balakrishnan [4], Henrion and et al [11], Li Yu and Jian Chu [15], Tuan, Apkarian, Hosoe and Tuy [21], Veselý [22]. Most of the above works present iterative algorithms in which a set of LMI problems are repeated until certain convergence criteria are met. The V-K iteration algorithm, proposed in El Ghaoui and Balakrishnan [4], is based on an alternative solution of two convex LMI optimization problems obtained by fixing the Lyapunov matrix or the gain controller matrix. This algorithm is guaranteed to converge, but not necessarily, to the global optimum of the problem depending on the starting conditions.

In this paper, new necessary and sufficient conditions to stabilize continuous time systems via static output feedback have been used to design a robust affine controller. For guaranteed cost and affine system this leads to a non iterative LMI based algorithm. The design procedure guarantees with sufficient conditions the robust affine quadratic stability for closed loop systems.

The paper is organized as follows. In Section 2 the problem formulation and some preliminary results are brought. The main results are given in Section 3. In Section 4 the obtained theoretical results are applied.

We have used the standard notation. A real symmetric positive (negative) definite matrix is denoted by $P > 0$ ($P < 0$). Much of the notation and terminology follows the references of Kučera and De Souza [14], and Gahinet, Apkarian and Chilali [5].

2. PRELIMINARIES AND PROBLEM FORMULATION

This paper is concerned with the class of uncertain linear systems that can be described as

$$\begin{aligned}\dot{x}(t) &= (A_{c0} + A_{c1}\theta_1 + \cdots + A_{ck}\theta_k)x(t) \\ &= A_c(\theta)x(t)\end{aligned}\tag{1}$$

where $\theta = [\theta_1, \dots, \theta_k] \in \mathbb{R}^k$ is a vector of uncertain and possibly time varying real parameters.

The system represented by (1) is a polytope of linear affine systems which can be described by a list of its vertices

$$\dot{x}(t) = D_{ci}x(t), \quad i = 1, 2, \dots, N\tag{2}$$

where $N = 2^k$.

The system represented by (2) is quadratically stable if and only if there is a Lyapunov matrix $P > 0$ such that

$$D_{ci}^T P + P D_{ci} < 0, \quad i = 1, 2, \dots, N.\tag{3}$$

A weakness of quadratic stability is that it guards against arbitrary fast parameter variations. As a result, this test tends to be conservative for constant or slow-varying

parameters θ . To reduce conservatism when (1) is affine in θ and the parameters of system are time invariant, in Gahinet, Apkarian and Chilali [5], the parameter-dependent Lyapunov functions $P(\theta)$ has been used in the form

$$P(\theta) = P_0 + P_1\theta_1 + \dots + P_k\theta_k. \quad (4)$$

Other types of parameter-dependent Lyapunov functions have been proposed in De Oliveira, Bernussou and Geromel [17] for the stability analysis of linear discrete time systems and for the analysis and the design of continuous time systems with affine type uncertainties in Henrion, Alzelier and Peaucelle [11] and in Takahashi, Ramos, and Peres [20].

We shall consider the following linear time invariant continuous time uncertain systems

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t) \\ y(t) &= C(\theta)x(t), \quad x(0) = x_0 \end{aligned} \quad (5)$$

where $x(t) \in \mathbb{R}^n$ is the plant state; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^\ell$ is the output vector of system; $A(\theta), B(\theta), C(\theta)$ are matrices of appropriate dimensions depending affinely on θ

$$A(\theta) = A_0 + A_1\theta_1 + \dots + A_k\theta_k$$

$$B(\theta) = B_0 + B_1\theta_1 + \dots + B_k\theta_k$$

$$C(\theta) = C_0 + C_1\theta_1 + \dots + C_k\theta_k.$$

Note that, in order to keep the polytope affine property, the matrix $B(\theta)$ or $C(\theta)$ must be precisely known. The following definition and theorem by Gahinet, Apkarian and Chilali [5] will be heavily exploited in the next development.

Definition 1. The linear system

$$\dot{x}(t) = A_c(\theta)x(t), \quad x(0) = x_0 \quad (6)$$

is affine quadratically stable if there exist $k + 1$ symmetric matrices P_0, P_1, \dots, P_k such that

$$P(\theta) = P_0 + P_1\theta_1 + \dots + P_k\theta_k > 0 \quad (7)$$

and

$$\frac{dV(x, \theta)}{dt} = x(t)^T \left(A_c^T(\theta)P(\theta) + P(\theta)A_c(\theta) + \frac{dP(\theta)}{dt} \right) x(t) < 0 \quad (8)$$

for $\theta = [\theta_1, \dots, \theta_k]$.

Note that quadratic stability corresponds to the case $P_1 = \dots = P_k = 0$. Sufficient affine quadratic stability conditions are given by the next theorem.

Theorem 1. (Gahinet, Apkarian and Chilali [5].) Consider the linear systems governed by (6), where $A_c(\theta)$ depends affinely on the uncertain parameter vector $\theta = [\theta_1, \dots, \theta_k]$ and θ_i satisfies

$$\theta_i \in \langle \underline{\theta}_i, \bar{\theta}_i \rangle, \quad \dot{\theta}_i \in \langle \underline{\nu}_i, \bar{\nu}_i \rangle, \quad \text{for } i = 1, 2, \dots, k \quad (9)$$

where $\underline{\theta}_i, \bar{\theta}_i, \underline{\nu}_i, \bar{\nu}_i$ are known lower and upper bounds. Let Γ and Λ denote the set of $N = 2^k$ vertices of the parameters box (9) and of the rate of variation box (9), respectively

$$\Gamma = \{(\gamma_1, \dots, \gamma_k) : \gamma_i = \underline{\theta}_i \text{ or } \gamma_i = \bar{\theta}_i\} \quad (10)$$

$$\Lambda = \{(\lambda_1, \dots, \lambda_k) : \lambda_i = \underline{\nu}_i \text{ or } \lambda_i = \bar{\nu}_i\}$$

and let

$$\theta_m = \left[\frac{\underline{\theta}_1 + \bar{\theta}_1}{2}, \dots, \frac{\underline{\theta}_k + \bar{\theta}_k}{2} \right]$$

denote the average value of the uncertain parameters vector. This system is affine quadratically stable if $A_c(\theta_m)$ is stable and there exist $k + 1$ symmetric matrices P_0, P_1, \dots, P_k such that $P(\theta) > 0$ satisfies

$$L(\gamma, \lambda) = A_c(\gamma)^T P(\gamma) + P(\gamma) A_c(\gamma) + P(\lambda) - P_0 < 0 \quad (11)$$

for all $(\gamma, \lambda) \in \Gamma \times \Lambda$ and

$$A_{ci}^T P_i + P_i A_{ci} \geq 0 \quad (12)$$

for $i = 1, 2, \dots, k$.

When (11) and (12) are met, a Lyapunov function for (6) and all trajectories $\theta(t)$ satisfying (9) is given by

$$V(x, \theta) = x^T(t) P(\theta) x(t).$$

The following performance index is associated with the system (5)

$$J = \int_0^\infty (x(t)^T Q x(t) + u(t)^T R u(t)) dt \quad (13)$$

where $Q = Q^T \geq 0, R = R^T > 0$ are matrices of compatible dimensions.

The problem studied in this paper can be formulated as follows:

For a continuous time system described by (5) design a static output feedback controller with the gain matrix F and control algorithm

$$u(t) = Fy(t) = FC(\theta)x(t) \quad (14)$$

so that the closed loop system

$$\dot{x} = (A(\theta) + B(\theta)FC(\theta))x(t) = A_c(\theta)x(t) \quad (15)$$

is affine quadratically stable with guaranteed cost.

Definition 2. Consider the system (5). If there exists a control law u^* and a positive scalar J^* such that closed loop system (15) is stable and the closed loop value cost function (13) satisfies $J \leq J^*$, then J^* is said to be the guaranteed cost and u^* is said to be the guaranteed cost control law for system (5).

3. THE MAIN RESULTS

In this section we present a new procedure to design a static output feedback controller for affine continuous time linear systems (5) which ensure the guaranteed cost and affine quadratic stability of closed loop system. The following theorem is one of the main results.

Theorem 2. For system (5) and Lyapunov function $V(\theta) = x(t)^T P(\theta)x(t) > 0$ the following statements are equivalent:

- There exist positive definite matrices Q and R that system (5) is static output feedback affine quadratic stabilizable (AQS) with guaranteed cost

$$\int_0^\infty x(t)^T (Q + C(\theta)^T F^T R F C(\theta)) x(t) dt \leq x_0^T P(\theta) x_0. \quad (16)$$

- There exist $k + 1$ symmetric matrices P_0, P_1, \dots, P_k that $P(\theta) > 0$, positive definite matrices Q and R , and matrix F such that the following inequality holds

$$\begin{aligned} & (A(\theta) + B(\theta)FC(\theta))^T P(\theta) + P(\theta)(A(\theta) + B(\theta)FC(\theta)) + \dot{P}(\theta) \\ & + Q + C(\theta)^T F^T R F C(\theta) < 0. \end{aligned} \quad (17)$$

- There exist $k + 1$ symmetric matrices P_0, P_1, \dots, P_k that (7) holds, positive definite matrices Q and R , and matrix F such that the following inequality holds

$$\begin{aligned} & A(\theta)^T P(\theta) + P(\theta)A(\theta) - P(\theta)B(\theta)R^{-1} \\ & B(\theta)^T P(\theta) + \dot{P}(\theta) + Q + G(\theta)^T R^{-1}G(\theta) < 0 \end{aligned} \quad (18)$$

where

$$G(\theta) = B(\theta)^T P(\theta) + R F C(\theta).$$

- There exist $k + 1$ symmetric matrices P_0, P_1, \dots, P_k that (7) holds, positive definite matrices Q and R , and matrix F such that the following inequality holds

$$A(\theta)^T P(\theta) + P(\theta)A(\theta) - P(\theta)B(\theta)R^{-1}B(\theta)^T P(\theta) + \dot{P}(\theta) + Q < 0 \quad (19)$$

$$G(\theta)\phi(\theta)^{-1}G(\theta)^T - R < 0 \quad (20)$$

where

$$\begin{aligned} \phi(\theta) &= A(\theta)^T P(\theta) + P(\theta) A(\theta) \\ &\quad - P(\theta) B(\theta) R^{-1} B(\theta)^T P(\theta) + \dot{P}(\theta) + Q. \end{aligned}$$

The proof of this theorem goes the same way as in [22, 23]. \square

Because of Theorem 1, inequalities (17), (18) and (19), (20) are negative in the box (10) if they take negative values at the vertices of (10); that is if they are negative for all γ in the vertex set Γ given by (10) and inequality (12) holds for all $i = 1, 2, \dots, k$. In the vertex set (10) define the polytope system in the form

$$\{(D_1, E_1, H_1), \dots, (D_N, E_N, H_N)\} \quad (21)$$

where $N = 2^k$ and

$$\begin{aligned} D_1 &= A_0 + A_1 \underline{\theta}_1 + \dots + A_k \underline{\theta}_k \\ E_1 &= B_0 + B_1 \underline{\theta}_1 + \dots + B_k \underline{\theta}_k \\ H_1 &= C_0 + C_1 \underline{\theta}_1 + \dots + C_k \underline{\theta}_k. \end{aligned} \quad (22)$$

In (21) each vertex is calculated for the k variables $\theta_i, i = 1, 2, \dots, k$ alternatively taken at maximum and minimum values. For polytopic system inequality (19) could not be solved within LMI therefore let us introduce the inverse Lyapunov matrix $S(\theta)$ as in Gahinet, Nemirovski, Laub and Chilali [6].

$$S(\theta) = P(\theta)^{-1} = S_0 + S_1 \theta_1 + \dots + S_k \theta_k. \quad (23)$$

Note that equality (23) holds for all vertices $(\gamma, \lambda) \in \Gamma \times \Lambda$. For the new variable $S(\theta)$, equations (11), (12) and (19) read as follows

$$L(\gamma, \lambda) = S(\gamma)A(\gamma)^T + A(\gamma)S(\gamma) - (S(\lambda) - S_0) < 0 \quad (24)$$

$$S_i A_{ci}^T + A_{ci} S_i \geq 0, \quad i = 1, 2, \dots, k \quad (25)$$

and

$$\begin{aligned} &S(\gamma)A(\gamma)^T + A(\gamma)S(\gamma) - B(\gamma)R^{-1}B(\gamma)^T \\ &\quad - (S(\lambda) - S_0) + S(\gamma)QS(\gamma) < 0. \end{aligned} \quad (26)$$

For reducing the conservatism of the AQS test [5] nonnegative matrices $M_i \geq 0$, $i = 1, 2, \dots, k$ are added to (26) and (25) as follows

$$S(\gamma)A(\gamma)^T + A(\gamma)S(\gamma) - B(\gamma)R^{-1}B(\gamma)^T$$

$$-(S(\lambda) - S_0) + S(\gamma)QS(\gamma) + \sum_{i=1}^k \theta_i^2 M_i < 0 \quad (27)$$

and

$$S_i A_{ci}^T + A_{ci} S_i + M_i \geq 0, \quad i = 1, 2, \dots, k. \quad (28)$$

The resulting test is generally less conservative for (27) and (28). However, this improvement is at the expense of higher computational needs since the number of optimization variables is increased in the new LMI problem (27), (28). Combining the results of (19), (27), (28) and (20) the following algorithm for computation of a robust output feedback controller with guaranteed affine quadratic stability has been proposed.

Algorithm.

Step 1. Find the solution of (27) at all vertices $(\gamma, \lambda) \in \Gamma \times \Lambda$ with respect to the variables $S_0, S_1, \dots, S_k, M_1, M_2, \dots, M_k$ from the following LMI inequalities

$$\begin{bmatrix} N_i(\gamma, \lambda) & S(\gamma)Q & V_i(\gamma) \\ QS(\gamma) & -Q & 0 \\ V_i^T(\gamma) & 0 & -R^{-1} \end{bmatrix} \leq 0 \quad (29)$$

where

$$N_i(\gamma, \lambda) = S(\gamma)D_i^T + D_i S(\gamma) - E_i R^{-1} E_i^T - (S(\lambda) - S_0) + \sum_{j=1}^k \theta_j^2 M_j$$

$$V_i(\gamma) = (E_i^T + R F_v H_i S(\gamma))^T R^{-1}$$

for $i = 1, 2, \dots, N$

$$K_j = S_j A_j^T + A_j S_j + M_j \geq 0 \quad (30)$$

$$M_j \geq \rho_1 I, \quad S(\gamma) \geq \rho_2 I, \quad S_j \leq \rho_3 I, \quad j = 1, 2, \dots, k$$

where I is identity matrix with corresponding dimensions and ρ_1, ρ_2, ρ_3 are some nonnegative constants. For the first iterative procedure $F_v = 0$.

Step 2. Calculate the value of the inverse Lyapunov matrix S_{ni} and P_{ni} , $i = 1, 2, \dots, N$ at all vertices of $\gamma \in \Gamma$.

Step 3. Compute the value of Riccati equation at all vertices of $(\gamma, \lambda) \in \Gamma \times \Lambda$.

$$\phi_i(\lambda) = D_i^T P_{ni} + P_{ni} D_i - P_{ni} E_i R^{-1} E_i^T P_{ni} + (P(\lambda) - P_0) + Q. \quad (31)$$

Step 4. Compute the gain matrix F from the following LMI inequalities

$$\begin{bmatrix} -R & E_i^T P_{ni} + R F H_i \\ (E_i^T P_{ni} + R F H_i)^T & \phi_i(\lambda) \end{bmatrix} \leq 0 \quad (32)$$

$$A_2 = \begin{bmatrix} 0 & 0.0125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0594 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0116 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0308 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0188 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0156 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0208 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0.3148 & 0 \\ 0.0478 & 0 \\ 0 & -0.1028 \\ 0 & -0.0091 \\ -0.0841 & 0 \\ -0.0287 & 0 \\ 0 & 0.3676 \\ 0 & 0.2448 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0.0625 & 0 \\ -0.0798 & 0 \\ 0 & -0.0462 \\ 0 & -0.0449 \\ 0.0016 & 0 \\ 0.0072 & 0 \\ 0 & 0.077 \\ 0 & -0.005 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -0.0094 & 0 \\ 0.0151 & 0 \\ 0 & 0.0019 \\ 0 & -0.003 \\ -0.0121 & 0 \\ -0.03 & 0 \\ 0 & -0.064 \\ 0 & 0.0189 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The number of polytope systems are equal to 4 and the polytope vertices are computed for two variables θ_1, θ_2 alternatively taken at their maximum $\bar{\theta}_i$ and minimum $\underline{\theta}_i, i = 1, 2$. The decentralized control structure for the two PI controllers can be obtained by the choice of the static output feedback gain matrix F structure. It is given as follows

$$F = \begin{bmatrix} f_{11} & 0 & f_{13} & 0 \\ 0 & f_{22} & 0 & f_{24} \end{bmatrix}$$

The results of calculation of a static output feedback gain matrix F for quadratically and affine quadratically stable system for different $Q = qI, R = rI, |\theta_1| = |\theta_2| = 1$

and $\rho_i, i = 1, 2, 3$ are summarized in the following table.

N	q	r	ρ_1	ρ_2	ρ_3	$quad$	$aff.quad$
1	1.5	1	1.5	1	0.166	-0.1342	-0.0954
2	5	1	1.5	1	0.166	+0.307	-0.1277*
3	10	1	1.5	1	0.166	-0.081	-0.1922**
4	20	1	1.5	1	0.166	+1.11	-0.1148
5	10^{-4}	1	1.5	1	0.166	-0.0164	+0.0011
6	10^{-4}	1	10^{-4}	1	0.166	-0.0164	-0.0136
7	0.1	1	1.5	1	0.166	-0.1383	-0.0386
8	0.1	1	10^{-4}	1	0.166	-0.1383	-0.1133
9	0.1	1	0	1	0.166	-0.1383	-0.1449
10	0.1	1	0	1	0	-0.1383	-0.1448
11	10^{-6}	1	0	1	0.166	-0.0015	-0.0015
12	10^{-6}	0.1	0	1	0.166	-0.0134	-0.0178

where $quad$ and $aff.quad$ denote the max (real (eigenvalue)) of the closed loop system for quadratic or affine quadratic stability, respectively. The solutions are feasible for 11 and 12 cases. For other cases the closed loop system is quadratically or affine quadratically stable but the minimal value of cost is not guaranteed. The static output feedback gain matrix for cases * and ** are given as follow

$$F^* = \begin{bmatrix} -0.3582 & 0 & -0.376 & 0 \\ 0 & -0.7927 & 0 & -0.7535 \end{bmatrix}$$

$$F^{**} = \begin{bmatrix} -1.0708 & 0 & -0.6317 & 0 \\ 0 & -2.6952 & 0 & -1.671 \end{bmatrix}$$

The second example has been borrowed from [1] to demonstrate the design of affine quadratic controller. It is known that the presented system is static output feedback stabilizable. Let (A, B, C) in (5) be defined as

$$A = \begin{bmatrix} -0.036 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.010 & 0.0024 & -4.0208 \\ 0.1002 & q_1(t) & -0.707 & q_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ q_3(t) & -7.59222 \\ -5.520 & 4.490 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

with parameters bounds $-0.6319 \leq q_1(t) \leq 1.3681, 1.22 \leq q_2(t) \leq 1.420$, and $2.7446 \leq q_3(t) \leq 4.3446$. Find a stabilizing output feedback matrix F . The nominal model of (A_0, B_0) is given by the above matrices when we substitute for the entries $A_0(3, 2) = 0.3681, A_0(3, 4) = 1.32$ and $B_0(2, 1) = 3.5446$. The structured model uncertainty (5) (A_1, A_2, B_1) are matrices with the following entries $A_1(3, 2) = 1, A_2(3, 4) = 0.1$ and $B_1(2, 1) = 0.8$ with $\theta_i \in \langle -1, 1 \rangle, i = 1, 2$. Other entries of the above uncertain matrices are equal to zero. The number of polytope

systems are equal to 4.

The nominal model is unstable with eigenvalues:

$$\text{eig} \{-2.0516, 0.2529 \pm 0.3247i, -0.2078\}.$$

For the first “iterative” procedure, $F_v = 0$, the closed-loop eigenvalues of 4 polytope systems are as follows

$$\text{eig } CL\{0.6885, 0.5726, 0.0713 \pm 1.1077i, \dots, -4.0422\}.$$

Closed-loop system is not stable. After using the iterative procedure the closed-loop eigenvalues of 4 polytope systems are as follows

$$\text{eig } CL\{-0.04504 \pm 0.3555i, -0.1472 \pm 0.4304i, -0.1896 \pm 0.4636i, \dots, -688.527\}.$$

The eigenvalues of closed-loop system for the case $\|\theta_1\| = \|\theta_2\| = 0$ are equal as follows

$$\text{eig } CL\{-0.1668 \pm 0.4432i, -0.403, -660.91\}$$

and output feedback gain matrix

$$F = \begin{bmatrix} 34.51 \\ 103.03 \end{bmatrix}$$

Hence, the affine quadratic stability conditions are met the closed-loop system is affine quadratically stable. Because, the available solution in the above design LMI procedure is only may be feasible but are not strictly (best value of $t = 5.17310^{-4}$) the cost minimal value (14) is not guaranteed. For the above example we obtain the value of cost as follows

$$J = \int_0^\infty (x(t)^T (Q + C^T F^T R F C) x(t)) dt \leq 33.82663 \|x_0\|^2.$$

5. CONCLUSIONS

In this paper, we have proposed a new procedure for robust output feedback controller design for linear systems with affine and possible time varying parameter uncertainty. The feasible solution of the output feedback controller with sufficient conditions guarantee the affine quadratic stability and guaranteed cost. The design procedure is based on new necessary and sufficient conditions for output feedback stabilizability of linear systems and a non-iterative LMI based algorithm. A valuable feature of the robust controller design procedure is that quantitative information about the rate of parameter variation is readily incorporated to reduce conservatism in the time varying case.

ACKNOWLEDGEMENT

The work has been supported by grant No. 1/0158/03 of the Scientific Grant Agency of the Ministry of Education of Slovak Republic and the Slovak Academy of Sciences.

The author is grateful to the referee for the careful reading of the manuscript and many helpful suggestions which improved the paper.

(Received October 3, 2002.)

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