

On the Tangential Velocity Arising in a Crystalline Approximation of Evolving Plane Curves

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Abstract: In a crystalline algorithm, a tangential velocity is used implicitly. In this short note, it is specified for the case of evolving plane curves, and is characterized by using the intrinsic heat equation.

Keywords: tangential velocity; intrinsic heat equation; crystalline algorithm; admissible polygonal curve;

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