KYBERNETIKA — VOLUME *43* (2007), NUMBER 6, PAGES 903-912

Asymptotic Behavior of Solutions to an Area-preserving Motion by Crystalline Curvature

Shigetoshi Yazaki

Abstract: Asymptotic behavior of solutions of an area-preserving crystalline curvature flow equation is investigated. In this equation, the area enclosed by the solution polygon is preserved, while its total interfacial crystalline energy keeps on decreasing. In the case where the initial polygon is essentially admissible and convex, if the maximal existence time is finite, then vanishing edges are essentially admissible edges. This is a contrast to the case where the initial polygon is admissible and convex: a solution polygon converges to the boundary of the Wulff shape without vanishing edges as time tends to infinity.

Keywords: essentially admissible polygon; crystalline curvature; the Wulff shape; isoperimetric inequality;

AMS Subject Classification: 34A26; 34A34; 34K25; 39A12; 53A04; 74N05; 82D25;

References

[1] F. Almgren and J. E. Taylor: Flat flow is motion by crystalline curvature for curves with crystalline energies. J. Differential Geom. 42 (1995), 1–22.

[2] S. Angenent and M. E. Gurtin: Multiphase thermomechanics with interfacial structure, 2. Evolution of an isothermal interface. Arch. Rational Mech. Anal. 108 (1989), 323–391.

[3] M. Gage: On an area-preserving evolution equations for plane curves. Contemp. Math. 51 (1986), 51–62.

[4] Y. Giga: Anisotropic curvature effects in interface dynamics. Sūgaku 52 (2000), 113–127; English transl., Sūgaku Expositions 16 (2003), 135–152.

[5] M. E. Gurtin: Thermomechanics of Evolving Phase Boundaries in the Plane. Clarendon Press, Oxford 1993.

[6] H. Hontani, M.-H. Giga, Y. Giga, and K. Deguchi: Expanding selfsimilar solutions of a crystalline flow with applications to contour figure analysis. Discrete Appl. Math. 147 (2005), 265–285.

[7] S. Roberts: A line element algorithm for curve flow problems in the plane. CMA Research Report 58 (1989); J. Austral. Math. Soc. Ser. B 35 (1993), 244–261.

[8] J. E. Taylor: Constructions and conjectures in crystalline nondifferential geometry. In: Proc. Conference on Differential Geometry, Rio de Janeiro, Pitman Monographs Surveys Pure Appl. Math. 52 (1991), 321–336, Pitman London.

[9] J. E. Taylor: Motion of curves by crystalline curvature, including triple junctions and boundary points. Diff. Geom.: partial diff. eqs. on manifolds (Los Angeles, CA, 1990), Proc. Sympos. Pure Math., 54 (1993), Part I, 417–438, AMS, Providence.

[10] J. E. Taylor, J. Cahn, and C. Handwerker: Geometric models of crystal growth. Acta Metall. 40 (1992), 1443–1474.

[11] S. Yazaki: On an area-preserving crystalline motion. Calc. Var. 14 (2002), 85–105.

[12] S. Yazaki: On an anisotropic area-preserving crystalline motion and motion of nonadmissible polygons by crystalline curvature. Sūrikaisekikenkyūsho Kōkyūroku 1356 (2004), 44–58.

[13] S. Yazaki: Motion of nonadmissible convex polygons by crystalline curvature. Publ. Res. Inst. Math. Sci. 43 (2007), 155–170.