

## Bilinear System as a Modelling Framework for Analysis of Microalgal Growth

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*Abstract:* A mathematical model of the microalgal growth under various light regimes is required for the optimization of design parameters and operating conditions in a photobioreactor. As its modelling framework, bilinear system with single input is chosen in this paper. The earlier theoretical results on bilinear systems are adapted and applied to the special class of the so-called intermittent controls which are characterized by rapid switching of light and dark cycles. Based on such approach, the following important result is obtained in the present paper: as the light/dark cycle frequency is going to infinity, the value of resulting production rate in the microalgal culture goes to a certain limit value, which depends on average irradiance in the culture only. As a case study, the so-called three-state model of photosynthetic factory, being a simple four-parameter model, is analyzed. The present paper shows various numerical simulations for the model parameters previously published and analyzed experimentally in the biotechnological literature. These simulation results are in a very good qualitative compliance with the well-known flashing light experiments, thereby confirming viability of the approach presented here.

*Keywords:* bilinear system; model of photosynthetic factory; microalgae; light/dark cycles; flashing light experiments;

*AMS Subject Classification:* 93C10; 37N25;

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