KYBERNETIKA — VOLUME *34* (1998), NUMBER 6, PAGES 625 – 634

THE FINITE INCLUSIONS THEOREM: A TOOL FOR ROBUST DESIGN

THEODORE E. DJAFERIS

Methods for robust controller design, are an invaluable tool in the hands of the control engineer. Several methodologies been developed over the years and have been successfully applied for the solution of specific robust design problems. One of these methods, is based on the Finite Inclusions Theorem (FIT) and exploits properties of polynomials. This has led to the development of FIT-based algorithms for robust stabilization, robust asymptotic tracking and robust noise attenuation design. In this paper, we consider SISO systems with parameter uncertainty and show how FIT can be used to develop algorithms for robust phase margin design.

1. INTRODUCTION

Over the last thirty years, a multitude of techniques have been suggested for robust controller design. Some the most popular are H_∞, LQG, Parameter Space Methods, QFT, each with its own special characteristics and strengths. In recent years, we have been promoting a robust control design method [1], which is Nyquist Theorem based and employs the Finite Inclusions Theorem. It exploits properties of polynomials and has been used to solve problems of robust stabilization, robust asymptotic tracking and disturbance rejection, for systems with parameter uncertainty. It has also been applied to problems with multi-objective performance specifications (see [1]). FIT design, takes a given design problem and formulates it as a simultaneous polynomial family stabilization problem. The controller is then computed iteratively, where at each iteration (for SISO systems) a set of *linear* inequalities is solved. In this paper, we demonstrate how a design problem with robust phase margin specifications can be solved using FIT design.

For a stable feedback loop, the phase margin is one of the most important system parameters. Firstly, it provides information on what it might take in order to destabilize the system. Secondly, it characterizes indirectly the transient response to external inputs. For this reason it is quite common for design requirements to include a specification on phase margin. Its importance has long been recognized by control engineers and for SISO systems without uncertainty, any good undergraduate control text (e. g., [4]) will include algorithms for design. Usually, these are frequency domain techniques which are implemented by trial and error. One can also find quite elegant results [2, 3], that in certain cases give formulae for achievable gain and phase margins and also provide constructive methods for controller design.

The design for guaranteed phase margin becomes much more complex, when the plant description involves uncertainty. The phase margin requirement can be taken into account by including additional parameter uncertainty in the plant dynamics. After incorporating this in the plant uncertainty, the overall structure may not conform to that required by existing robust design techniques for "tight" results. One is then faced with a decision: Either express the design problem in an "exact" manner but then have no computationally efficient techniques for solving it, or introduce some type of "overbounding," which destroys "exactness," but makes the design problem computationally tractable. It is clear from this discussion, that it is best to develop problem formulations that both reduce conservatism and lead to computationally attractive controller design methods.

In this paper, we deal with the SISO problem of controller design for guaranteed phase margin, when the plant includes parameter uncertainty. We first formulate the problem in terms of polynomial family stabilization. The resulting family of polynomials can be thought of as having real parameter uncertainty and complex coefficients. Having posed the design as a robust polynomial stabilization problem, we then show how the Finite Inclusions Theorem [1] can be used to develop algorithms for robust controller design. These are iterative algorithms, initialized by a certain controller that achieves some phase margin, which is less than the desired. At each iteration, a new controller is computed that achieves larger phase margin. The procedure is terminated when (if) the desired margin is reached, or some "stopping" criteria are met. The robust phase margin design problem can certainly be attacked using H_∞ techniques (in addition to others). This however, will require rather conservative overbounding. When no plant uncertainty is present, very precise results have been reported in [2] (based on techniques developed in [3]).

In Section 2 we show how to formulate the phase margin problem as a robust polynomial stabilization problem. In Section 3 we state the Finite Inclusions Theorem which provides the foundation of our design algorithms. In Section 4, a FIT-based design algorithm is presented. In Section 5 we apply these algorithms to examples, and in Section 6 present some conclusions.

2. FORMULATION

Consider the feedback system shown in Figure 1:

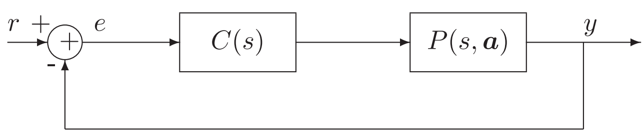

Fig. 1. Unity feedback configuration, parametric uncertainty.

The plant family is strictly proper and given by:

$$P(s,\boldsymbol{a}) = \frac{n_p(s,\boldsymbol{a})}{d_p(s,\boldsymbol{a})}, \tag{1}$$

where $d_p(s,\boldsymbol{a})$ is monic of degree $\bar{n}$, and $\boldsymbol{a}$ is a k-vector of parameters, taking values in some given set $\Omega_a \subset \boldsymbol{R}^k$, ($\boldsymbol{R}$ denotes the set of real numbers). Specifically, let $\Omega_a = \left\{\boldsymbol{a} \in \boldsymbol{R}^k | a_i^- \le a_i \le a_i^+, 1 \le i \le k\right\}$, $a_i^- < 0,\ a_i^+ > 0,\ 1 \le i \le k$. The coefficients of $n_p(s,\boldsymbol{a}), d_p(s,\boldsymbol{a})$ are in general polynomial expressions of the uncertain parameters. The numerator and denominator polynomials are coprime for all values of $\boldsymbol{a} \in \Omega_a$ and the plant, when $\boldsymbol{a} = \boldsymbol{0}$, will be referred to as "nominal." The controller is given by $C(s) = \frac{n_c(s)}{d_c(s)}$, and the loop transfer function is: $L(s,\boldsymbol{a}) = \frac{n_l(s,\boldsymbol{a})}{d_l(s,\boldsymbol{a})} = \frac{n_p(s,\boldsymbol{a})n_c(s)}{d_p(s,\boldsymbol{a})d_c(s)}$. The closed loop characteristic polynomial can then be expressed as:

$$\phi(s,\boldsymbol{a}) = d_l(s,\boldsymbol{a}) + n_l(s,\boldsymbol{a}). \tag{2}$$

Let $\boldsymbol{b}$ be the complex parameter which takes values on the unit circle in the set: $\Omega_b = \left\{e^{-j\theta} | -\theta_1 \le \theta \le \theta_1\right\}$, where θ_1 is some given angle in the range $0 < \theta_1 \le \pi$. One can immediately state the following result:

Proposition 1. The feedback loop in Figure 1 is robustly stable and each plant has phase margin greater than θ_1 if and only if the polynomial family $\phi_{ph}(s,\boldsymbol{a},\boldsymbol{b}) = d_l(s,\boldsymbol{a}) + \boldsymbol{b}n_l(s,\boldsymbol{a})$ is robustly stable for all $\boldsymbol{a} \in \Omega_a$ and $\boldsymbol{b} \in \Omega_b$.

Proof. Let us first show that the result is true for the nominal plant and nominal characteristic polynomial. The proof for the entire plant family follows directly, as we can repeat the "nominal" arguments for each member of the plant family.
Suppose first that $\phi_{ph}(s,\boldsymbol{0},\boldsymbol{b}) = d_l(s,\boldsymbol{0}) + \boldsymbol{b}n_l(s,\boldsymbol{0})$ is stable for all $\boldsymbol{b} \in \Omega_b$. Since the set Ω_b includes the value 1 the closed loop will be stable. Let us first assume no poles or zeros of the loop transfer function on the imaginary axis. Stability of the nominal implies that the image of the Nyquist Path encircles the -1 point an appropriate number of times. Suppose then that the phase margin is less than θ_1. This implies that the image of the Nyquist Path (under $L(j\omega,\boldsymbol{0})$) intersects the unit circle at a point with phase in the range $(\pi-\theta_1, \pi+\theta_1)$. In particular there exists a frequency ω_1 and a point $\boldsymbol{b}_1$ in Ω_b such that:

$$\frac{n_l(j\omega_1,\boldsymbol{0})}{d_l(j\omega_1,\boldsymbol{0})} = -\boldsymbol{b}_1. \tag{3}$$

This implies that ("*" denotes complex conjugate):

$$\boldsymbol{b}_1^* \frac{n_l(j\omega_1,\boldsymbol{0})}{d_l(j\omega_1,\boldsymbol{0})} = -1 \tag{4}$$

or equivalently that:

$$d_l((j\omega_1,\boldsymbol{0}) + \boldsymbol{b}_1^* n_l((j\omega_1,\boldsymbol{0}) = 0. \tag{5}$$

This means that the polynomial $\phi_{ph}(s,\mathbf{0},\boldsymbol{b}_1^*)$ has a root on the imaginary axis which is a contradiction. A similar argument can be used for the case of imaginary axis poles or zeros.

We now prove the converse. Suppose that $d_l(s,\mathbf{0})\ +n_l(s,\mathbf{0})$ is stable and the loop has phase margin greater than θ_1, but assume that $\phi_{ph}(s,\mathbf{0},\boldsymbol{b}) = d_l(s,\mathbf{0}) + \boldsymbol{b}n_l(s,\mathbf{0})$ is not stable for all $\boldsymbol{b}\in\Omega_b$. Since $\phi_{ph}(s,\mathbf{0},1) = d_l(s,\mathbf{0}) + n_l(s,\mathbf{0})$ is stable and the coefficients of $\phi_{ph}(s,\mathbf{0},\boldsymbol{b})$ depend continuously on $\boldsymbol{b}$, there must exist a $\boldsymbol{b}_1\in\Omega_b$ and a frequency ω_1 such that:

$$d_l(j\omega_1,\mathbf{0}) + \boldsymbol{b}_1 n_l(j\omega_1,\mathbf{0}) = 0. \tag{6}$$

If $d_l(j\omega_1,\mathbf{0}) = 0$ then $n_l(j\omega_1,\mathbf{0}) = 0$ and d_l, n_l would not be coprime. Therefore, $d_l(j\omega_1,\mathbf{0}) \neq 0$ and we must have:

$$\frac{n_l(j\omega_1,\mathbf{0})}{d_l(j\omega_1,\mathbf{0})} = -1/\boldsymbol{b}_1. \tag{7}$$

But this contradicts the assumption that the phase margin is greater than θ_1. □

3. THE FINITE INCLUSIONS THEOREM

Consider a polynomial family $\phi(s,\boldsymbol{a})$ with real parameter uncertainty where the coefficients may lie in $\boldsymbol{C}$, the set of complex numbers. The Finite Inclusions Theorem can be used to investigate robust $\mathcal{D}$-stability. Specifically, let

$$\phi(s,\boldsymbol{a}) = \phi_0(s) + \alpha_1(\boldsymbol{a})\,\phi_1(s) + \alpha_2(\boldsymbol{a})\,\phi_2(s) + \ldots + \alpha_u(\boldsymbol{a})\,\phi_u(s), \tag{8}$$

where the $\phi_i(s)$, $0 \le i \le u$ are given polynomials. Suppose that the parameter $\boldsymbol{a}$ takes values in the hypercube Ω_a. Further assume that $\alpha_i(\boldsymbol{a})$, $1 \le i \le u$ are *polynomic* in $\boldsymbol{a}$ and such that $\alpha_i(\mathbf{0}) = 0$. Denote by intΓ the interior of some set Γ. For such a family the following result (see [1]) holds:

Theorem 1. (The Finite Inclusions Theorem, FIT) Let $\phi(s,\boldsymbol{a}) = \sum_{j=0}^{n}\alpha_j(\boldsymbol{a})s^j$, $\boldsymbol{a}\in\Omega_a$ $n\ge 0$, and $\alpha_j:\Omega_a\to\boldsymbol{C}$. Further, let $\Gamma\subset\boldsymbol{C}$ be a closed Jordan curve such that int Γ is convex. Then for all $\boldsymbol{a}\in\Omega_a, \phi(s,\boldsymbol{a})$ is of degree n and has all its roots in int Γ if there exists $m\ge 1$ intervals $(c_k,d_k)\subset\boldsymbol{R}$ and a counterclockwise sequence of points $s_k\in\Gamma$, $1\le k\le m$, such that

$$\begin{gathered}\forall\, 1\le k<m \max\{d_{k+1}-c_k, d_k-c_{k+1}\}\le\pi\\ \max\{d_m-(c_1+2\pi n),\,(d_1+2\pi n)-c_m\}\le\pi\\ \forall\, 1\le k\le m\ \phi(s_k,\Omega_a)\subset S_k=\{re^{j\theta}|r>0,\theta\in(c_k,d_k)\}.\end{gathered}$$

As stated, the Finite Inclusions Theorem is much broader than what is needed for robust phase margin design. Here we are just interested in Hurwitz stability.

Furthermore, one can immediately see that it can be used to express conditions for simultaneous stability of a finite number of polynomial families.

It is evident from the above discussion, that FIT leads to conditions for robust stability that are expressed in terms of fitting polynomial value sets in sectors. There are no restrictions as to what should be the shape of these value sets. However, checking whether a value set with a "curved" boundary lies in a sector would in general involve a fair amount of computation and would certainly complicate the design. If, on the other hand, the value set is a *polygon* checking whether it lies in a sector can be done by examining just the value set vertices. This fact introduces significant simplification and will be exploited in the development of design algorithms.

4. AN ALGORITHM FOR ROBUST DESIGN

We are now in a position to state a FIT based algorithm for robust phase margin controller design. Let the controller in the feedback loop of Figure 1 be given by:

$$C(s) = \frac{n_c(s)}{d_c(s)} = \frac{x_{2q+1}s^q + x_{2q}s^{q-1} + \ldots + x_{q+1}}{s^q + x_q s^{q-1} + x_{q-1}s^{q-2} + \ldots + x_1}. \tag{9}$$

Let the controller parameters be grouped in the vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_{2q+1}) \in \boldsymbol{R}^d$ where $d = 2q+1$. Now, polynomial family $\phi_{ph}(s, \boldsymbol{a}, \boldsymbol{b})$ will have degree $n = \bar{n}+q$ and one can immediately see that it has coefficients that are *affine* expressions in $\boldsymbol{x}$. This fact is crucial in our development, as it will allow us to use FIT and carry out an iterative controller design by solving *linear inequalities*.

As mentioned above, FIT does not place any restrictions on the structure of the parameter uncertainty in the plant. However, if we desire to use FIT as the basis of a controller design procedure, it becomes advantageous to make some assumptions. Specifically, we will assume that the parameter uncertainty $\boldsymbol{a}$, appears in the numerator and denominator of the plant family in a *multiaffine* manner. We also overbound the set Ω_b by some polygon Ω_{bp} (the simplest and most conservative being a single rectangle). This will ensure that the corresponding value sets can be easily overbounded by convex polygons, and the polynomial $\phi_{ph}(s, \boldsymbol{a}, \boldsymbol{b})$, can be thought as having real parameter uncertainty and complex coefficients. In view of these two assumptions, locations of value sets can be deduced from the location of their extreme points (a *finite set*). This dramatically reduces the computational burden of the controller design procedure. However, this comes at the expense of introducing conservatism in the solution. Let $\Omega_{ab} = \Omega_a \times \Omega_{bp}$. Let us now pose the following robust controller design problem for the system in Figure 1:

- Robust Phase Margin Problem: Find (if possible) a controller of order q that robustly stabilizes $\phi_{ph}(s, \boldsymbol{a}, \boldsymbol{b})$, for $(\boldsymbol{a}, \boldsymbol{b}) \in \Omega_{ab}$.

This can be attacked directly using FIT, as it is a robust polynomial stabilization problem. We do this, using an algorithm suggested in [1]. One needs to first choose an initial controller $\boldsymbol{x}^{(1)}$, which robustly stabilizes $\phi_{ph}(s, \Omega_{ab}^{(1)}, \boldsymbol{x}^{(1)})$, where

$\Omega_{ab}^{(1)} = \Omega_a^{(1)} \times \Omega_{bp}^{(1)} \subset \Omega_{ab}$. The controller can be designed by employing any design technique. This controller is then iteratively improved upon using FIT. At each iteration, frequencies $\omega_k^{(j)}$ are found which place the value sets $\phi_{ph}(j\omega_k^{(j)}, \Omega_{ab}^{(j)}, \boldsymbol{x}^{(j)})$ in their corresponding sectors $S_k^{(j)}$. By FIT, this guarantees the robust stability of $\phi_{ph}(s, \Omega_{ab}^{(j)}, \boldsymbol{x}^{(j)})$. At each iteration the uncertainty set $\Omega_{ab}^{(j)}$ is enlarged. Initially, only the $\boldsymbol{b}$ parameters are affected until (if possible) for some j, the desired phase margin is attained $\Omega_b^{(j)} = \Omega_{bp}$. Once this is accomplished the procedure continues this time with an enlargement of $\Omega_{ab}^{(j)}$ where the $\boldsymbol{a}$ parameters are affected and the $\boldsymbol{b}$ parameters remain unchanged. The algorithm terminates when (if) $\Omega_{ab}^{(j)} \supset \Omega_{ab}$, with $\boldsymbol{x}^{(j)}$ being the desired controller. In what follows, $Ext\, S$, denotes the extreme points of the set S.

Robust Phase Margin Design Algorithm

1. Let $\boldsymbol{x}^{(1)} \in \boldsymbol{R}^d$ and $\Omega_{ab}^{(1)} \subset \Omega_{ab}$ be such that $\phi_{ph}(s, \Omega_{ab}^{(1)}, \boldsymbol{x}^{(1)})$ is stable and set $j := 1$.
2. Determine $m^{(j)} \geq 1$ sectors $S_k^{(j)}$, $1 \leq k \leq m^{(j)}$, and frequencies $\omega_k^{(j)}$ along the $j\omega$ axis such that $\phi_{ph}(\omega_k^{(j)},\ Ext\,\Omega_{ab}^{(j)}, \boldsymbol{x}^{(j)}) \subset S_k^{(j)}$. By FIT, $\phi_{ph}(s, \Omega_{ab}^{(j)}, \boldsymbol{x}^{(j)})$ is stable. Each $\omega_k^{(j)}$ should roughly center (angularly) the set $\phi_{ph}(\omega_k^{(j)},\ Ext\,\Omega_{ab}^{(j)}, \boldsymbol{x}^{(j)})$ in $S_k^{(j)}$.
3. Choose a slightly larger set $\Omega_{ab}^{(j+1)} \supset \Omega_a^{(j)}$. First this should affect the $\boldsymbol{b}$ parameters. When (if) $\Omega_b^{(j)} \supset \Omega_b$, the enlargement in the $\boldsymbol{b}$-direction terminates and the enlargement in the $\boldsymbol{a}$-direction commences.
4. Compute new vector of controller parameters $\boldsymbol{x}^{(j+1)}$ such that $\phi_{ph}(j\omega_k^{(j)}, Ext\,\Omega_{ab}^{(j+1)}, \boldsymbol{x}^{(j+1)}) \subset S_k^{(j)}$ for all k. Note, this is equivalent to solving a system of linear inequalities in $\boldsymbol{x}^{(j+1)}$. If no solutions exist to this system of inequalities, return to Step 3 and choose a smaller $\Omega_{ab}^{(j+1)}$.
5. Let $j := j + 1$, and if $\Omega_{ab}^{(j)} \supset \Omega_{ab}$, stop; otherwise, go to Step 2.

This is one of several FIT-based algorithms that can be suggested. Clearly, as stated, the algorithm may never terminate but appropriate "stopping" conditions can be added. We should also note, that several possibilities exist for the choice of the polygon that overbounds Ω_b. Some choices do allow for a design procedure that guarantees a robust phase margin as well as some robust gain margin. This is immediately true if one uses a rectangular overbound of Ω_b (see Example 1, in Section 5).

Very frequently in control design, one is faced with having to satisfy a number of requirements (including one on phase margin) *simultaneously*. The FIT-based approach, would formulate each as a robust polynomial stabilization problem and then employ the Simultaneous Stability Finite Inclusions Theorem (see [1]).

5. EXAMPLES

Example 1. In our first example, the plant does not include any uncertainty. It is given by the transfer function:

$$P(s) = \frac{n_p(s)}{d_p(s)} = \frac{s-1}{s^2 - s - 2} \tag{10}$$

which is unstable and nonminimum phase (see [2]). Our objective is to design (if possible) a first order controller that has phase margin of at least 25° (either positive or negative as required for stability). This implies that the set $\Omega_b = \left\{e^{-j\theta} | -25^\circ \le \theta \le 25^\circ\right\}$. The problem will be solved, if a first order controller can be found, that robustly stabilizes the polynomial family:

$$\phi_{ph}(s, \boldsymbol{b}) = d_l(s) + \boldsymbol{b} n_l(s) \tag{11}$$

for all $\boldsymbol{b} \in \Omega_b$. In order to apply the FIT-based algorithm suggested earlier, we need to overbound the set Ω_b by some polygon. The simplest, but most conservative way, is to overbound it by a single rectangle, as shown in Figure 2.

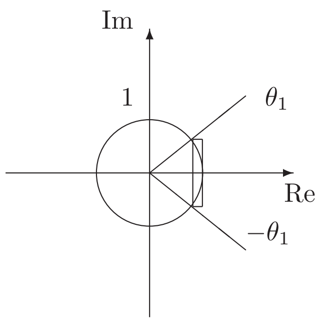

Fig. 2. Rectangular overbound of the set Ω_{bp}.

Note, that in robustly stabilizing $\phi_{ph}(s, \Omega_{bp})$, we will not only be achieving the desired phase margin, but impacting the gain margin as well. The FIT-based algorithm suggested in Section 4, can certainly be used. A number of possibilities exist for implementing the suggested algorithm and the one used here, takes a "reduced" set Ω_{bp}, and iteratively expands it. In addition, we employ software written for implementing a version of this algorithm (see [1]). To initiate the process, the algorithm requires an initial controller. Using pole placement, we designed the controller:

$$C_0(s) = \frac{50.25s + 45.25}{s - 37.25} \tag{12}$$

that places the closed loop poles at: $-1.5 \pm j1, -9$. With this controller the phase margin (using MATLAB) is: -13.25°. With this initial controller, the software computed the controller:

$$C(s) = \frac{51.75s + 68.22}{s - 40.11}. \tag{13}$$

One can determine that this controller guarantees $-23.6°$ of phase margin. However, overbounding has been introduced and the actual phase margin is larger: $-25.31°$ at $\omega = 1.293$, which achieves our design goal. The Nyquist plot of the loop transfer function is given in Figure 3.

Fig. 3. Nyquist plot of loop transfer function.

Some remarks are in order: First, there is no guarantee that a first order controller exists that provides the required phase margin. Second, even if one did exist, there is no guarantee that our iterative algorithm would have computed it. It is clear that we have introduced conservatism in the solution using the single rectangular overbound. Other, arbitrarily less conservative overbounds can also be used, which will however lead to additional computations. Third, suppose that a higher order controller was allowed, would it improve the phase margin? Examining Figure 3, we can speculate that this could be possible. In fact, one can show [2], that $\theta_{\sup} = -2\sin^{-1}(1/3) = -38.94°$. A word of caution should be stated at this point. Indeed, one can envision a controller design that "stretches" the Nyquist plot in order to achieve a larger phase margin. Since phase margin is the only requirement, this "stretching" could reduce the gain margin to unreasonable levels. Therefore, the overall robustness properties of the loop could be compromised. Care must be taken so that this does not happen.

Example 2. In this example we introduce plant uncertainty in the transfer function of Example 1, and solve the following problem. Let

$$P(s, a_1) = \frac{n_p(s, a_1)}{d_p(s, a_1)} = \frac{s - 1}{s^2 + (-1 + a_1)s - 2 - 2a_1} \tag{14}$$

be an unstable and nonminimum phase plant family, where $-.2 \leq a_1 \leq .2$. Our design objective is to design (if possible) a first order controller that provides a robust phase margin of at least $15°$ (either positive or negative as required for stability). This implies that the set $\Omega_b = \left\{e^{-j\theta} | -15° \leq \theta \leq 15°\right\}$. This problem will be solved, if a first order controller can be found, that robustly stabilizes the polynomial family:

$$\phi_{ph}(s, a_1, \boldsymbol{b}) = d_l(s, a_1) + \boldsymbol{b} n_l(s) \tag{15}$$

for all $\boldsymbol{b} \in \Omega_b$ and all a_1. To apply the FIT-based algorithm we overbound the set Ω_b by a rectangle as in Figure 2. As an initial controller we use the one computed in Example 1, (13). Running the FIT software algorithm generates the controller:

$$C(s) = \frac{49.76s + 60.83}{s - 40.28} \tag{16}$$

which guarantees a robust phase margin of $-14.48°$. Again because of the overbounding introduced, the actual robust phase margin is: $-15.5°$, which meets the robust phase margin objective set. Figure 4, displays the loop transfer function Nyquist Plots for a number of a_1 parameter values.

Fig. 4. Nyquist plots of loop transfer functions.

6. CONCLUSIONS

In this paper we formulated and solved the robust phase margin design problem. The formulation exploited the fact that this problem can be posed as robust polynomial stabilization problem. Once that was done, a robust design algorithm was suggested which is based on the Finite Inclusions Theorem. The algorithm was demonstrated on academic examples. Other design objectives could also have been included and the interested reader is directed to [1] for more details on how this can be accomplished.

(Received April 8, 1998.)

REFERENCES

[1] T. E. Djaferis: Robust Control Design: A Polynomial Approach. Kluwer Academic Publishers, Boston 1995.

[2] J. C. Doyle, B. A. Francis and A. L. Tannenbaum: Feedback Control Theory. MacMillan, New York 1992.

[3] P. P. Khargonekar and A. Tannenbaum: Non–Euclidean metrics and the robust stabilization of systems with parameter uncertainty. IEEE Trans. Automat. Control *30* (1985), 10, 1005–1013.

[4] K. Ogata: Modern Control Engineering. Third edition. Prentice Hall, Englewood Cliffs, N. J. 1997.

Theodore E. Djaferis, Associate Professor, Department of Electrical and Computer Engineering, University of Massachusetts, Amherst, MA 01003. U. S. A.
e-mail: djaferis@ecs.umass.edu