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A STUDY ON DECENTRALIZED H_∞ FEEDBACK CONTROL SYSTEMS WITH LOCAL QUANTIZERS

Guisheng Zhai, Ning Chen and Weihua Gui

In this paper, we study decentralized H_∞ feedback control systems with quantized signals in local input-output (control) channels. We first assume that a decentralized output feedback controller has been designed for a multi-channel continuous-time system so that the closed-loop system is Hurwitz stable and a desired H_∞ disturbance attenuation level is achieved. However, since the local measurement outputs are quantized by a general quantizer before they are passed to the controller, the system's performance is not guaranteed. For this reason, we propose a local-output-dependent strategy for updating the quantizers' parameters, so that the closed-loop system is asymptotically stable and achieves the same H_∞ disturbance attenuation level. We also extend the discussion and the result to the case of multi-channel discrete-time H_∞ feedback control systems.

Keywords: decentralized H_∞ feedback control system, quantizer, quantization, matrix inequality, output feedback

AMS Subject Classification: 93C15, 93C55, 93C83, 93D15, 93D25

1. INTRODUCTION

In classical feedback control theory, various signals or data in the control loop have been assumed to be passed directly without data loss, except in saturated systems. However, this is not true in many real applications. For example, in networked control systems [2, 4] where all signals are transferred through network, package dropouts or data transfer rate limitations always happen. Another important aspect, which is well known in signal processing area, is signal quantization. Since quantization always exists in computer based control systems, many researchers have begun to study the analysis and design problems for control systems involving various quantization methods. Ref. [3] addressed the problem of stabilizing an unstable linear system by means of quantized state feedback, i.e., state feedback where the measurements of the system state are quantized. The quantizer in [3] takes value in a countable set. Ref. [1] defined a quantizer taking value in a finite set and considered quantized feedback stabilization for linear systems. It has been shown there that if it is possible to change the sensitivity of the quantizer on the basis of available quantized measurements, then a hybrid control strategy, for both

continuous- and discrete-time systems, can be designed to guarantee global asymptotic stability. While the approach in [1] relies on the possibility of making discrete online adjustments of quantizer parameters, Ref. [7] extended the approach for more general nonlinear systems with general types of quantizers involving the states of the system, the measured outputs, and the control inputs. The idea and results in [7] are applied for stabilization of discrete-time LTI systems with quantized measurement outputs in [8].

Later, Ref. [9] considered the stabilization problem for a discrete-time LTI system via state feedback involving both quantized states and control inputs. As assumed in [7], the system considered in [9] is supposed to be stabilizable and a stabilizing state feedback has been designed without taking quantization into account. However, the system's states are quantized before they are passed to the controller, and the control inputs are quantized before they are passed to the system. This is a natural setting in networked control systems, where all informations (reference inputs, plant outputs, control inputs, etc.) are exchanged through a network among control system components (sensors, controllers, actuators, etc.). Due to the quantization effects, the desired system stability can not be guaranteed. For this reason, Ref. [9] defined the two quantizers with general forms as in [7] and then proposed a hybrid quantized state feedback strategy where the values of the quantizer parameters are updated at discrete instants of time. Further, they extended the results to H_∞ feedback control systems in [10], dealing with both state feedback and dynamic output feedback. The key point is to propose a state-dependent (or output-dependent) strategy for updating the quantizer's parameter, so that the system is asymptotically stable and achieves the same H_∞ disturbance attenuation level. It was also noted in [10] that the control strategies of updating the quantizer's parameter are dependent on time in the existing works [1, 7, 8, 9], and such control strategies can not be applied for the case of H_∞ control systems since the value of the disturbance inputs is not available and thus we can not drive the state into an invariant region, as done in [7, 8, 9]. As a great contrast, the control strategy in [10] is state or output dependent, which is usually regarded to have more robustness.

In this paper, we extend the discussion in [10] to decentralized H_∞ feedback control systems, as depicted in Figure. As also noted later, although the discussion and the result are valid for the case where there are more than three input-output channels, we assume for notation simplicity that the number of input-output (control) channels is two. Each channel has its own measurement output y_i and generates its own control input u_i through a local controller. The word "local" here means that although the controller's structure (coefficient matrices) is computed in an offline manner using the whole system's various information, the controller's output u_i only depends on the measurement output y_i of the ith channel, while independent on other channel's measurement output.

Now, as in our previous work, we assume that for the system, a decentralized static output feedback controller has been designed such that the closed-loop system is stable and some H_∞ disturbance attenuation level is achieved. However, the measurement outputs y_i are quantized before they are passed to the controller, and due to the quantization effects, the desired system stability and H_∞ disturbance

attenuation level can not be guaranteed. Here, we suppose that the quantizers are in a generalized form and there is a zoom parameter which can adjusted. Then, we propose to update the quantizers' parameters in a reasonable decentralized online manner, i.e., to change the parameter's value depending on each control channel's measurement output information. We show that under some flexible sufficient condition, there exists a decentralized control strategy for updating each quantizer's zoom parameter, such that the closed-loop system is asymptotically stable and the same H_∞ disturbance attenuation level is achieved.
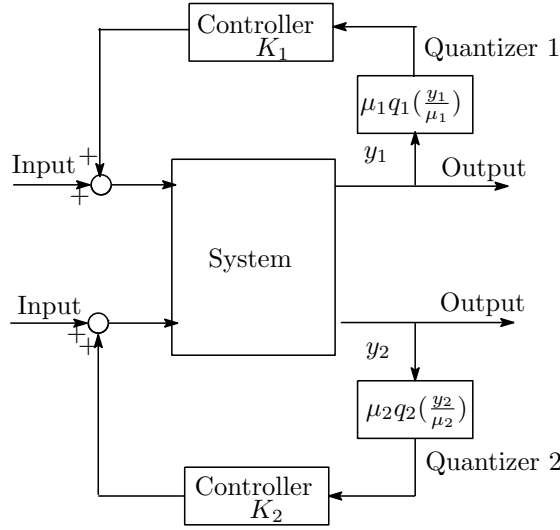
Fig. Decentralized feedback control system with quantized local measurement outputs.

The rest of this paper is organized as follows. Section 2 gives the definition and the property of generalized quantizer. Section 3 formulates the control problem in the case of decentralized continuous-time system and proposes a local-output-dependent strategy for updating the quantizers' parameters, so that the closed-loop system is asymptotically stable and achieves the same H_∞ disturbance attenuation level. Section 4 extends the discussion to the case of decentralized discrete-time system, and obtain nontrivial parallel results. Section 5 gives some important remarks and observations, and finally Section 6 concludes the paper.

2. QUANTIZER DESCRIPTION

First, we give the definition of a quantizer with general form as introduced in [7]. Let $z \in \mathbb{R}^l$ be the variable being quantized. A *quantizer* is defined as a piecewise constant function $q : \mathbb{R}^l \to \mathcal{D}$, where $\mathcal{D}$ is a finite subset of $\mathbb{R}^l$. This leads to a partition of $\mathbb{R}^l$ into a finite number of quantization regions of the form $\{z \in \mathbb{R}^l : q(z) = i\}$, $i \in \mathcal{D}$. These quantization regions are not assumed to have any particular shapes. We assume that there exist positive real numbers M and Δ such that the following conditions hold:

1. If
$$|z| \leq M \tag{1}$$

then
$$|q(z) - z| \leq \Delta. \tag{2}$$

2. If $|z| > M$, then $|q(z)| > M - \Delta$.

Throughout this paper, we denote by $|\cdot|$ the standard Euclidean norm in the n-dimensional vector space $\mathbb{R}^n$, and denote by $\|\cdot\|$ the corresponding induced matrix norm in $\mathbb{R}^{n \times n}$. Condition 1 gives a bound on the quantization error when the quantizer does not saturate. Condition 2 provides a way to detect the possibility of saturation. We will refer to M and Δ as *the range of q* and the *quantization error*, respectively. We also assume that $q(x) = 0$ for x in some neighborhood of the origin. The example of satisfying the above requirements is given by the quantizer with rectangular quantization regions in [1, 6].

In the control strategy to be developed below, we will use quantized measurements of the form
$$q_\mu(z) \overset{\triangle}{=} \mu q \left(\frac{z}{\mu} \right), \tag{3}$$

where $\mu > 0$ is the parameter. The extreme case of $\mu = 0$ is regarded as setting the output of the quantizer as zero. The range of this quantizer is $M\mu$ and the quantization error is $\Delta\mu$. We can view μ as a "zoom" variable: increasing μ corresponds to zooming out and essentially obtaining a new quantizer with larger range and larger quantization error, while decreasing μ corresponds to zooming in and obtaining a quantizer with smaller range but also smaller quantization error. We will update μ later depending on the system local state (or the local measurement output). In this sense, it can be considered as another state of the resultant closed-loop system.

3. CONTINUOUS–TIME SYSTEM DESIGN

Although the discussion in this paper can be easily extended to the case where there are more than two control channels in the system, we focus our attention on the case of two channels (as in Figure) which are described by

$$\begin{cases} \dot{x} = Ax + B_1 w + B_{21} u_1 + B_{22} u_2 \\ z = C_1 x + Dw \\ y_1 = C_{21} x \\ y_2 = C_{22} x \end{cases} \tag{4}$$

where $x \in \mathbb{R}^n$ is the system's state, $u_1 \in \mathbb{R}^{m_1}$ and $u_2 \in \mathbb{R}^{m_2}$ are the control inputs of channel 1 and 2, respectively, $w \in \mathbb{R}^h$ is the disturbance input, $z \in \mathbb{R}^p$ is the controlled output, $y_1 \in \mathbb{R}^{q_1}$ and $y_2 \in \mathbb{R}^{q_2}$ are the local measurement outputs of channel 1 and 2, respectively. The matrices A, B_1, B_{21}, B_{22}, C_1, C_{21}, C_{22} and D are constant and of appropriate dimension. We assume that the triple $(A, [B_{21}\ B_{22}], [C_{21}^T\ C_{22}^T]^T)$ is stabilizable and detectable.

Suppose that for the system (4), we have designed a decentralized controller composed of two local static output feedbacks

$$u_1 = K_1 y_1 \,, \quad u_2 = K_2 y_2 \tag{5}$$

so that the closed-loop system, composed of (4) and (5), is stable and the H_∞ norm of the transfer function from w to z is less than a specified level γ. More precisely, the closed-loop system is written as

$$\begin{cases} \dot{x} = A_{cl}x + B_1 w \\ z = C_1 x + Dw \end{cases} \tag{6}$$

where $A_{cl} = A + B_{21}K_1 C_{21} + B_{22}K_2 C_{22}$. Then, the hypothesis is that, without taking quantization into consideration, the gains K_1 and K_2 in (5) are designed so that A_{cl} is Hurwitz stable and $\|D + C_1(sI - A_{cl})^{-1}B_1\|_\infty < \gamma$. Therefore, according to the well known Bounded Real Lemma [5], there exists a positive definite matrix P satisfying the matrix inequality

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} & P B_1 & C_1^T \\ B_1^T P & -\gamma I & D^T \\ C_1 & D & -\gamma I \end{bmatrix} < 0 \tag{7}$$

or equivalently,

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} + \frac{1}{\gamma}C_1^T C_1 & P B_1 + \frac{1}{\gamma}C_1^T D \\ B_1^T P + \frac{1}{\gamma}D^T C_1 & -\gamma I + \frac{1}{\gamma}D^T D \end{bmatrix} < 0 \,. \tag{8}$$

Since (8) is a matrix inequality, we can always find a positive definite matrix R such that

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} + \frac{1}{\gamma}C_1^T C_1 + R & P B_1 + \frac{1}{\gamma}C_1^T D \\ B_1^T P + \frac{1}{\gamma}D^T C_1 & -\gamma I + \frac{1}{\gamma}D^T D \end{bmatrix} < 0 \,. \tag{9}$$

Throughout this paper, we will let $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the smallest and the largest eigenvalue of a symmetric matrix, respectively. Then, for any positive definite matrix W, the inequalities

$$\lambda_m(W)\,|x|^2 \le x^T W x \le \lambda_M(W)\,|x|^2 \tag{10}$$

hold for any x.

Here, as depicted in Figure, we deal with the case where only quantized local output information is available. For this reason, we modify the static output feedback (5) using the quantized information of y_1 and y_2 as

$$u_1 = K_1 \mu_1 q_1\left(\frac{y_1}{\mu_1}\right) \,, \quad u_2 = K_2 \mu_2 q_2\left(\frac{y_2}{\mu_2}\right) \,. \tag{11}$$

For any fixed positive scalars μ_1 and μ_2, the closed-loop system composed of the system (4) and the quantized output feedback (11) is given by

$$\begin{cases} \dot{x} = A_{cl}x + B_1 w + F(\mu, y) \\ z = C_1 x + Dw \,, \end{cases} \tag{12}$$

where
$$F(\mu, y) = F_1(\mu_1, y_1) + F_2(\mu_2, y_2)$$
$$F_1(\mu_1, y_1) \triangleq \mu_1 B_{21} K_1 \left(q_1(\tfrac{y_1}{\mu_1}) - \tfrac{y_1}{\mu_1} \right) \tag{13}$$
$$F_2(\mu_2, y_2) \triangleq \mu_2 B_{22} K_2 \left(q_2 \left(\tfrac{y_2}{\mu_2} \right) - \tfrac{y_2}{\mu_2} \right).$$

Now, the control problem is very natural. Due to the existence of quantization error, the stability of the closed-loop system and the desired H_∞ disturbance attenuation level γ is not guaranteed. For this reason, we formulate our control problem as follows:

Decentralized Quantizer Design Problem. Design a decentralized control strategy which adjusts μ_1 depending on the local output y_1 and adjusts μ_2 depending on the local output y_2 appropriately, so that the stability of the closed-loop system and the same H_∞ disturbance attenuation level γ is achieved.

We are in the position to state and prove the first main result in this paper.

Theorem 1. Assume that for the two quantizers M_i is chosen large enough compared to Δ_i so that

$$M_i > 4\Delta_i \frac{\|PB_{2i}K_i\|\|C_{2i}\|}{\lambda_m(R)}, \quad i = 1, 2. \tag{14}$$

Then, there exists a control strategy for updating μ_i, which is dependent on the local measurement output y_i, such that the closed-loop system (12) is asymptotically stable and the H_∞ disturbance attenuation level γ is achieved.

P r o o f. Since $\frac{y_i}{\mu_i} = \frac{C_{2i}x}{\mu_i}$ $(i = 1, 2)$ is quantized before being passed to the feedback, we obtain by using the properties of general quantizers in (1) and (2) that whenever $|y_i| \le M_i \mu_i$, the inequality

$$\left| \frac{y_i}{\mu_i} - q \left(\frac{y_i}{\mu_i} \right) \right| \le \Delta_i \tag{15}$$

is true. We consider the Lyapunov function candidate

$$V(x) = x^T P x \tag{16}$$

for the closed-loop system (12). By using the matrix inequality (9), we obtain that when $|y_i| \le M_i \mu_i$, the derivative of $V(x)$ along solutions of (12) satisfies

$$\begin{aligned}
\dot{V} &= (A_{cl}x + B_1 w + F(\mu, y))^T Px + x^T P (A_{cl}x + B_1 w + F(\mu, y)) \\
&= \begin{bmatrix} x^T & w^T \end{bmatrix} \begin{bmatrix} A_{cl}^T P + PA_{cl} & PB_1 \\ B_1^T P & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + 2x^T P F(\mu, y) \\
&\le \begin{bmatrix} x^T & w^T \end{bmatrix} \begin{bmatrix} -R - \tfrac{1}{\gamma} C_1^T C_1 & -\tfrac{1}{\gamma} C_1^T D \\ -\tfrac{1}{\gamma} D^T C_1 & \gamma I - \tfrac{1}{\gamma} D^T D \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + 2x^T P F(\mu, y)
\end{aligned}$$

$$\leq -\frac{1}{\gamma} z^T z + \gamma w^T w - x^T R x + 2x^T P(F_1(\mu_1, y_1) + F_2(\mu_2, y_2))$$

$$\leq -\frac{1}{\gamma} z^T z + \gamma w^T w$$
$$-\frac{1}{2}\lambda_m(R)|x|\left(|x| - 4\Delta_1 \frac{\|PB_{21}K_1\|}{\lambda_m(R)}\mu_1\right)$$
$$-\frac{1}{2}\lambda_m(R)|x|\left(|x| - 4\Delta_2 \frac{\|PB_{22}K_2\|}{\lambda_m(R)}\mu_2\right)$$

$$\leq -\frac{1}{\gamma} z^T z + \gamma w^T w$$
$$-\frac{1}{2}\lambda_m(R)\frac{|x|}{\|C_{21}\|}\left(|y_1| - 4\Delta_1 \frac{\|PB_{21}K_1\|\|C_{21}\|}{\lambda_m(R)}\mu_1\right)$$
$$-\frac{1}{2}\lambda_m(R)\frac{|x|}{\|C_{22}\|}\left(|y_2| - 4\Delta_2 \frac{\|PB_{22}K_2\|\|C_{22}\|}{\lambda_m(R)}\mu_2\right). \tag{17}$$

According to (14), we can always find a scalar $\varepsilon \in (0,1)$ such that

$$M_i > 4\Delta_i \frac{\|PB_{2i}K_i\|\|C_{2i}\|}{\lambda_m(R)} \times \frac{1}{1-\varepsilon}, \tag{18}$$

which is equivalent to

$$\frac{1}{1-\varepsilon} \times 4\Delta_i \frac{\|PB_{2i}K_i\|\|C_{2i}\|}{\lambda_m(R)}\mu_i < M_i\mu_i. \tag{19}$$

Therefore, for any nonzero y_i, we can find a positive scalar μ_i such that

$$\frac{1}{1-\varepsilon} \times 4\Delta_i \frac{\|PB_{2i}K_i\|\|C_{2i}\|}{\lambda_m(R)}\mu_i \leq |y_i| \leq M_i\mu_i. \tag{20}$$

This is also true in the case of $y_i = 0$, where we set $\mu_i = 0$ as an extreme case and consider the output of the quantizer as zero.

In other words, since we can always choose μ_i so that (20) is satisfied, (17) holds and thus

$$\dot{V} \leq -\frac{1}{\gamma}\Gamma(t) - \frac{1}{2}\varepsilon\lambda_m(R)\left(\frac{|x|}{\|C_{21}\|}|y_1| + \frac{|x|}{\|C_{22}\|}|y_2|\right), \tag{21}$$

where $\Gamma = z^T z - \gamma^2 w^T w$.

First, by setting $w = 0$ in (21), we see clearly that the closed-loop system is asymptotically stable.

Next, since $V(t) \geq 0$, we obtain from (21) that $\dot{V} \leq -\frac{1}{\gamma}\Gamma(t)$, and thus for any $t > t_0$,

$$V(t) - V(t_0) \leq -\frac{1}{\gamma}\int_{t_0}^t \Gamma(\tau)\,\mathrm{d}\tau. \tag{22}$$

Using $V(t) \geq 0$ again, we obtain

$$\int_{t_0}^t z^T(\tau)z(\tau)\,\mathrm{d}\tau \leq \gamma V(t_0) + \gamma^2 \int_{t_0}^t w^T(\tau)w(\tau)\,\mathrm{d}\tau, \tag{23}$$

which implies that the H_∞ disturbance attenuation level γ is achieved. This completes the proof. $\square$

Since decentralized state feedback is the special case of decentralized static output feedback, we briefly summarize the result as follows.

Suppose we have designed a decentralized state feedback

$$u_1 = K_1 x, \quad u_2 = K_2 x \tag{24}$$

so that the closed-loop system, composed of (4) and (24), is stable and the H_∞ norm of the transfer function from w to z is less than a specified level γ. Now, the inputs to the quantizers are the system state instead of local measurement outputs.

If we use the same notation $A_{cl} = A + B_{21}K_1 + B_{22}K_2$ as before (without causing confusion) to denote the closed-loop system matrix, the same matrix inequality (8) holds and thus (9) holds with some positive definite matrix R. Then, using similar calculation as for Theorem 1, we obtain easily the following corollary.

Corollary 1. Assume that for the two quantizers M_i is chosen large enough compared to Δ_i so that

$$M_i > 4\Delta_i \frac{\|PB_{2i}K_i\|}{\lambda_m(R)}, \quad i = 1, 2. \tag{25}$$

Then, there exists a control strategy for updating μ_i, which is dependent on the system state x, such that the closed-loop system is asymptotically stable and the H_∞ disturbance attenuation level γ is achieved.

4. DISCRETE–TIME SYSTEM DESIGN

Consider the multi-channel discrete-time system described by

$$\begin{cases} x(k+1) = Ax(k) + B_1 w(k) + B_{21} u_1(k) + B_{22} u_2(k) \\ \quad z(k) = C_1 x(k) + Dw(k) \\ \quad y_1(k) = C_{21} x(k) \\ \quad y_2(k) = C_{22} x(k) \end{cases} \tag{26}$$

where all the notations have the same meaning as before but now all vectors are in discrete-time form.

Suppose that for the system (26), we have designed a decentralized static output feedback (5) such that the closed-loop system, composed of (26) and (5), is Schur stable and the H_∞ norm of the transfer function from w to z is less than a specified level γ. Therefore, A_{cl} is Schur stable and $\|D + C_1(zI - A_{cl})^{-1}B_1\|_\infty < \gamma$. According to the Bounded Real Lemma for discrete-time systems [5], there exists a positive definite matrix P satisfying the matrix inequality

$$\begin{bmatrix} -P & PA_{cl} & PB_1 & 0 \\ A_{cl}^T P & -P & 0 & C_1^T \\ B_1^T P & 0 & -\gamma I & D^T \\ 0 & C_1 & D & -\gamma I \end{bmatrix} < 0. \tag{27}$$

Since this is a matrix inequality, we can always find a positive matrix R such that

$$
\begin{bmatrix}
-P & PA_{cl} & PB_1 & 0 \\
A_{cl}^T P & -P+R & 0 & C_1^T \\
B_1^T P & 0 & -\gamma I & D^T \\
0 & C_1 & D & -\gamma I
\end{bmatrix} < 0 \, .
\tag{28}
$$

The above matrix inequality is equivalent to

$$
\begin{bmatrix}
-P & PA_{cl} & PB_1 \\
A_{cl}^T P & -P+R+\frac{1}{\gamma}C_1^T C_1 & \frac{1}{\gamma}C_1^T D \\
B_1^T P & \frac{1}{\gamma}D^T C_1 & -\gamma I + \frac{1}{\gamma}D^T D
\end{bmatrix} < 0
\tag{29}
$$

and

$$
\begin{bmatrix}
A_{cl}^T P A_{cl} - P + R + \frac{1}{\gamma}C_1^T C_1 & A_{cl}^T P B_1 + \frac{1}{\gamma}C_1^T D \\
B_1^T P A_{cl} + \frac{1}{\gamma}D^T C_1 & -\gamma I + B_1^T P B_1 + \frac{1}{\gamma}D^T D
\end{bmatrix} < 0 \, .
\tag{30}
$$

Furthermore, since $P > 0$, we can always find a positive number $\xi > 1$ such that

$$
\begin{bmatrix}
(1-\xi^2)P & PA_{cl} & PB_1 \\
A_{cl}^T P & 0 & 0 \\
B_1^T P & 0 & 0
\end{bmatrix} \le 0 \, ,
\tag{31}
$$

or equivalently,

$$
\begin{bmatrix}
P & PA_{cl} & PB_1 \\
A_{cl}^T P & 0 & 0 \\
B_1^T P & 0 & 0
\end{bmatrix} \le
\begin{bmatrix}
\xi^2 P & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \, .
\tag{32}
$$

Again, we assume that only quantized local output information is available in the feedback, and thus the static output feedback (5) is modified using the quantized information of y as in (11). For any fixed positive scalars μ_1 and μ_2, the closed-loop system composed of the system (26) and the quantized output feedback (11) is given by

$$
\begin{cases}
x(k+1) = A_{cl}x(k) + B_1 w(k) + F(\mu, y(k)) \\
z(k) = Cx(k) + Dw(k) \, .
\end{cases}
\tag{33}
$$

We now state and prove the second main result in this paper.

Theorem 2. Assume that for the two quantizers M_i is chosen large enough compared to Δ_i so that

$$
M_i > 2\xi\Delta_i \|C_{2i}\| \sqrt{\frac{\|K_i^T B_{2i}^T P B_{2i} K_i\|}{\lambda_m(R)}} \, , \quad i = 1, 2 \, .
\tag{34}
$$

Then, there exists a control strategy for updating μ_i, which is dependent on the local measurement output y_i, such that the closed-loop system (33) is asymptotically stable and the H_∞ disturbance attenuation level γ is achieved.

P r o o f . Since $\frac{y_i}{\mu_i} = \frac{C_{2i}x}{\mu_i}$ $(i = 1, 2)$ is quantized before being passed to the feedback, we obtain that whenever $|y_i| \le M_i\mu_i$, the inequality

$$\left| \frac{y_i(k)}{\mu_i(k)} - q\left(\frac{y_i(k)}{\mu_i(k)}\right) \right| \le \Delta_i \tag{35}$$

is true. We consider the same Lyapunov function candidate (16) for the closed-loop system (33). By using the matrix inequalities (30) and (32), we obtain that when $|y_i(k)| \le M_i\mu_i(k)$, the difference of $V(x)$ along solutions of (33) satisfies

$$
\begin{aligned}
V(x(k&+1)) - V(x(k)) \\
&= (A_{cl}x + B_1w + F(\mu, y))^T P (A_{cl}x + B_1w + F(\mu, y)) - x^T P x \\
&= \begin{bmatrix} x^T & w^T \end{bmatrix} \begin{bmatrix} A_{cl}^T P A_{cl} - P & A_{cl}^T P B_1 \\ B_1^T P A_{cl} & B_1^T P B_1 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \\
&\quad + 2F^T(\mu, y) P (A_{cl}x + B_1w) + F^T(\mu, y) P F(\mu, y) \\
&\le \begin{bmatrix} x^T & w^T \end{bmatrix} \begin{bmatrix} -R - \frac{1}{\gamma}C_1^T C_1 & -\frac{1}{\gamma}C_1^T D \\ -\frac{1}{\gamma}D^T C_1 & \gamma I - \frac{1}{\gamma}D^T D \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \\
&\quad + 2F^T(\mu, y) P (A_{cl}x + B_1w) + F^T(\mu, y) P F(\mu, y) \\
&\le -\frac{1}{\gamma}z^T z + \gamma w^T w - x^T R x \\
&\quad + 2F^T(\mu, y) P (A_{cl}x + B_1w) + F^T(\mu, y) P F(\mu, y) \\
&= -\frac{1}{\gamma}z^T z + \gamma w^T w - x^T R x \\
&\quad + \begin{bmatrix} F \\ x \\ w \end{bmatrix}^T \begin{bmatrix} P & P A_{cl} & P B_1 \\ A_{cl}^T P & 0 & 0 \\ B_1^T P & 0 & 0 \end{bmatrix} \begin{bmatrix} F \\ x \\ w \end{bmatrix} \\
&\le -\frac{1}{\gamma}z^T z + \gamma w^T w - x^T R x + \xi^2 F^T P F \\
&= -\frac{1}{\gamma}z^T z + \gamma w^T w - x^T R x + \xi^2 (F_1^T + F_2^T) P (F_1 + F_2) \\
&\le -\frac{1}{\gamma}z^T z + \gamma w^T w - x^T R x + 2\xi^2 (F_1^T P F_1 + F_2^T P F_2) \\
&\le -\frac{1}{\gamma}z^T z + \gamma w^T w - x^T R x + 2\xi^2 \mu_1^2 \Delta_1^2 \|K_1^T B_{21}^T P B_{21} K_1\| \\
&\quad + 2\xi^2 \mu_2^2 \Delta_2^2 \|K_2^T B_{22}^T P B_{22} K_2\|
\end{aligned}
\tag{36}
$$

$$\leq -\frac{1}{\gamma}z^T z + \gamma w^T w$$

$$-\frac{\lambda_m(R)}{2}\left[|x|^2 - \left(2\xi\Delta_1\sqrt{\frac{\|K_1^T B_{21}^T P B_{21} K_1\|}{\lambda_m(R)}}\mu_1\right)^2\right]$$

$$-\frac{\lambda_m(R)}{2}\left[|x|^2 - \left(2\xi\Delta_2\sqrt{\frac{\|K_2^T B_{22}^T P B_{22} K_2\|}{\lambda_m(R)}}\mu_2\right)^2\right]. \tag{37}$$

Since we need using the measurement output information, we use $|y_i| = |C_{2i}x| \leq \|C_{2i}\||x|$ in the above inequality to obtain

$$V(x(k+1)) - V(x(k)) \leq -\frac{1}{\gamma}z^T z + \gamma w^T w$$

$$-\frac{\lambda_m(R)}{2\|C_{21}\|^2}\left[|y_1|^2 - \left(2\xi\Delta_1\|C_{21}\|\sqrt{\frac{\|K_1^T B_{21}^T P B_{21} K_1\|}{\lambda_m(R)}}\mu_1\right)^2\right]$$

$$-\frac{\lambda_m(R)}{2\|C_{22}\|^2}\left[|y_2|^2 - \left(2\xi\Delta_2\|C_{22}\|\sqrt{\frac{\|K_2^T B_{22}^T P B_{22} K_2\|}{\lambda_m(R)}}\mu_2\right)^2\right]. \tag{38}$$

According to (34), we can always find a scalar $\varepsilon \in (0,1)$ such that

$$M_i > 2\xi\Delta_i\|C_{2i}\|\sqrt{\frac{\|K_i^T B_{2i}^T P B_{2i} K_i\|}{\lambda_m(R)}} \times \frac{1}{\sqrt{1-\varepsilon}}, \tag{39}$$

which is equivalent to

$$\frac{1}{\sqrt{1-\varepsilon}} \times 2\xi\Delta_i\|C_{2i}\|\sqrt{\frac{\|K_i^T B_{2i}^T P B_{2i} K_i\|}{\lambda_m(R)}}\mu_i < M_i\mu_i. \tag{40}$$

Therefore, for any nonzero y_i, we can find a positive scalar μ_i such that

$$\frac{1}{\sqrt{1-\varepsilon}} \times 2\xi\Delta_i\|C_{2i}\|\sqrt{\frac{\|K_i^T B_{2i}^T P B_{2i} K_i\|}{\lambda_m(R)}}\mu_i \leq |y_i| \leq M_i\mu_i. \tag{41}$$

In other words, since we can always choose μ_i so that (41) is satisfied, (38) holds and thus

$$\dot{V} \leq -\frac{1}{\gamma}\Gamma(t) - \frac{1}{2}\varepsilon\lambda_m(R)\left(\frac{|y_1|^2}{\|C_{21}\|^2} + \frac{|y_2|^2}{\|C_{22}\|^2}\right). \tag{42}$$

The remaining proof, concerning the asymptotic stability and the H_∞ disturbance attenuation level, is the same as in Theorem 1, and is thus omitted. $\square$

Similarly to Corollary 1, the following result is obtained easily from (37), corresponding to the case of state feedback.

Corollary 2. Assume that for the two quantizers M_i is chosen large enough compared to Δ_i so that

$$M_i > 2\xi\Delta_i\sqrt{\frac{\|K_i^T B_{2i}^T P B_{2i} K_i\|}{\lambda_m(R)}}\,, \quad i = 1, 2\,. \tag{43}$$

Then, there exists a control strategy for updating μ_i, which is dependent on the system state x, such that the closed-loop system is asymptotically stable and the H_∞ disturbance attenuation level γ is achieved.

5. REMARKS AND OBSERVATION

In this section, we provide two important remarks and two observations concerning the results in the previous sections.

Remark 1. Both the condition (14) in Theorem 1 and the condition (34) in Theorem 2 are flexible, in the sense that we can choose the matrices P, R (and K_i) so that these conditions are satisfied. These matrices are not independent and they must satisfy the matrix inequality (9) or (29), but we still have much design freedom since these are inequalities and we can incorporate some optimization requirement when solving (9) or (29).

Remark 2. The extension from continuous-time system to discrete-time system is not trivial. To deal with the quantization error term, we introduce a parameter ξ in (32) to estimate the final term in (36) and to deal with the interference between F and (x, w). The parameter ξ is also flexible, and we can choose it appropriately such that the condition (34) is satisfied.

Observation 1. In Theorems 1 and 2, in order to obtain the conditions and the updating strategies of the zooming parameters, we divide the term $\lambda_m(R)|x|^2$ into $\frac{1}{2}\lambda_m(R)|x|^2 + \frac{1}{2}\lambda_m(R)|x|^2$ and then use each of them to deal with the local quantization error term in (17) and (37). However, in real applications, we can have more flexibility by dividing the term $\lambda_m(R)|x|^2$ into $\beta_1\lambda_m(R)|x|^2 + \beta_2\lambda_m(R)|x|^2$ with $\beta_i > 0$ and $\beta_1 + \beta_2 \leq 1$. In doing so, the condition (14) can be modified as

$$M_i > 2\Delta_i\frac{\|PB_{2i}K_i\|\|C_{2i}\|}{\beta_i\lambda_m(R)}\,, \quad i = 1, 2\,, \tag{44}$$

and the condition (34) can be modified as

$$M_i > \frac{\xi\Delta_i\|C_{2i}\|}{\beta_i}\sqrt{\frac{\|K_i^T B_{2i}^T P B_{2i} K_i\|}{\lambda_m(R)}}\,, \quad i = 1, 2\,. \tag{45}$$

When there are more than three control channels, the above idea of introducing β_i's will gain much more flexibility so as to satisfy (14) or (34) for all channels. In fact, this can be interpreted as a kind of cooperation among all control channels in a pre-designed decentralized control system.

Observation 2. As also mentioned in Section 3, we note that all the discussion and the results can be easily extended to the case where there are more than two control channels in the system (4), i.e.,

$$
\begin{cases}
\dot{x} = Ax + B_1 w + \displaystyle\sum_{i=1}^{N} B_{2i} u_i \\
z = C_1 x + Dw \\
y_i = C_{2i} x, \quad i = 1, \cdots, N
\end{cases}
\tag{46}
$$

where $N \geq 2$ is the number of control channels, $u_i \in \mathbb{R}^{m_i}$ is the control input of the ith channel, and $y_i \in \mathbb{R}^{q_i}$ is the local measurement output of the ith channel. This is a decentralized system since the control input u_i of the ith channel is only dependent on the measurement output y_i of the same channel.

Specifically, if we set the coefficient matrices in (46) as

$$
A = \begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1N} \\
A_{21} & A_{22} & \cdots & A_{2N} \\
\cdots & \cdots & \cdots & \cdots \\
A_{N1} & A_{N2} & \cdots & A_{NN}
\end{bmatrix}, \quad
B_1 = \begin{bmatrix}
B_{11} & 0 & \cdots & 0 \\
0 & B_{12} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & B_{1N}
\end{bmatrix},
$$

$$
B_{2i} = \begin{bmatrix}
0 \\
\cdots \\
\bar{B}_{2i} \\
0 \\
0
\end{bmatrix}, \quad
C_1 = \begin{bmatrix}
C_{11} & 0 & \cdots & 0 \\
0 & C_{12} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & C_{1N}
\end{bmatrix}, \quad
D = \begin{bmatrix}
D_1 & 0 & \cdots & 0 \\
0 & D_2 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & D_N
\end{bmatrix},
\tag{47}
$$

where all the matrices have appropriate size, and divide the state x, the disturbance input w and the controlled output z appropriately, then the system (46) becomes

$$
\begin{cases}
\dot{x}_i = A_{ii} x_i + B_{1i} w_i + \bar{B}_{2i} u_i + \displaystyle\sum_{j=1, j \neq i}^{N} A_{ij} x_j \\
z_i = C_{1i} x_i + D_i w_i \\
y_i = C_{2i} x_i, \qquad\qquad i = 1, \cdots, N
\end{cases}
\tag{48}
$$

which is obviously the well known large scale interconnected system. Therefore, it is an easy matter to apply the discussion and the results in this paper to quantized decentralized H_∞ feedback control for the above large scale interconnected dynamical systems.

6. CONCLUSION

In this paper, we have studied stabilization and H_∞ disturbance attenuation problem for multi-channel feedback control systems where the states or the measurement outputs are quantized before they go to the controller. We have proposed a local-state-dependent (or local-output-dependent) control strategy for updating the quantizers' parameters on line so that the overall closed-loop system is asymptotically stable and achieves the same H_∞ disturbance attenuation level as in the case where no quantization is involved.

Our next interest is the H_∞ disturbance attenuation problem for multi-channel feedback control systems with two local quantizers (quantization of both states/outputs and control inputs) for each control channel, as mentioned in [9]. Furthermore, the application of these results for design of large-scale networked control systems is an interesting and challenging problem.

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Guisheng Zhai, Department of Mechanical Engineering, Osaka Prefecture University, Sakai, Osaka 599-8531. Japan.
e-mail: zhai@me.osakafu-u.ac.jp

Ning Chen and Weihua Gui, School of Information Science & Engineering, Central South University, Changsha, Hunan 410083. China.
e-mails: ningchen, gwh@mail.csu.edu.cn