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GLOBAL CONTROL OF FLEXURAL AND TORSIONAL DEFORMATIONS OF ONE–LINK MECHANICAL SYSTEM

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Usual way in controlling infinite dimensional dynamical systems such as deformable mechanical one is to project onto a finite dimensional manifold $(\mathcal{M})$ and apply previous, mainly PD-type, controllers, plus conditions for non-spillover. This approximate method rests upon representing needed system information for controller onto $(\mathcal{M})$, with associated difficulties unsolvable in case of mixed flexural and torsional origin. To avoid them, another procedure is proposed here, which uses directly measurable and collocated bending moment at link origin representing global force due to deformations onto link rigid motion, into the feedback loop for controlling deformations by stiffening selectively the system to all these modes. The corresponding global control accounts for all deformation modes, flexural and torsional, so no projection is needed as usual, and is easily implementable and efficient.

1. INTRODUCTION

1.1 — Improvement of mechanical systems performances is now hitting a breakpoint where internal material structure cannot be neglected, due to excitation of deformation modes impairing their behavior [33]. "Simple" passive approach may be first proposed by upgrading material qualities [17], but this way is expensive and not always adapted [6]. Next methods of control are "active" [3], and use adequate power dumping into the system. But to be applicable, they correspondingly require accurate enough models for faithful system representation [24, 32].

Interactions of both rigid body and deformations dynamics result in intriquate highly nonlinear system of ODE and PDE [5, 7, 16, 22, 30, 36]. Usual attempts to simplify total system reduce PDE's to ODE's by projection methods to select a finite number of modes [2, 14, 18, 25]. But even if mode selection can be justified by adapted convergence results, a difficulty still remains in adequate observation of these modes for reinjection into system feedback loops, and also because power flow extends over a frequency band larger than controlled actuator's one.

1.2 — More fundamentally, the system exhibits specific structure of "complex system" [10] with torques applied to generalized (rigid) variables, in other words, without direct action on deformation modes. So a mismatch between internal natural power cascade and outer power cascade imposed by added feedback loops is always possible, see Figure 1, especially when the two power flows are of similar amplitude. Arrow 1 is the source effect from rigid onto deformation modes, and arrow 2 is the drift effect from feedback interaction of deformation modes onto rigid dynamics. It is important for avoiding spillover effects to account for both effects, as all power inside the system circulates along this exchange structure.

Fig. 1. Complex system structure of deformable N-link mechanical system.

To avoid this difficulty, the way to deal with such system is to exactly project complete system dynamics onto rigid dynamics driving system behavior [11], which gives a modified rigid "core dynamics". The operation can be explicitely analyzed for rigid displacement dynamics characteristic time τ_q and deformation dynamics characteristic time τ_d such that $\tau_d \simeq \epsilon\tau_q$, and deformation amplitudes small compared

to displacement amplitudes, i. e. $\Delta P \simeq \eta|OP|$, for any point P of the body by asymptotic expansion in ϵ small enough compared to 1 [4, 27].

1.3 — This approach leads to another type of control, recognizing that, under deformation effects, exact trajectory knowledge is no longer accessible as long as trajectories become more and more complicated, and at the same time less and less information is related to a single trajectory as it cannot be distinguished from neighbouring ones. In this case only global structure parameters, which are now the natural invariants instead of trajectory initial values as before, have a physical meaning. So rather than the precise actual trajectory, the natural setting becomes a class of trajectories belonging to a well defined function space, and control now expresses this belonging by a fixed point property in this space, hence the name "functional" control given to the new control.

The simple, robust and very global "functional" control is not as directive as usual PD tracking control, because acting only on rigid variables, and provides asymptotic, but not always exponential, stability. It is however interesting to look for this global control. General results [12] will be illustrated here by application on the simple example of a one-link mechanical

system with flexion and torsion deformations, the equations of which are given

in next paragraph within Euler–Bernoulli approximation. Comparison will be next made between classical control approach developed in third paragraph and present functional control discussed in the fourth paragraph.

2. DYNAMICAL EQUATIONS OF ONE–LINK DEFORMABLE SYSTEM

Dynamical equations follow from application of elementary Newton's law, but Lagrange formalism [15, 34] simply gives complete equations of one-link deformable mechanical system in the following form

$$\rho A\left(x\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+\frac{\partial^2 u(t,x)}{\partial t^2}\right)=-\frac{\partial^2}{\partial x^2}\,EI\,\frac{\partial^2 u(t,x)}{\partial x^2} \tag{1}$$

$$\rho K^2\,\frac{\partial^2\gamma(t,x)}{\partial t^2}=GJ\,\frac{\partial^2\gamma(t,x)}{\partial x^2} \tag{2}$$

$$mX=\frac{\partial}{\partial x}\,EI\,\frac{\partial^2 u(t,x)}{\partial x^2}|_{x=L} \tag{3}$$

$$ml_fX+J_f\left(x\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+\frac{\partial^3 u(t,x)}{\partial t^2\partial x}\right)\Bigg|_{x=L}=-EI\,\frac{\partial^2 u(t,x)}{\partial x^2}\Bigg|_{x=L} \tag{4}$$

$$ml_tX+J_t\,\frac{\partial^2\gamma(t,x)}{\partial t^2}\Bigg|_{x=L}=-GJ\,\frac{\partial\gamma(t,x)}{\partial x}\Bigg|_{x=L} \tag{5}$$

$$J_a\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}+F_a\frac{\mathrm{d}\theta}{\mathrm{d}t}+K_m(\theta-\theta_m)=F_{ext} \tag{6}$$

$$J_m\frac{\mathrm{d}^2\theta_m}{\mathrm{d}t^2}+F_m\frac{\mathrm{d}\theta_m}{\mathrm{d}t}+K_m(\theta_m-\theta)=\tau+EI\,\frac{\partial^2 u(t,x)}{\partial x^2}\Bigg|_{x=0} \tag{7}$$

with θ, θ_m, $u(t,x)$, $\gamma(t,x)$ respectively the articular and actuator variables, and the deformation, flexion and torsion, variables, (l_f, l_t) the coordinates of the tip mass m with respect to the end of the link, see Figure 2, and the various other coefficients characterizing the beam as usual within Euler–Bernoulli approximation [8]. Note immediatly that on actuator eqn (7), are acting both applied input torque τ and bending moment $M_a = EI(\partial^2 u(t,0)/\partial x^2)$, which is the only term through which deformations are seen by rigid part of the system.

Boundary Conditions are given by eqns $(3, 4, 5)$ which are simply expressing the equality between generalized force vector

$$F = col\left\{-EI\left(\frac{\partial^2 u(t,x)}{\partial x^2}\right), \frac{\partial}{\partial x}\left(EI\frac{\partial^2 u(t,x)}{\partial x^2}\right), -GI\left(\frac{\partial \gamma(t,x)}{\partial x}\right)\right\}$$

and inertial force produced by the motion of mass m in absolute frame $\mathbb{R}_{(0)}$ derived from kinetic potential

Fig. 2. Mass representation and associated frame.

$$K_S = \frac{1}{2}mX^2 + J_f\left(\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\partial^2 u(t,x)}{\partial x \partial t}\right)^2_{x=L} + J_t\left(\frac{\partial \gamma(t,x)}{\partial x}\right)^2_{x=L} \tag{8}$$

with

$$X = (L + l_f)\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\partial u(t,x)}{\partial x} + l_f\frac{\partial^2 u(t,x)}{\partial x \partial t} + l_t\frac{\partial \gamma(t,x)}{\partial x}\bigg|_{x=L}. \tag{9}$$

3. CLASSICAL CONTROL OF COMPLIANT DEFORMABLE ONE LINK SYSTEM

3.1 — It is first convenient to restate eqns $(1,2,3,4,5)$ into functional evolution equation in normalized form. Consider Hilbert space $\mathcal{H} = \mathcal{L}_2(0,1) \times \mathcal{L}_2(0,1) \times \mathrm{I\!R}_3$ with inner product

$$\begin{aligned} & [(u_1,u_2,u_3,u_4,u_5)^T,(v_1,v_2,v_3,v_4,v_5)]_{\mathcal{H}} \\ = & \int_0^1 [u_1(x)\,v_1(x) + u_2(x)\,v_2(x)]\,\mathrm{d}x + \sum_{j=3}^{5} u_j v_j. \end{aligned} \tag{10}$$

Define $u_1(t,x) = u(t,x),\ u_2(t,x) = \gamma(t,x),\ u_3(t) = u(t,1),\ u_4(t) = \partial u(t,L)/\partial x$ $u_5(t) = \gamma(t,1)$ and subspace $\mathcal{V} \subset \mathcal{H}$ by

$$\begin{aligned} \mathcal{V} = & \Big\{ u = col(u_1,u_2,u_3,u_4,u_5) |\ u_1(.) \in \mathcal{H}_2(0,1),\ u_2(.) \in \mathcal{H}_1(0,1), \\ & u_3(t) = u(t,1),\ u_4(t) = \left.\frac{\partial u(t,x)}{\partial x}\right|_{x=1},\ u_5(t) = \gamma(t,1),\ u_1(t,0) = 0, \\ & u_2(t,0) = 0,\ \omega_f^2 \frac{\partial^2 u_1(t,x)}{\partial x^2} + \kappa_1 \left.\frac{\partial u(t,x)}{\partial x}\right|_{x=0} = 0 \Big\} \end{aligned} \tag{11}$$

with $\mathcal{H}_m(0,1)$ the Sobolev space of order m [1] and $\omega_f^2 = EI/\rho AL^4$, $\omega_t^2 = GJ/\rho K^2 L^2$ the two natural flexion and torsion system frequencies. In $\mathcal{V}$ inner product is

$$\begin{aligned} [u,v]_{\mathcal{V}} = & \int_0^1 \Big\{ \omega_f^2 \left[\frac{\partial^2 u_1(t,x)}{\partial x^2}\right] \left[\frac{\partial^2 v_1(t,x)}{\partial x^2}\right] \\ & + \omega_t^2 \left[\frac{\partial u_2(t,x)}{\partial x}\right] \left[\frac{\partial v_2(t,x)}{\partial x}\right] \Big\} \mathrm{d}x + \sum_{j=3}^{5} u_j v_j. \end{aligned} \tag{12}$$

Define operator Π by

$$\begin{aligned} \Pi u = & col\big(u_1, u_2, \mu(u_3 + \lambda_f u_4 + \lambda_t u_5), \mu\lambda_f(u_3 + \lambda_f u_4 + \lambda_t u_5) + J_f u_4, \\ & \mu\lambda_t(u_3 + \lambda_f u_4 + \lambda_t u_5)\big) \end{aligned} \tag{13}$$

of the form $\Pi = diag(1,1,M)$ with symmetric positive definite

$$M = \begin{bmatrix} \mu & \mu\lambda_f & \mu\lambda_t \\ \mu\lambda_f & J_f + \mu\lambda_f^2 & \mu\lambda_f\lambda_t \\ \mu\lambda_t & \mu\lambda_f & J_t + \mu\lambda_t^2 \end{bmatrix} \tag{14}$$

with $\mu = m/M_0$, $\lambda_f = l_f/L$, $\lambda_t = l_t/L$, and M_0 the mass of the link.

3.2 — Because Π is positive definite, one can construct from $\mathcal{H}$ another Hilbert space $\mathcal{H}^*$ with inner product

$$[u,v]_{H^*} = [\Pi u, v]_H = \int_0^1 [u_1(x)\,v_1(x) + u_2(x)\,v_2(x) + \sum_{j=3}^{5} u_j M v_j \tag{15}$$

and same topology (any ball of each space can be embedded into balls of the other one).

Now define the operator K and its domain by

$$Ku = diag\bigg(\frac{\partial^2}{\partial x^2}\omega_f^2\frac{\partial^2 u_1}{\partial x^2},\ -\omega_t^2\frac{\partial^2 u_2(t,x)}{\partial x^2}, -\frac{\partial}{\partial x}\omega_f^2\frac{\partial^2 u_1}{\partial x^2},$$

$$\omega_f^2\frac{\partial^2 u(t,1)}{\partial x^2},\ \omega_t^2\frac{\partial\gamma(t,1)}{\partial x}\bigg) \tag{16}$$

with

$$D(K) = \bigg\{u|\ u \in \mathcal{V},\ \frac{\partial^2 u(t,.)}{\partial x^2} \in \mathcal{H}_2(0,1), \frac{\partial\gamma(t,.)}{\partial x} \in \mathcal{H}_1(0,1)\bigg\}. \tag{17}$$

Let the 5-vector Θ with remaining terms in $\theta, u(t,x), u(t,L)$ from eqn (1). Then finally eqns $(1,2,3,4,5)$ take the simple operational differential form

$$\frac{\mathrm{d}^2 u}{\mathrm{d}t^2} + \Pi^{-1}D\frac{\mathrm{d}u}{\mathrm{d}t} + \Pi^{-1}Ku = \Pi^{-1}\Theta. \tag{18}$$

in Hilbert space $\mathcal{H}^*$ which can be studied in this framework [31]. So initial system transforms into "canonical" normalized form above with "enlarged" vector u containing all informations on it.

3.3 — Most properties of eqn (18) are now relying upon operator $\Phi = \Pi^{-1}K$. Now selfadjointness and positive definiteness are physically related to nonexistence of potential instability occuring in deformation system from direct calculation of inner product in H^* with previous definitions. Though M is manifestly symmetric, independently of κ characterizing stiffness (or compliance) of deformable segment bearing linkage, $[Fu,v]_{H*}$ is not generally coercive, unless $\kappa = \infty$ (infinitely stiff bearing), because of the non passivity of compliant system.

However $[Fu,v]_{H*}$ is extendable by addition of a term $k(u,v)$ to a coercive form, allowing analysis of existence and uniqueness of solution of eqn (18) in weak sense [21]. But further conditions are required for selfadjointness of M [13]. In this case, simple eigenfunction study can be developed for M in H from eqn (18) with u given by a sum of sin, cos, cosh and sinh functions, and using boundary conditions to eliminate the coefficients. Then as for classical case, eqn (18) is finally obtained in projected form, out of which control analysis can be developed and its coefficients be designed [28].

So in principle the control problem reduces to a classical PD-type form within the context of the defined function spaces. However, it is important to realize that there is a defect in the method, in that system output is given by bending moment M_a, which is directly measurable by strain gage cemented at link origin for instance, whereas here this quantity has to be projected on base functions, and necessarily only a finite number of them is kept. When eigenvalues decay fast enough with their rank, complete initial system may be expected to "almost" reduce to the retained finite dimensional subset. Discussion of this "almost" characterizes equations which are homeomorphic to finite dimensional systems.

For regular enough boundaries, nonlinearity in system equations can be shown to be controlled by a finite dimensional control constructed on this subset, by using extension of Popov's theorem [26, 20, 9]. For linear system, the control is obtainable by $\mathcal{H}_\infty$ optimization method [23], but it is easily understandable that the solution along this approach is always suffering from antagonistic requirements of both restricting reasonnably the base subset and forcing the system to exhibit a preassigned time dependent behavior.

4. GLOBAL CONTROL OF DEFORMABLE COMPLIANT ONE LINK SYSTEM

4.1 — Though appealing, the previous result is nevertheless impaired by the difficulty of constructing actual control law requiring higher order, generally unaccessible deformation modes. On the other hand, unless specifically required, there is no real need to determine deformation and error parameters as long as system behavior follows prescribed trajectory. More importantly, observable bending moment is not directly used, but has to be processed in classical PD control approach where all knowledge about system dynamics is going through mode representation, of course poorly suited for this situation.

So one should step back from this approach, and take more advantage of the very specific structure of complete (rigid + deformable) system, in order to follow a more global approach where the objective of trajectory behavior control is more directly reached without diverting, as above, into (side) problem of deformations determination and associated representation. It is already clear from eqn (8) that deformation effects summarize to simple mass motion located off link tip at end of vector (l_f, l_t), suggesting that few global informations are really necessary for controlling the system, i. e. that its motion can be restrained to smaller subspace.

To show that system motion can be constrained, consider simple linearized form of eqns (6, 7)

$$\begin{aligned} J_a \frac{\mathrm{d}^2\Delta\theta}{\mathrm{d}t^2} + F_a \frac{\mathrm{d}\Delta\theta}{\mathrm{d}t} + K(\Delta\theta - \Delta\theta_m) &= M_a + \delta_a(t) \\ J_m \frac{\mathrm{d}^2\Delta\theta_m}{\mathrm{d}t^2} + F_m \frac{\mathrm{d}\Delta\theta_m}{\mathrm{d}t} + r^2 K(\Delta\theta_m - \Delta\theta) &= U \end{aligned} \tag{19}$$

rejecting in the term $\delta_a(t)$ the effect of external perturbations and

of nonlinear (small) terms, with $\Delta\theta = \theta - \theta_d(t)$, $\Delta\theta_m = \theta_m - \theta_d(t)$, and define around desired trajectory $\theta(t) = \theta_d(t)$ the discontinuity surface $S = 0$ [35] by

$$\begin{aligned} S \quad = \quad & b_1 \frac{\mathrm{d}\Delta\theta}{\mathrm{d}t} + b_2 \frac{\mathrm{d}\Delta\theta_m}{\mathrm{d}t} + b_3\Delta\theta + b_4\Delta\theta_m + b_5 \int_0^t \Delta\theta \mathrm{d}t \\ & + b_6 \int_0^t \Delta\theta_m \, \mathrm{d}t + b_7 \int_0^t M_a \, \mathrm{d}t. \end{aligned} \tag{20}$$

To show that system trajectory goes to S = 0 and stays on it, let Lyapunov function

$V = S^2/2$, and take its derivative along system trajectory. One gets

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = S\left\{b^2 J_m^{-1} U + (b_1 J_a^{-1} + b_7) M_a + b_1 J_a^{-1}\delta_a(t)\right. \tag{21}$$

$$\left. + c_1\frac{\mathrm{d}\Delta\theta}{\mathrm{d}t} + c_2\frac{\mathrm{d}\Delta\theta_m}{\mathrm{d}t} + c_3\Delta\theta + c_4\Delta\theta_m\right\} \tag{22}$$

with

$$c_1 = b_5 + b_2 J_m^{-1} r^2 K - b_1 J_a^{-1} K, \quad c_2 = b_3 - b_1 J_a^{-1} B_a, \quad c_3 = b_4 - b_2 J_m^{-1} B_m$$
$$c_4 = b_6 - b_2 J_m^{-1} r^2 K + b_1 J_a^{-1} K.$$

Defining the control

$$U = k_1\frac{\mathrm{d}\Delta\theta}{\mathrm{d}t} + k_2\frac{\mathrm{d}\Delta\theta_m}{\mathrm{d}t} + k_3\Delta\theta + k_4\Delta\theta_m + k_m M_a + D\, sgn\, S \tag{23}$$

with $D > \|\delta_a(t)\|_{L_\infty} + \eta$, $\eta > 0$ small, coefficients k_j can be determined so that

$$V'(t) < b_2 J_m^{-1}\eta|S| < 0 \tag{24}$$

showing the property that system trajectory is forced to remain on $S = 0$ once reached with the only assumption of boundedness of perturbation term.

4.2 — Now with previous result expressing the layered structure of system motion organized along the "sheets" $S = 0$, asymptotic tracking stability for initial system will follow. Considering weak enough compliance case for simplicity, eqns $(6, 7)$ reduce to only one equation by addition with $\theta_m \simeq \theta = q$, and their shift, because K is finite very large but not strictly infinite, can be included in the bounded term $\delta_a(t)$.

Consider now Lyapunov function

$$W = K_S + \frac{1}{2}p_1\left(\frac{\mathrm{d}\Delta q}{\mathrm{d}t}\right)^2 + \frac{1}{2}p_2\Delta q^2 + \frac{p_3}{2}\left\{\int \rho\left(xh_1\frac{\mathrm{d}\Delta q}{\mathrm{d}t} + \frac{\partial u(t,x)}{\partial t}\right)^2\right.$$
$$\left. + \rho K^2\left(\frac{\partial\gamma(t,x)}{\partial t}\right)^2 + \omega_f^2\left(\frac{\partial^2 u(t,x)}{\partial x^2}\right)^2 + \omega_t^2\left(\frac{\partial\gamma(t,x)}{\partial x}\right)^2\right\}\mathrm{d}x \tag{25}$$

manifestly positive definite for positive p_j.

Taking into account boundary conditions from eqns $(3, 4, 5)$, its derivative along solution of eqn (18) can, after some algebraic manipulations, be cast in the form

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}\Delta q}{\mathrm{d}t}p_1 J_t^{-1}\left[U + M_a + \delta_a - B_t\frac{\mathrm{d}\Delta q}{\mathrm{d}t}\right] + p_2\Delta q + p_3 M_a \tag{26}$$

with $Q_a = 0$ and $M_a = \omega_f^2(\partial^2 u_1/\partial x^2(0))$. It is first possible to choose gain coefficients r_j in extended PD control form

$$U = r_1\Delta q + r_2\frac{\mathrm{d}\Delta q}{\mathrm{d}t} + r_3 M_a + r_4\delta_a \tag{27}$$

so that dW/dt is negative definite. But using control expression which makes $V' = 0$, i. e. $S' = 0$ in eqn (21), one even gets

$$\frac{\mathrm{d}W}{\mathrm{d}t} < -P\left(\frac{\mathrm{d}\Delta q}{\mathrm{d}t}\right)^2 \tag{28}$$

with P positive definite. The invariant set of $\mathrm{d}W_j/\mathrm{d}t$ is thus $\mathrm{d}\Delta q/\mathrm{d}t \simeq 0$, at closed loop system equilibrium all other higher order derivatives are also 0. On the other hand, from eqns $(1, 2)$ second order space derivatives of $\partial^2 y(x,t)/\partial x^2$ and $\gamma(x,t)$ are 0 at equilibrium, implying when solving for x-dependence and accounting for boundary conditions that $\partial^2 u(0,t)/\partial x^2 = 0$ as well, and, from actuator and control eqns $(6, 7)$, it follows in turn that $\Delta q = 0$ and $U = 0$. Hence the largest invariant set of $\mathrm{d}W_j/\mathrm{d}t$ is the null solution of the system. So with extended PD type control in the form of eqn (26), the system asymptotically tracks a prescribed trajectory with at the same time asymptotic decay of deformations because here $u(x,t)$ and $u'(x,t)$ are both going to 0 by LaSalle theorem [19]. The result is not modified by existence of external perturbations, and exact knowledge of system parameters is not even required because of the flexibility of possible choice of gain parameters.

So the present result is a robust one. On the other hand, because M_a is directly measurable by strain gages at link origin, control implementation, of global nature, is evidently much simpler than classical PD one requiring deformation modes analysis, projection onto these modes, and signals reconstitution through observer. This is confirmed by elementary analysis of one-link flexible system with steplike input, see Figure 3.

5. DISCUSSION AND CONCLUSION

Complicated general N-link mechanical systems, with appropriate generalized coordinates, can be cast in the form of conventional operational differential equation in adapted functional space, on which classical PD control analysis directly applies. But this approach relies upon state space representation requiring development of modes analysis for projection of observables on representation frame. As usual, this implies solving approximation problem, with consequences on power flow accuracy of resulting model. More fundamentally, system observables are forcedly represented by their projection and are painfully reconstituted afterward by theory, generally limited to linear case.

From observation that bending force acting onto each link is directly measurable, another approach is proposed here, which uses this information as a whole without any representation filtering. A motivation is in the remark that system dynamics are not completely free, but organized in layered structure with well defined representation. Then explicit conditions for asymptotic trajectory tracking on this layered surface are expressed in terms of only addition of bending force acting on the link on top of classical rigid variables PD terms, giving an extended hybrid PDF form.

Though algebraically involved to obtained, the meaning of actual result is elementarily understood by noting that for any system, the variation of its energy is equal to the work done by applied exterior forces. This is exactly observed here, where for a link, on top of actuators and exterior applied forces, deformations are acting only through bending moment at link origin. So only these intrinsic elements are really needed for system dynamics control, thus called a “global” control. On the other hand, the role of the bending moment part added to the classical PD feedback is, similar to usual force feedback, to physically change inertia-damping-stiffness parameters in the system for this exterior force, and so, to selectively change system response to these effects.

Fig. 3. Typical response of a flexible link with tip mass to step input.

It is thus important to properly design actuators response to deformation frequencies, with an advantage to micro-Macro structure [29] splitting apart in a natural way large amplitude-low frequency and small amplitude-high frequency responses required here. Finally, note that the present result is easily generalizable to nonlinear dynamical case as well, because it is never needed more than lagrangian structure of the equations and their associated “natural” boundary conditions [12]. Present results may even be improved by using additional asymptotic robust controller to drive faster error to zero.

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