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On a Class of Perimeter-Type Distances of Probability Distributions

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Abstract: The class $I_{f_p}, p \in (1,\infty]$, of f-divergences investigated in this paper generalizes an f-divergence introduced by the author in [9] and applied there and by Reschenhofer and Bomze [11] in different areas of hypotheses testing. The main result of the present paper ensures that, for every $p \in (1,\infty)$, the square root of the corresponding divergence defines a distance on the set of probability distributions. Thus it generalizes the respecting statement for $p = 2$ made in connection with Example 4 by Kafka, Österreicher and Vincze in [6].

From the former literature on the subject the maximal powers of f-divergences defining a distance are known for the subsequent classes. For the class of Hellinger-divergences given in terms of $f^{(s)}(u) = 1+u-(u^s+u^{1-s}), s \in (0,1)$, already Csiszár and Fischer [3] have shown that the maximal power is $\min(s, 1-s)$. For the following two classes the maximal power coincides with their parameter. The class given in terms of $f_{(\alpha)}(u) = |1-u^\alpha|^{\frac{1}{\alpha}}, \alpha \in (0,1]$, was investigated by Boekee [2]. The previous class and this one have the special case $s = \alpha = \frac{1}{2}$ in common. This famous case is attributed to Matusita [8]. The class given by $\varphi_\alpha(u) = |1-u|^{\frac{1}{\alpha}}(1+u)^{1-\frac{1}{\alpha}}, \alpha \in (0,1]$, and investigated in [6], Example 3, contains the wellknown special case $\alpha = \frac{1}{2}$ introduced by Vincze [13].

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