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Monogenicity of Probability Measures Based on Measurable Sets Invariant under Finite Groups of Transformations

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Abstract: Let A denote a σ-algebra of subsets of a set Ω, G a finite group of (A, A)-measurable transformations $g : \Omega \to \Omega$, $F(G)$ the set consisting of all $\omega \in \Omega$ such that $g(\omega) = \omega$, $g \in G$, is fulfilled, and let $B(G, A)$ stand for the σ-algebra consisting of all sets $A \in A$ satisfying $g(A) = A$, $g \in G$. Under the assumption $f(B) \in A^{|G|}$, $B \in B(G, A)$, for $f : \Omega \to \Omega^{|G|}$ defined by $f(\omega) = (g_1(\omega), \dots, g_{|G|}(\omega))$, $\omega \in \Omega$, $\{g_1, \dots, g_{|G|}\} = G$, where $|G|$ stands for the number of elements of G, $\Omega^{|G|}$ for the $|G|$-fold Cartesian product of Ω, and $A^{|G|}$ for the $|G|$-fold direct product of A, it is shown that a probability measure P on A is uniquely determined among all probability measures on A by its restriction to $B(G, A)$ if and only if $P^*(F(G)) = 1$ holds true and that $F(G) \in A$ is equivalent to the property of A to separate all points $\omega_1, \omega_2 \in F(G)$, $\omega_1 \neq \omega_2$, and $\omega \in F(G)$, $\omega' \notin F(G)$, by a countable system of sets contained in A. The assumption $f(B) \in A^{|G|}$, $B \in B(G, A)$, is satisfied, if Ω is a Polish space and A the corresponding Borel σ-algebra.

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