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Criteria for Optimal Design of Small-Sample Experiments with Correlated Observations

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Abstract: We consider observations of a random process (or a random field), which is modeled by a nonlinear regression with a parametrized mean (or trend) and a parametrized covariance function. Optimality criteria for parameter estimation are to be based here on the mean square errors (MSE) of estimators. We mention briefly expressions obtained for very small samples via probability densities of estimators. Then we show that an approximation of MSE via Fisher information matrix is possible, even for small or moderate samples, when the errors of observations are normal and small. Finally, we summarize some properties of optimality criteria known for the noncorrelated case, which can be transferred to the correlated case, in particular a recently published concept of universal optimality.

Keywords: optimal design; correlated observations; random field; spatial statistics; information matrix;

AMS Subject Classification: 62K05; 62M10;

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