# On The 3D-Pollution Dispersion Over 2D-Hill Including Turbulence Modelling

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#### Abstract

The paper deals with a concentration-validation study performed on the mathematical/numerical model of atmospheric boundary layer flow. The mathematical model is based on the system of RANS equations closed by the two-equation  $k - \varepsilon$  turbulence model together with wall functions. The finite volume method and the explicit Runge–Kutta time integration method are utilized for the numerics. The test–case is related to a neutral boundary layer 2D-flow over an isolated hill with a rough wall. 2D- and 3D- concentration results have been compared.

## 1 Mathematical formulation

The flow itself is assumed to be a turbulent, viscous, incompressible, stationary and indifferently stratified as well. The mathematical model is based on the RANS approach and the governing equations modified according to the method of artificial compressibility can be re-casted in the conservative and vector form

$$\vec{W_t} + \begin{pmatrix} u \\ u^2 + \frac{p}{\varrho} \\ uv \\ uw \\ uC \end{pmatrix}_x + \begin{pmatrix} v \\ vu \\ v^2 + \frac{p}{\varrho} \\ vw \\ vC \end{pmatrix}_y + \begin{pmatrix} w \\ wu \\ wv \\ w^2 + \frac{p}{\varrho} \\ wC \end{pmatrix}_z = \begin{pmatrix} 0 \\ Ku_x \\ Kv_x \\ Kw_x \\ \tilde{K}C_x \end{pmatrix}_x + \begin{pmatrix} 0 \\ Ku_y \\ Kv_y \\ Kw_y \\ \tilde{K}C_y \end{pmatrix}_y + \begin{pmatrix} 0 \\ Ku_z \\ Kv_z \\ Kw_z \\ \tilde{K}C_z \end{pmatrix}_z$$
(1)

where  $\vec{W} = (p/\beta^2, u, v, w, C)^T$  stands for the vector of unknown variables: the pressure p, the velocity vector  $\vec{V} = (u, v, w)^T$ , the passive pollutant concentration C (measured in  $[kg/m^3]$ ) and the parameters K,  $\tilde{K}$  refer to the turbulent diffusion coefficients and  $\beta$  is related to the artificial sound speed. The governing system (1) is closed by a conventional two-equation  $k - \varepsilon$  turbulence model as briefly described in Sládek *et.al.* (2008) [3].

#### **2** Boundary conditions

The system (1) and  $(k - \varepsilon)$  turbulence model is solved with the following boundary conditions, Castro (1981) [2]

Inlet:  $u = \frac{u^*}{\kappa} \ln\left(\frac{z}{z_0}\right)$ , v = 0, w = 0,  $k = \frac{u^{*2}}{\sqrt{C_{\mu}}} \left(1 - \frac{z}{D}\right)^2$ ,  $\varepsilon = \frac{C_{\mu}^{3/4} \cdot k^{3/2}}{\kappa \cdot z}$ ; Outlet: homogeneous Neumann conditions for all quantities; Top:  $u = U_0$ , v = 0,  $\frac{\partial w}{\partial z} = 0$ ,  $\frac{\partial k}{\partial z} = 0$ ; Sides: symmetric conditions; Wall: standard wall functions are applied at  $\sim 50$  wall-units from wall; where  $u^*$  is the friction velocity,  $\kappa = 0.40$  denotes the von Karman constant,  $z_0$  represents the roughness parameter.

#### **3** Validation case

• The reference experimental data due to Khurshudyan (1981) [1] is also available in the ERCOFTAC database. Moreover, Castro (1996) [2] performed flow and pollution dispersion reference numerical study.

• The 2D-computational domain symmetric about hill summit is  $9 imes 1.6 \, m$  long x high and

non-uniformly discretized by  $400 \times 80$  cells. Two 3D-computational domains were tested starting at hill summit: (domain-1)  $4.5 \times 0.11 \times 1.6 m \log \times \text{wide} \times \text{high}$  and discretized by  $200 \times 11 \times 80$  cells and (domain-2)  $4.5 \times 0.44 \times 1.6 m \log \times \text{wide} \times \text{high}$  and discretized by  $200 \times 40 \times 80$  cells. The 2D flow-field was uniformly redistributed in the lateral *y*-direction for all 3D-computations. The free-stream air velocity  $U_0 = 4 m/s$  and boundary layer depth of D = 1 m. The Reynolds number based on  $U_0$  and hill height H = 117 mm is  $Re \sim 3.1 \cdot 10^4$ , Sládek *et.al.* (2008) [3].

• The concentration input data: two different heights of a source in 2D/3D-runs have been assumed at the downwind side of the hill summit:  $x_s = 3H$  and  $z_s = 0.25H$ , 0.5H, both in the recirculation zone. A normalization of the concentration field is performed as  $C \cdot U_0 H^2/Q$ 



Fig 1: Ground-level-concentrations with source at heights 0.25H and 0.5H showing the effect of terrain amplification factor and pollutant plume decay in 2D-case (upper two profiles) versus 3D-case on domain-1 (lower two profiles).



Fig 2: Castro's reference data (solid line) based on 3D-calculation from [2] pp.847: Profile of ground-level-concentrations profiles with source at heights 0.25H and 0.5H, normalized as  $C \cdot U_0 H^2/Q$ .

## 4 Conclusion

Both 2D- and 3D-concentration simulations well captured the terrain amplification effect leading to a near ground upstream increase of the concentration level compared to source location  $x_s$ . It has been confirmed much slower 2D-pollutant plume downstream decay compared to 3D-case, see figure 1. A quantitative agreement can be found between our and the reference Castro's near-ground concentration predictions.

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## References

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- [2] Castro I.P., Apsley P.P. (1996): Flow and dispersion over topography: A comparison between numerical and laboratory data for two-dimensional flows, Atmospheric Environment, Vol.31, No.6.
- [3] Sládek I., Kozel K., Jaňour Z. (2008): On the 2D-validation study of turbulence  $k \varepsilon$ model including pollution dispersion, In: "Topical Problems Of Fluid Mechanics 2008", Institute of Thermo-mechanics, ISBN 978-80-87012-09-3, pp. 101–104, Prague.

# 5 Some numerical results



Fig 3: 2D-Computational domain  $9.36 m \times 1.6 m$  for 2D-pressure-velocity field, discretized be 400x80 cells.



Fig 5: Zoom to separation zone behind hill, computed reattachment  $x_r = 6.84H$ , measured from experiment  $x_r = 6.5H$ .



Fig 7: Rotated 3D-Computational domain-1  $4.68 \ m \times 0.11 \ m \times 1.6 \ m$  for 3D-concentration field.



Fig 4: Zoom to non-uniform grid near hill.



Fig 6: 2D-Computational domain  $4.68 \, m \times 1.6 \, m$  for 2D-concentration field.



Fig 8: Zoom to 3D-computational domain-1 for 3D-concentration field discretized by 200x11x80 cells.



Fig 9: Computed contours of normalized 2D-concentration field  $C \cdot U_0 H^2/Q$  with exponential scale for source height 0.25H.



Fig 10: Computed contours of normalized 3D-concentration field  $C \cdot U_0 H^2/Q$  with exponential scale for source height 0.25H, middle XZ-plane at index J=6.

A much slower concentration plume decay is clearly visible when comparing both figures 9 and 10, especially the scales. The contours have practically the same shape, however both concentration fields differ from quantitative point of view.



Fig 11: Reference data taken from [2] pp.847: Computed contours of normalized concentration field  $C \cdot U_0 H^2/Q$  with exponential scale starting at value=1 (upper contour) for source height 0.25H.

There is a quantitative matching between our computed 3D-concentration field and the reference Castro's data.