

## Many-dimensional Observables on Łukasiewicz Tribe: Constructions, Conditioning and Conditional Independence

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*Abstract:* Probability on collections of fuzzy sets can be developed as a generalization of the classical probability on  $\sigma$ -algebras of sets. A Łukasiewicz tribe is a collection of fuzzy sets which is closed under the standard fuzzy complementation and under the pointwise application of the Łukasiewicz t-norm to countably many fuzzy sets. An observable is a fuzzy set-valued mapping defined on a  $\sigma$ -algebra of sets and satisfying some additional properties; formally, the role of an observable is in a sense analogous to that of a random variable in classical probability theory. This article aims at studying and surveying some properties of observables on a Łukasiewicz tribe of fuzzy sets with a special focus on many-dimensional observables. Namely, the definition and basic construction techniques of observables are discussed. A method for a reasonable construction and interpretation of a joint observable is proposed. Further, the contribution contains results concerning conditioning of observables. We continue in our study [?] of conditional independence in this framework and conclude that semi-graphoid properties are preserved.

*Keywords:* state; observable; tribe of fuzzy sets; conditional independence;

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