Program for Generating Fuzzy Logical Operations and Its Use in Mathematical Proofs.

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Abstract: Fuzzy logic is one of the tools for management of uncertainty; it works with more than two values, usually with a continuous scale, the real interval $[0, 1]$. Implementation restrictions in applications force us to use in fact a finite scale (finite chain) of truth degrees. In this paper, we study logical operations on finite chains, in particular conjunctions. We describe a computer program generating all finitely-valued fuzzy conjunctions (t-norms). It allows also to select these t-norms according to various criteria. Using this program, we formulated several conjectures which we verified by theoretical proofs, thus obtaining new mathematical theorems. We found out several properties of t-norms that are quite surprising. As a consequence, we give arguments why there is no "satisfactory" finitely-valued conjunction. Such an operation is desirable, e. g., for search in large databases. We present an example demonstrating both the motivation and the difficulties encountered in using many-valued conjunctions. As a by-product, we found some consequences showing that the characterization of diagonals of finitely-valued conjunctions differs substantially from that obtained for t-norms on $[0, 1]$.