A Spectral Theorem for σ MV-Algebras

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Abstract: MV-algebras were introduced by Chang, 1958 as algebraic bases for multi-valued logic. MV stands for "multi-valued" and MV algebras have already occupied an important place in the realm of nonstandard (mathematical) logic applied in several fields including cybernetics. In the present paper, using the Loomis–Sikorski theorem for σ-MV-algebras, we prove that, with every element a in a σ-MV algebra M, a spectral measure (i.e. an observable) $\Lambda_a : \mathcal{B}([0,1]) \to \mathcal{B}(M)$ can be associated, where $\mathcal{B}(M)$ denotes the Boolean σ-algebra of idempotent elements in M. This result is similar to the spectral theorem for self-adjoint operators on a Hilbert space. We also prove that MV-algebra operations are reflected by the functional calculus of observables.