

On Entropies for Random Partitions of the Unit Segment

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Abstract: We prove the complete convergence of Shannon's, paired, genetic and α -entropy for random partitions of the unit segment. We also derive exact expressions for expectations and variances of the above entropies using special functions.

Keywords: genetic entropy; α -entropy; random partitions; complete convergence;

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References

- [1] L. E. Baum and M. Katz: Convergence rates in the law of large numbers. *Trans. Amer. Math. Soc.* 120 (1968), 108–123.
- [2] M. Bieniek and D. Szynal: A contribution to results on random partitions of the segment. *Internat. J. Pure and Appl. Math.* 13 (2004), 3, 337–378.
- [3] N. Burbea and N. Rao: Entropy differential metrics and divergence measures in probability spaces: a unified approach. *J. Multivariate Anal.* 12 (1982), 575–596.
- [4] D. A. Darling: On a class of problems related to the random division of an interval. *Ann. Math. Statist.* 24 (1953), 239–253.
- [5] P. Erdős: On a theorem of Hsu and Robbins. *Ann. Math. Statist.* 20 (1949), 286–291.
- [6] W. Feller: *An Introduction to Probability Theory and its Applications*. Vol. II. Wiley, New York 1966.
- [7] S. Goldstein: On entropy of random partitions of the segment $[0, 1]$. *Bull. Soc. Sci. Lett. Łódź XXIV* 4 (1974), 1–7.
- [8] I. S. Gradstein and I. M. Ryzhik: *Tables of Integrals, Sums, Series and Products*. Fourth edition. Academic Press, New York–London 1965.
- [9] R. L. Graham, D. E. Knuth, and O. Patashnik: *Concrete Mathematics*. Addison–Wesley Publishing Company Advanced Book Program, Reading, MA 1989.

- [10] M. Ekstörn: Sum-functions of spacings of increasing order. *J. Statist. Plann. Inference* 136 (2006), 2535–2546.
- [11] P. Hall: Limit theorems for sums of general functions of m -spacings. *Math. Proc. Cambridge Philos. Soc.* 96 (1984), 517–532.
- [12] P. Hall: On power distributional tests based on sample spacings. *J. Multivariate Anal.* 19 (1986), 201–224.
- [13] E. R. Hansen: *A Table of Series and Products*. Prentice-Hall, Englewood Cliffs, N. J. 1975.
- [14] J. Havrda and F. Charvát: Quantification method in classification process: Concept of structural α -entropy. *Kybernetika* 3 (1967), 30–35.
- [15] C. C. Heyde: A supplement to the strong law of large numbers. *J. Appl. Probab.* 12 (1975), 173–175.
- [16] P. L. Hsu, and H. Robbins: Complete convergence and the law of large numbers. *Proc. Nat. Acad. Sci. U. S. A.* 33 (1947), 25–31.
- [17] B. D. H. Latter: Measures of genetic distance between individuals and populations. *Publ. Univ. Hawai, Honolulu, Genetic Structure of Populations* (1973), 27–39.
- [18] M. L. Menendez, D. Morales, L. Pardo, and M. Salicrú: Asymptotic distribution of $\{h, \phi\}$ -entropies. *Comm. Statist. – Theory Methods* 22 (1993), 7, 2015–2031.
- [19] N. Misra: A new test of uniformity based on overlapping sample spacings. *Comm. Statist. – Theory Methods* 30 (2001), 7, 1435–1470.
- [20] A. Renyi: New nonadditive measures of entropy for discrete probability distributions. In: *Proc. 4th Berkeley Symp. Math. Statist. and Prob.* Vol. 1, 1961, pp. 547–561.
- [21] C. E. Shannon: A mathematical theory of communications. *Bell System Tech. J.* 27 (1948), 379–425, 623–656.
- [22] Y. Shao and R. Jimenez: Entropy for random partitions and its applications. *J. Theoret. Probab.* 11 (1998), 417–433.
- [23] E. Slud: Entropy and maximal spacings for random partitions. *Z. Warsch. verw. Gebiete* 41 (1978), 341–352.
- [24] N. M. Temme: *Special Functions: An Introduction to the Classical Functions of Mathematical Physics*. Wiley, New York 1996.
- [25] H. M. Srivastava, S.-T. Tu, and T.-C. Wu: Some combinatorial series identities associated with the Digamma function and harmonic numbers. *Appl. Math. Lett.* 13 (2000), 101–106.