KYBERNETIKA — VOLUME 44 (2008), NUMBER 1, PAGES 61-74

Nonlinear State Prediction by Separation Approach for Continuous-Discrete Stochastic Systems

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Abstract: The paper deals with a filter design for nonlinear continuous stochastic systems with discrete-time measurements. The general recursive solution is given by the Fokker–Planck equation (FPE) and by the Bayesian rule. The stress is laid on the computation of the predictive conditional probability density function from the FPE. The solution of the FPE and its integration into the estimation algorithm is the cornerstone for the whole recursive computation. A new usable numerical scheme for the FPE is designed. In the scheme, the separation technique based on the upwind volume method and the finite difference method for hyperbolic and parabolic part of the FPE is used. It is supposed that separation of the FPE and choice of a suitable numerical method for each part can achieve better estimation quality comparing to application of a single numerical method to the unseparated FPE. The approach is illustrated in some numerical examples.

Keywords: stochastic systems; state estimation; nonlinear filters; Fokker–Planck equation; numerical solutions; finite volume method; finite difference method;

AMS Subject Classification: 93E11;

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