ROBUST AND RELIABLE H_∞ OUTPUT FEEDBACK CONTROL FOR LINEAR SYSTEMS WITH PARAMETER UNCERTAINTY AND ACTUATOR FAILURE[1]

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The robust and reliable H_∞ output feedback controller design problem is investigated for uncertain linear systems with actuator failures within a prespecified subset of actuators. The uncertainty considered here is time-varying norm-bounded parameter uncertainty in the state matrix. The output of a faulty actuator is assumed to be any arbitrary energy-bounded signal. An observer-based output feedback controller design is presented which stabilizes the plant and guarantees an H_∞-norm bound on attenuation of augmented disturbances, for all admissible uncertainties as well as actuator failures. The construction of the observer-based output feedback control law requires the positive-definite solutions of two algebraic Riccati equations. The result can be regarded as an extension of existing results on robust H_∞ control and reliable H_∞ control of uncertain linear systems.

1. INTRODUCTION

The relationship between H_∞ optimization and robust stabilization of uncertain linear systems has been established in [1]. Since then, interests have focused on the problem of robust H_∞ control for linear systems with parameter uncertainties (see [2, 4] and [5] for example). The objective is to design a controller which stabilizes an uncertain system while satisfying an H_∞-norm bound constraint on disturbance attenuation for all admissible uncertainties. However, these control designs may result in unsatisfactory performances or even unexpected instabilities in the event of control component failures, e.g., actuator failures, sensor failures, etc. In practice, failures of control components are often found in the real world. Hence, it should be taken into account when a practical control system is designed. Recently, a methodology for the design of reliable control systems using observer-based output feedback was introduced in [3]. The resultant control system provides guaranteed stability and satisfies an H_∞-norm disturbance attenuation bound in normal condition as well as in the event of actuator or sensor failures in the system. However, in [3], the

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system uncertainty is not considered when the control system is designed. Hence the desired closed-loop behaviors may not be guaranteed if the system uncertainty exists in the system under consideration.

In this paper, interest is focused on systems with practical control environments where both system uncertainties and control component failures may exist. Especially, attention is concentrated on uncertain linear systems with time-varying norm-bounded parameter uncertainties in the state matrix and actuator failures among various control components. The output of a faulty actuator is assumed to be any arbitrary energy bounded signal. It is a generalization for the actuator failure mode in [3], where the output of a faulty actuator is assumed to be zero. Robust and reliable H_∞ control methodology is developed using observer-based output feedback under the assumption that all information of the plant state is not available for feedback. The approach adopted here relies on the notion of quadratic stabilization with an H_∞-norm bound which was introduced in [5]. An observer-based output feedback control law is constructed by solving two parameter-dependent algebraic Riccati equations. This control methodology guarantees satisfactory closed-loop behavior despite the appearance of parameter uncertainties and actuator failures, which is an extension of existing results on robust H_∞ control [2, 4] and reliable H_∞ control [3].

2. SYSTEMS AND DEFINITION

Consider a class of uncertain linear systems described by state-space models of the form

$$
\begin{align}
\dot{x}(t) &= [A + \Delta A(t)]\, x(t) + Bu(t) + Gw_1(t) \tag{1a}\\
y(t) &= Cx(t) + w_2(t) \tag{1b}\\
z(t) &= \left[\begin{array}{c} z_1(t) \\ z_2(t) \end{array}\right] = \left[\begin{array}{c} Hx(t) \\ u(t) \end{array}\right] \tag{1c}
\end{align}
$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $y(t) \in R^p$ is the measured output, $w_1(t) \in R^r$ and $w_2(t) \in R^s$ are the disturbance inputs which belong to $L_2[0, \infty)$, and $z(t) \in R^q$ is the controlled output. A, B, G, C and H are known real constant matrices of appropriate dimensions describing the nominal system. $\Delta A(\cdot)$ is a real-valued matrix function representing time-varying parameter uncertainty, which is of the form

$$
\Delta A(t) = DF(t)\, E \tag{2}
$$

where $D \in R^{n \times i}$ and $E \in R^{j \times n}$ are known real constant matrices and $F(t) \in R^{i \times j}$ is an unknown matrix function satisfying $F^T(t)\, F(t) \le I$ with the elements of $F(\cdot)$ being Lebesgue measurable. Note that this kind of uncertainty structure has been analyzed in [1] and [6], and also used in numerous papers (see [4] and [5] for example).

The following concept of quadratic stabilization with an H_∞-norm bound will be essentially used in deriving robust and reliable output feedback H_∞ controller for the uncertain system (1), which was introduced in [5].

Definition 1. Let the constant $\gamma > 0$ be given. The uncertain system (1) is said to be *quadratically stabilizable with an H$_\infty$-norm bound γ (via linear output-feedback)* if there exist a fixed linear time-invariant proper output-feedback law $u = K(s)\,y$, where s is a complex variable, and a real symmetric positive definite matrix $Q \in R^{n \times n}$ such that the inequality

$$A_c^T(t)Q + QA_c(t) + \frac{1}{\gamma^2}QB_cB_c^TQ + C_c^TC_c < 0 \tag{3}$$

holds for any admissible uncertainty $F(\cdot)$, where $(A_c(t), B_c, C_c)$ is a state-space realization of the closed-loop system.

Note that Definition 1 implies the following facts. The proof is similar to that of Lemma 2.1 in [5] and thus is omitted.

Lemma 1. Suppose the uncertain system (1) is quadratically stabilized with an H_∞-norm bound $\gamma > 0$ by linear output feedback. Then, the closed-loop system is uniformly asymptotically stable. Moreover, with the zero initial condition, $\|z\|_2 < \gamma\|w\|_2$ for all admissible uncertainty $F(\cdot)$ and all nonzero $w \in L_2[0,\infty)$, where $w = [w_1^T \;\; w_2^T]^T$ and $\| \cdot \|_2$ denotes the usual $L_2[0,\infty)$-norm.

We conclude this section by introducing a decomposition of a matrix that will be used in the control design. Let M be an $n \times m$ matrix and S be a subset of the set U constructed to column numbers of M, that is,

$$S \subseteq \{1, 2, \ldots, m\} \equiv U. \tag{4}$$

Let $\overline{S}$ denote the complement set of S, that is, $\overline{S} = U - S$. We define *the decomposition of M for S* as follows:

$$M = M_S + M_{\overline{S}} \tag{5}$$

where M_S and $M_{\overline{S}}$ are $n \times m$ matrices formed from M by replacing only columns of M corresponding to $\overline{S}$ and S with null vectors, respectively. M_S(respectively, $M_{\overline{S}}$) will be called a 'decomposition matrix of M for S(respectively, $\overline{S}$)'. This decomposition has the following properties.

$$M_SM_{\overline{S}}^T = M_{\overline{S}}M_S^T = 0. \tag{6}$$

Let s be a subset of S. Then

$$M_SM_S^T = M_sM_s^T + M_{S-s}M_{S-s}^T \tag{7}$$

and

$$M_sM_s^T \leq M_SM_S^T. \tag{8}$$

Note that the notation $M \geq N$(respectively, $M > N$) where M and N are symmetric matrices, refers to the fact that the matrix $M - N$ is positive semidefinite(respectively, positive definite).

3. PROBLEM FORMULATION

We classify actuators of a given system into two groups. One is a set of actuators susceptible to failures, which is denoted by $\Omega \subseteq \{1, 2, \ldots, m\}$. These actuators may fail occasionally. This set of actuators is redundant in view of the stabilization of the system while it may contribute and is necessary to improving a control system performance. The other is a set of actuators robust to failures, which is denoted by $\overline{\Omega} = \{1, 2, \ldots, m\} - \Omega$. We assume that these actuators never fail, and also that $\overline{\Omega}$ contains the minimum set of actuators required to stabilize a given system.

The actuators play a role to transmit the controller outputs to the plant. Without loss of generality, the transfer function of an actuator is assumed to be 1. Generally, the outputs of faulty actuators may have arbitrary signals different from normal controller outputs, and these signals will act on the system as unexpected control inputs. It is desirable that both the effects of failure are reduced to be negligible by control feedback, and the stability of closed-loop system is maintained. In this paper, the output of a faulty actuator is assumed to be any arbitrary energy bounded signal, that is, the output of a faulty actuator belongs to $L_2[0, \infty)$. The outputs of faulty actuators are regarded as disturbance inputs. Attempts are made to suppress the signals on the system outputs caused by faulty actuators as well as disturbance inputs, below a given level.

Problem 1.   (robust and reliable H_∞ output-feedback control problem) Assume that not all states are available for feedback. Let $(A, B_{\overline{\Omega}})$ be a controllable pair, where $B_{\overline{\Omega}}$ is the decomposition matrix of B for $\overline{\Omega}$, and also let (A, C) be an observable pair. When a constant $\gamma > 0$ is given, design a fixed linear output-feedback controller to stabilize the system (1) and guarantee the given H_∞-norm constraint γ on attenuation of augmented disturbances including failure signals, for actuator failures within an actuator set corresponding to Ω as well as all admissible uncertainties satisfying $F^T(t)\, F(t) \leq I$.

4. ROBUST AND RELIABLE H_∞ CONTROLS

Based on Definition 1, we will solve the Problem 1 for the design of a robust and reliable H_∞ controller for the uncertain linear system (1) that is robust for parameter uncertainties and exogenous disturbances, and is reliable despite possible actuator failures. Let $\omega \subseteq \Omega$ correspond to a particular subset of susceptible actuators that actually experience failures. When the actuators corresponding to ω actually experience failures, the control input is represented as

$$u(t) = u_{\overline{\omega}}^N(t) + u_\omega^F(t). \tag{9}$$

where $u_{\overline{\omega}}^N(t)$ is the normal control input vector only concerned by normal actuators, whose elements corresponding to $\overline{\omega}$, which is the set of $\{1, 2, \ldots, m\} - \omega$, have normal actuator output and the other elements are zero, and $u_\omega^F(t)$ is the abnormal control input vector only concerned by faulty actuators, whose elements corresponding to ω have faulty actuator output and the other elements are zero, where the superscripts

N and F mean 'normality' and 'failure', respectively. The controlled output is described by

$$z(t) = \left[\begin{array}{c} Hx(t) \\ u(t) \end{array} \right] = \left[\begin{array}{c} Hx(t) \\ u_{\overline{\omega}}^N(t) \end{array} \right] + \left[\begin{array}{c} 0 \\ u_{\omega}^F(t) \end{array} \right] = z_{\overline{\omega}}^N(t) + z_{\omega}^F(t) \qquad (10)$$

where $z_{\overline{\omega}}^N(t) \equiv \left[\begin{array}{c} Hx(t) \\ u_{\overline{\omega}}^N(t) \end{array} \right]$ and $z_{\omega}^F(t) \equiv \left[\begin{array}{c} 0 \\ u_{\omega}^F(t) \end{array} \right]$. Since $z_{\omega}^F(t)$ is out of closed-loop system, only $z_{\overline{\omega}}^N(t)$ can be considered in the closed-loop system. Hence, the signals on the system output which should be suppressed are $z_{\overline{\omega}}^N(t)$.

The output-feedback control law for the uncertain linear system (1) is based on a state observer of the form:

$$\dot{\zeta} = A\zeta + B_{\overline{\Omega}}u + B_{\Omega}\hat{u} + D\hat{f} + G\hat{w}_1 + L(y - C\zeta) \qquad (11)$$

where

$$\hat{u} = K_a\zeta, \qquad (12)$$
$$\hat{f} = K_u\zeta \qquad (13)$$

and

$$\hat{w}_1 = K_d\zeta. \qquad (14)$$

L is the observer gain, K_a is the actuator output estimation gain, K_u is the uncertainty estimation gain, and K_d is the disturbance estimation gain. $\hat{u}$, $\hat{f}$ and $\hat{w}_1$ account for the actuator output u, the uncertainty $F(t)\,Ex$, and the disturbance input w_1, respectively. Then the control law becomes

$$\dot{\zeta} = (A + B_{\Omega}K_a + DK_u + GK_d - LC)\zeta + B_{\overline{\Omega}}u + Ly \qquad (15a)$$
$$u = K\zeta \qquad (15b)$$

where K is the control feedback gain. Let actuators corresponding to any set $\omega \subseteq \Omega$ be failed. The control input, that is, the actuator output becomes

$$u = (K^T)_{\overline{\omega}}^T\zeta + u_{\omega}^F \qquad (16)$$

where $(K^T)_{\overline{\omega}}$ is the decomposition matrix of K^T for $\overline{\omega}$. Applying the controller (15) with (16) to the system (1) gives a closed-loop system of order $2n$ described by

$$\dot{x}_e = F_e x_e + G_e w_{\omega}^F, \qquad z_{\overline{\omega}}^N = H_e x_e \qquad (17)$$

where $x_e = [x^T\ \zeta^T]^T$, $w_{\omega}^F = [w_1^T\ w_2^T\ (u_{\omega}^F)^T]^T$, and

$$F_e = \left[\begin{array}{cc} A + DF(t)\,E & B_{\overline{\omega}}(K^T)_{\overline{\omega}}^T \\ LC & A + B_{\overline{\Omega}}(K^T)_{\overline{\Omega}}^T + B_{\Omega}K_a + DK_u + GK_d - LC \end{array} \right],$$

$$G_e = \left[\begin{array}{ccc} G & 0 & B_{\omega} \\ 0 & L & 0 \end{array} \right], \qquad H_e = \left[\begin{array}{cc} H & 0 \\ 0 & (K^T)_{\overline{\omega}}^T \end{array} \right] \qquad (18)$$

where $(K^T)_{\overline{\Omega}}$ is the decomposition matrix of K^T for $\overline{\Omega}$. Transforming coordinates of (17) such that the last n state variables are the observer error $e = \zeta - x$, gives

$$\dot{\tilde{x}}_e = \tilde{F}_e \tilde{x}_e + \tilde{G}_e w_\omega^F, \quad z_{\overline{\omega}}^N = \tilde{H}_e \tilde{x}_e \tag{19}$$

where

$$\tilde{F}_e = \begin{bmatrix} A + DF(t)\,E + B_{\overline{\omega}}(K^T)_{\overline{\omega}}^T & B_{\overline{\omega}}(K^T)_{\overline{\omega}}^T \\ \{\ B_\Omega K_a + DK_u + GK_d & \{\ A + B_\Omega K_a + DK_u + GK_d \\ -DF(t)\,E - B_{\Omega-\omega}(K^T)_{\Omega-\omega}^T\ \} & -B_{\Omega-\omega}(K^T)_{\Omega-\omega}^T - LC\ \} \end{bmatrix},$$

$$\tilde{G}_e = \begin{bmatrix} G & 0 & B_\omega \\ -G & L & -B_\omega \end{bmatrix}, \quad \tilde{H}_e = \begin{bmatrix} H & 0 \\ (K^T)_{\overline{\omega}}^T & (K^T)_{\overline{\omega}}^T \end{bmatrix}. \tag{20}$$

where $(K^T)_{\Omega-\omega}$ is the decomposition matrix of K^T for $\Omega - \omega$. Now the problem is reduced to selecting K, L, K_a, K_u and K_d in (15) such that the augmented system (19) is quadratically stable with an H_∞-norm bound γ.

Theorem 1. Let a scalar $\gamma > 0$ be given. Suppose

$$K = -B^T X, \quad K_a = \frac{1}{\gamma^2} B^T X, \quad K_u = \frac{1}{\gamma^2} D^T X, \quad K_d = \frac{1}{\gamma^2} G^T X \tag{21}$$

where $X > 0$ satisfies

$$A^T X + XA - X B_{\overline{\Omega}} B_{\overline{\Omega}}^T X + \frac{1}{\gamma^2} X(GG^T + DD^T + B_\Omega B_\Omega^T)\,X$$
$$+ \gamma^2 E^T E + H^T H + \delta_1 I = 0 \tag{22}$$

for a positive scalar δ_1. Suppose also

$$L = \gamma^2 (W - X)^{-1} C^T \tag{23}$$

where $W > X$ satisfies

$$A^T W + WA - \gamma^2 C^T C + \frac{1}{\gamma^2} W(GG^T + DD^T + B_\Omega B_\Omega^T)W$$
$$+ W B_\Omega B_\Omega^T W + \gamma^2 E^T E + H^T H + \delta_2 I = 0 \tag{24}$$

for a positive scalar $\delta_2 > \delta_1$. Then for actuator failures corresponding to any $\omega \subseteq \Omega$, the observer-based controller (15) quadratically stabilizes the system (1) with an H_∞-norm bound γ in the sense of $\|z_{\overline{\omega}}^N\|_2 < \gamma \|w_\omega^F\|_2$.

P r o o f. Consider actuator failures corresponding to $\omega \subseteq \Omega$. With all assumptions in Theorem 1, if we can find a $2n \times 2n$ matrix $X_e > 0$ such that

$$\tilde{F}_e^T X_e + X_e \tilde{F}_e + \frac{1}{\gamma^2} X_e \tilde{G}_e \tilde{G}_e^T X_e + \tilde{H}_e^T \tilde{H}_e < 0, \tag{25}$$

the proof will be completed due to Definition 1.

Substituting (21) into (20), we obtain

$$
\tilde{F}_e = \begin{bmatrix} A + DF(t)\,E - B_{\overline{\omega}}B_{\overline{\omega}}^T X & -B_{\overline{\omega}}B_{\overline{\omega}}^T X \\ \left\{ \begin{array}{c} \frac{1}{\gamma^2}(GG^T + DD^T + B_\Omega B_\Omega^T)\,X \\ -DF(t)\,E + B_{\Omega-\omega}B_{\Omega-\omega}^T X \end{array} \right\} & \left\{ \begin{array}{c} A + \frac{1}{\gamma^2}(GG^T + DD^T + B_\Omega B_\Omega^T)\,X \\ +B_{\Omega-\omega}B_{\Omega-\omega}^T X - LC \end{array} \right\} \end{bmatrix},
$$

$$
\tilde{G}_e = \begin{bmatrix} G & 0 & B_\omega \\ -G & L & -B_\omega \end{bmatrix}, \qquad \tilde{H}_e = \begin{bmatrix} H & 0 \\ -B_{\overline{\omega}}^T X & -B_{\overline{\omega}}^T X \end{bmatrix}. \tag{26}
$$

Define $X_e > 0$ by

$$
X_e = \begin{bmatrix} X & 0 \\ 0 & X_1 \end{bmatrix}. \tag{27}
$$

Let the left-hand side of (25) be

$$
R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}. \tag{28}
$$

Calculating R using (26), we obtain

$$
\begin{aligned}
R_{11} &= A^T X + XA - XB_{\overline{\omega}}B_{\overline{\omega}}^T X + \frac{1}{\gamma^2}X(GG^T + B_\omega B_\omega^T)\,X \\
&\quad + E^T F^T(t)D^T X + XDF(t)\,E + H^T H \\
R_{12} &= \frac{1}{\gamma^2}X(DD^T + B_{\Omega-\omega}B_{\Omega-\omega}^T)\,X_1 + XB_{\Omega-\omega}B_{\Omega-\omega}^T X_1 - E^T F^T(t)D^T X_1 \\
R_{21} &= \frac{1}{\gamma^2}X_1(DD^T + B_{\Omega-\omega}B_{\Omega-\omega}^T)\,X + X_1 B_{\Omega-\omega}B_{\Omega-\omega}^T X - X_1 DF(t)\,E \\
R_{22} &= A^T X_1 + X_1 A + \frac{1}{\gamma^2}X(GG^T + DD^T + B_\Omega B_\Omega^T)\,X_1 \\
&\quad + \frac{1}{\gamma^2}X_1(GG^T + DD^T + B_\Omega B_\Omega^T)\,X + \frac{1}{\gamma^2}X_1(GG^T + B_\omega B_\omega^T)\,X_1 \\
&\quad - C^T L^T X_1 - X_1 LC + \frac{1}{\gamma^2}X_1 LL^T X_1 + XB_{\overline{\omega}}B_{\overline{\omega}}^T X \\
&\quad + XB_{\Omega-\omega}B_{\Omega-\omega}^T X_1 + X_1 B_{\Omega-\omega}B_{\Omega-\omega}^T X
\end{aligned}
$$

R_{11} can be modified to

$$
\begin{aligned}
R_{11} &= A^T X + XA - XB_{\overline{\Omega}}B_{\overline{\Omega}}^T X + \frac{1}{\gamma^2}X(GG^T + DD^T + B_\Omega B_\Omega^T)\,X \\
&\quad + \gamma^2 E^T E + H^T H + \delta_1 I - \delta_1 I - \left(\frac{1}{\gamma^2} + 1 \right) XB_{\Omega-\omega}B_{\Omega-\omega}^T X \\
&\quad + \gamma^2 E^T F^T(t)\,F(t)\,E - \gamma^2 E^T E - \frac{1}{\gamma^2}XDD^T X \\
&\quad + E^T F^T(t)D^T X + XDF(t)\,E - \gamma^2 E^T F^T(t)\,F(t)\,E.
\end{aligned}
$$

Using (22) gives

$$R_{11} = -\delta_1 I - \left(\frac{1}{\gamma^2} + 1\right) X B_{\Omega-\omega} B_{\Omega-\omega}^T X + \gamma^2 E^T F^T(t) F(t) E - \gamma^2 E^T E - \frac{1}{\gamma^2} X D D^T X$$
$$+ E^T F^T(t) D^T X + X D F(t) E - \gamma^2 E^T F^T(t) F(t) E.$$

R_{22} can be modified to

$$R_{22} = A^T(X + X_1) + (X + X_1) A + \frac{1}{\gamma^2}(X + X_1)(GG^T + DD^T + B_\Omega B_\Omega^T)(X + X_1)$$
$$+ (X + X_1) B_\Omega B_\Omega^T (X + X_1) - \gamma^2 C^T C + \gamma^2 E^T E + H^T H + \delta_2 I$$
$$+ \left(\gamma C^T - \frac{1}{\gamma} X_1 L\right)\left(\gamma C - \frac{1}{\gamma} L^T X_1\right) - (\delta_2 - \delta_1) I - \left(\frac{1}{\gamma^2} + 1\right) X_1 B_{\Omega-\omega} B_{\Omega-\omega}^T X_1$$
$$- \frac{1}{\gamma^2} X_1 D D^T X_1 - (X + X_1) B_\omega B_\omega^T (X + X_1).$$

By choosing $W = X + X_1$ and using (23) and (24), we obtain

$$R_{22} = -(\delta_2 - \delta_1) I - \left(\frac{1}{\gamma^2} + 1\right) X_1 B_{\Omega-\omega} B_{\Omega-\omega}^T X_1 - \frac{1}{\gamma^2} X_1 D D^T X_1 - W B_\omega B_\omega^T W. \quad (29)$$

Hence, it follows that

$$R = -\begin{bmatrix} \delta_1 I & 0 \\ 0 & (\delta_2 - \delta_1) I \end{bmatrix} - \gamma^2 \begin{bmatrix} E^T \\ 0 \end{bmatrix} \begin{bmatrix} I - F^T(t) F(t) \end{bmatrix} \begin{bmatrix} E & 0 \end{bmatrix}$$
$$- \left(\frac{1}{\gamma^2} + 1\right) \begin{bmatrix} X B_{\Omega-\omega} \\ -X_1 B_{\Omega-\omega} \end{bmatrix} \begin{bmatrix} B_{\Omega-\omega}^T X & -B_{\Omega-\omega}^T X_1 \end{bmatrix}$$
$$- \begin{bmatrix} \gamma E^T F^T(t) - \frac{1}{\gamma} X D \\ \frac{1}{\gamma} X_1 D \end{bmatrix} \begin{bmatrix} \gamma F(t) E - \frac{1}{\gamma} D^T X & \frac{1}{\gamma} D^T X_1 \end{bmatrix}$$
$$- \begin{bmatrix} 0 \\ W B_\omega \end{bmatrix} \begin{bmatrix} 0 & B_\omega^T W \end{bmatrix}.$$

From the assumptions that $\delta_1 < \delta_2$ and $F^T(t) F(t) \leq I$, we conclude that $R < 0$.

Theorem 1 gives a method to design a controller for the uncertain system (1) which guarantees robust and reliable stability and disturbance attenuation of the closed-loop system despite the appearance of actuator failures as well as time-varying parameter uncertainties in the state matrix. Note that in the event of actuator failures corresponding to ω, the controlled output to be achieved by Theorem 1 satisfies

$$\|z\|_2 < \gamma \|w_{\bar{\omega}}^N\|_2 + \|u_\omega^F\|_2, \quad (30)$$

which is due to each element of u_ω^F belonging to $L_2[0, \infty)$. Theorem 1 is an extension of the result for a reliable centralized controller design in [3], to allow for time-varying parameter uncertainty in the state matrix, and soft-type failures as well as hard-type failures studied in [3].   □

5. EXAMPLE

Consider the following linear system with the parameter uncertainty in the state matrix

$$
\dot{x}(t) = \left\{ \begin{bmatrix} -2 & 1 & 1 & 1 \\ 3 & 0 & 0 & 2 \\ -1 & 0 & -2 & -3 \\ -2 & -1 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} F(t) \begin{bmatrix} 0.01 & 0 & -0.01 & 0 \\ 0 & 0 & 0.01 & 0 \end{bmatrix} \right\} x(t)
$$

$$
+ \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} w_1(t) \tag{30a}
$$

$$
y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) + w_2(t) \tag{30b}
$$

$$
z(t) = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} x(t) \\ u(t) \end{bmatrix}. \tag{30c}
$$

where the uncertain matrix $F(t)$ is time-varying as follows:

$$
F(t) = \begin{bmatrix} 0 & 1 \\ \sin(2t) & 0 \end{bmatrix}. \tag{31}
$$

The nominal open-loop system, which is considered in [3], is unstable, since not all poles are in the left-half plane. The uncertain matrix $F(t)$ satisfies $F^T(t) F(t) \leq I$. Two cases of control designs are compared under the same environment. In the first case (Case 1), the controller is designed assuming that all actuators are well operational. In the other case (Case 2), the controller is designed where the first actuator failure is taken into account using the result in Theorem 1. The simulation environment is as follows:

Design parameter:

$\gamma = 20$, $\delta_1 = 0.01$, $\delta_2 = 0.1$.

Initial state:

$x(0) = \begin{bmatrix} 3 & -2 & 2 & -3 \end{bmatrix}^T$, $\zeta(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$.

Disturbance input:

$$
w(t) = \begin{bmatrix} w_1(t)^T & w_2(t)^T \end{bmatrix}^T = \begin{cases} \begin{bmatrix} 2 & -2 & 1 \end{bmatrix}^T & 5 \leq t \leq 10 \\ \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T & \text{otherwise.} \end{cases}
$$

First actuator failure:

$$
u_1(t) = \begin{cases} \text{not failed} & 0 \leq t < 5 \\ 2 & 5 \leq t \leq 15 \\ 0 & t > 15. \end{cases}
$$

Figure 1 shows that the control system of Case 1 is unstable in the presence of actuator failure even though it is robust for the uncertainties before the presence of first actuator failure. On the other hand, Figure 2 shows that the control system of Case 2 is robust and reliable for the uncertainties and first actuator failure.
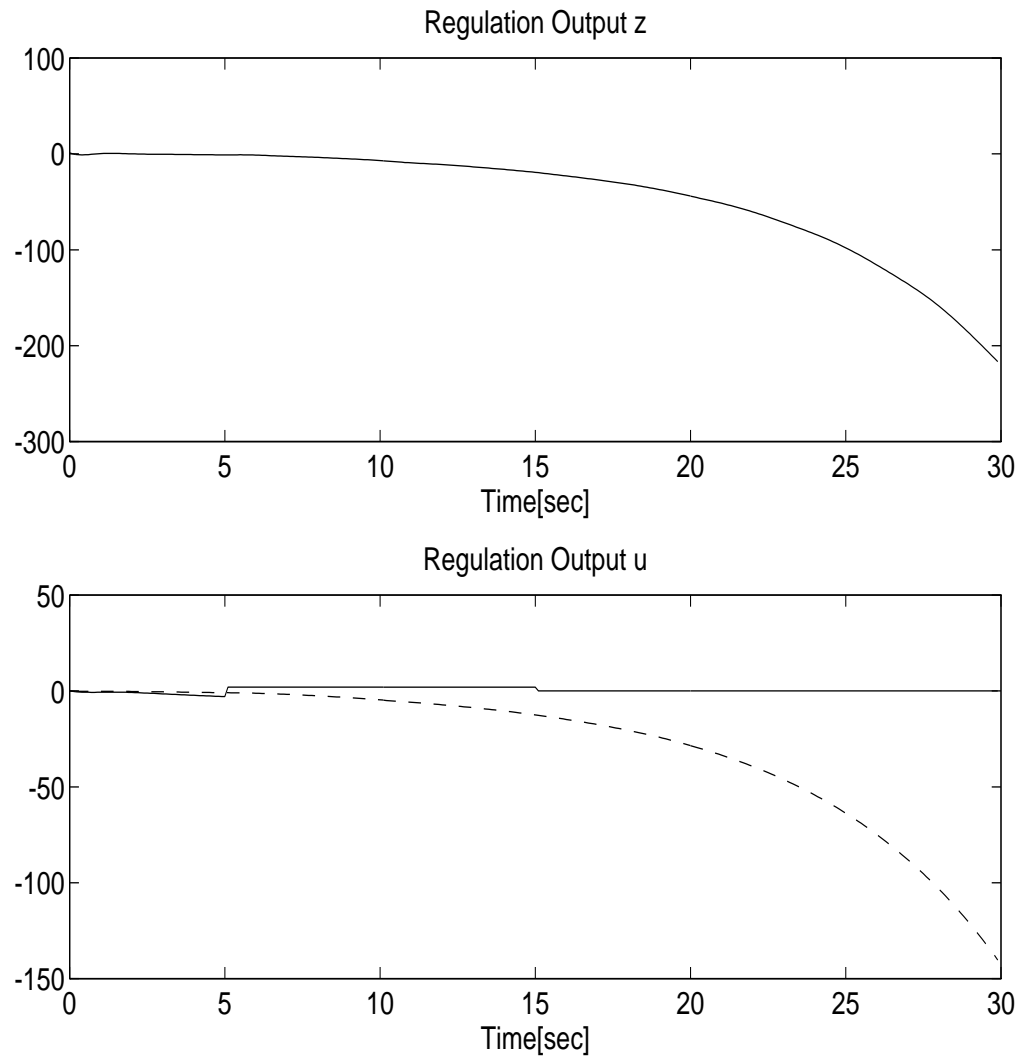

Fig. 1. System responses for robust H_∞ control – Case 1
(solid-line: $z(t)$, $u_1(t)$; dashed-line: $u_2(t)$).

Regulation Output z
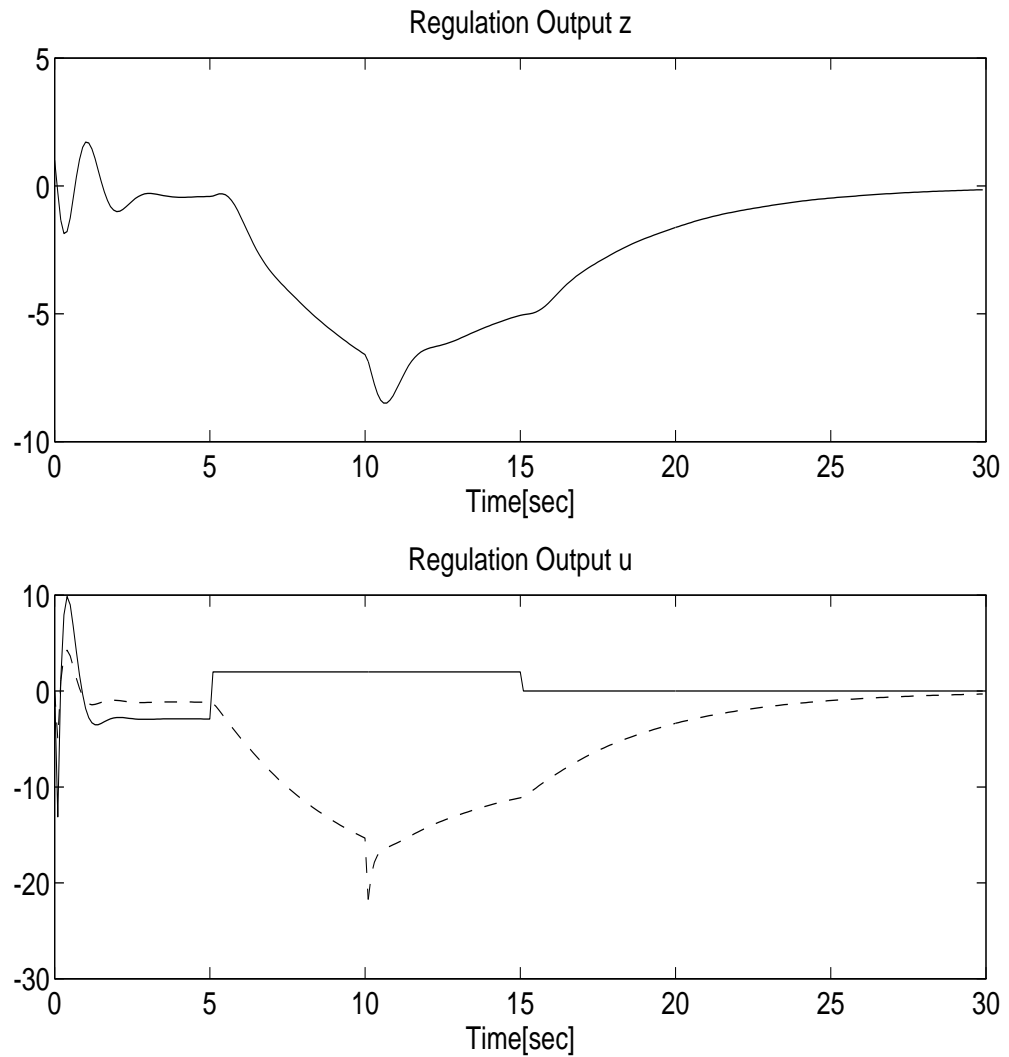

Regulation Output u

Fig. 2. System responses for robust and reliable H_∞ control – Case 2
(solid-line: $z(t)$, $u_1(t)$; dashed-line: $u_2(t)$).

6. CONCLUSIONS

For linear systems with time-varying parameter uncertainty in the state matrix, this paper has presented a robust and reliable H_∞ control design methodology to achieve quadratic stability and H_∞-disturbance attenuation, not only when the system is

operating properly, but also in the presence of certain actuator failures. Actuator failures are considered as arbitrary energy-bounded disturbance signals to the system. A set of actuators considered for reliable control is assumed to be susceptible to failures and redundant in view of the stabilization of the system. A construction for the desired observer-based output feedback control law is given in terms of the positive definite solutions of two parameter-dependent algebraic Riccati equations. The existence of an appropriate solution to the equations is sufficient to guarantee that the controller tolerates actuator failures within a prespecified set of susceptible actuators, and suppresses the effects of exogenous disturbance inputs and unexpected actuator outputs by failures under a predefined level. The result of this paper provides an unified solution for both robust control and reliable control. And also the result can be regarded as an extension of existing results on robust H_∞ control and reliable H_∞ control of uncertain linear systems.

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