

## A Second-Order Stochastic Dominance Portfolio Efficiency Measure

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*Abstract:* In this paper, we introduce a new linear programming second-order stochastic dominance (SSD) portfolio efficiency test for portfolios with scenario approach for distribution of outcomes and a new SSD portfolio inefficiency measure. The test utilizes the relationship between CVaR and dual second-order stochastic dominance, and contrary to tests in Post [14] and Kuosmanen [7], our test detects a dominating portfolio which is SSD efficient. We derive also a necessary condition for SSD efficiency using convexity property of CVaR to speed up the computation. The efficiency measure represents a distance between the tested portfolio and its least risky dominating SSD efficient portfolio. We show that this measure is consistent with the second-order stochastic dominance relation. We find out that this measure is convex and we use this result to describe the set of SSD efficient portfolios. Finally, we illustrate our results on a numerical example.

*Keywords:* stochastic dominance; CVaR; SSD portfolio efficiency measure;

*AMS Subject Classification:* 91B28; 91B30;

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