SIMULTANEOUS OUTPUT–FEEDBACK STABILIZATION FOR CONTINUOUS SYSTEMS

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A design technique for the stabilization of M linear systems by one constant output-feedback controller is developed. The design equations are functions of the state and the control weighting matrices. An example of the stabilization of an aircraft at different operating points is given.

1. INTRODUCTION

The robust control problem can be stated as follows: Design a single controller $C(s)$ which achieves prescribed performance over a region of operating conditions of the plant $P(s)$. This performance can be achieved by designing a controller for a representative plant $P_0(s)$ and maximizing the perturbation, around $P_0(s)$, that can be tolerated by the controller $C(s)$. The goal of the maximization is to cover the entire region of operation. This problem has been addressed in [4, 5, 8]. Another version of this problem is to look at a 'discrete' representation of the region of operation. In this approach, a finite number, say M, of representative plants are chosen. A controller $C(s)$ is then designed to control all of the M systems [1, 2, 3, 6, 7]. We will refer to this problem as the *simultaneous control problem*.

One of the motivations behind the simultaneous control problem is to control a nonlinear system, represented by linear models at different operating points, using one controller. In this case, each linear model represents an operating point. The parameter variations of the system model form a low-frequency upper bound on the singular values of the loop gain transfer function. Some of the robust controller design methods, LQG/LTR for example, have no mechanism for dealing with this upper bound. On the other hand, the simultaneous control technique can make gain scheduling easier to implement by reducing the number of operating points to be scheduled. This is accomplished by grouping the total number of pre-specified operating points into classes. A different controller is then designed for each of these classes [1].

Another use for the simultaneous control is in the design of controllers that are robust against sensor or actuator failures. When a sensor or an actuator fails during

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operation, the system characteristics change, effectively generating a new system. Simultaneous control design can be used to design a single controller that gives good performance for both the original system and the new system generated by the failure.

The simultaneous control problem has attracted many researchers over the last few years, but most of their work has dealt with the problem of simultaneous stabilization, without meeting any specified performance objectives. Some of the results for single-input/single-output (SISO) transfer functions were reported in [2, 3, 6, 7]. The continuous multi-input/multi-output case was addressed in [2]. In [2] a dynamical output-feedback controller is designed to achieve the stabilization of more than one system. A nonlinear feedback controller for SISO systems was used in [6]. A state-feedback controller was proposed in [7]. The design in [7] is a two stage design. In one stage a state-feedback gain K is designed to make all of the systems minimum-phase and make the pole/zero excess of each system equal to 1. Once this is achieved, a constant output-feedback controller can be designed to stabilize all of the 'new' systems. The work of Looze [3] deals with the problem of simultaneous attainment of performance objectives. The objective there is to find a state-feedback controller K which minimizes an objective function composed of the sum of M standard LQR cost functions. Each one of these cost functions penalizes the states and the controls of one of the M systems.

In this paper we develop a design technique for the stabilization of M continuous systems by one output-feedback controller. We follow the approach in [1] developed by the authors for the state-feedback case.

2. SIMULTANEOUS CONTROL RESULTS

We consider the system

$$\begin{aligned} \dot{x} &= A(q)x + B(q)u \\ y &= C(q)x \end{aligned} \tag{1}$$

where q is a vector of dimension v, and the value of q determines the system matrices. The above system may represent a linear approximation of a nonlinear system at different operating points. It may also represent an uncertain plant. In the first case, q is used as an index for identifying operating points while in the second case q may represents intervals of uncertainties of the system's parameters.

Definition 1. The *region of operation* of the above system is the domain of q.

Definition 2. The linear systems

$$\begin{aligned} \dot{x} &= A_i x + B_i u \\ y &= C_i x \end{aligned} \tag{2}$$

$i = 1, \ldots M$, *represent* the system in (1) if the stabilization of the M systems in (2) guarantees the stabilization of (1).

We assume that the M systems in (2) *represent* the system in (1). We also assume that each of the M systems in (2) is stabilizable by output-feedback controller, and we seek an output-feedback controller K which stabilizes the system in (1) for some values of q. The following theorem shows how to construct such a controller.

Theorem. An output-feedback controller $u = Ky$ which stabilizes the system in (1) exists if there exist Q_i's and R_i's such that

$$Q_i + C_i^t K_i^t R_i K_i C_i + C_i^t \sum_{j \neq i} K_j^t B_i^t P_i + P_i B_i \sum_{j \neq i} K_j C_i > 0, \quad i = 1, \dots, M \tag{3}$$

where K_i is given by

$$K_i = R_i^{-1} B_i^t P_i N_i C_i^t (C_i N_i C_i^t)^{-1}, \tag{4}$$

P_i is given by

$$(A_i - B_i K_i C_i)^t P_i + P_i (A_i - B_i K_i C_i) + Q_i + C_i^t K_i^t R_i K_i C_i = 0 \tag{5}$$

and N_i is the solution of

$$N_i (A_i - B_i K_i C_i)^t + (A_i - B_i K_i C_i) \quad N_i + I = 0. \tag{6}$$

P r o o f. Define M Lyapunov functions

$$V_i = x^t P_i x$$

where P_i is a positive definite matrix. Using the controller in (4), system (1) is stable if the derivatives of V_i with respect to time

$$\dot{V}_i = x \left[A_i^t P_i + P_i A_i \right] x + u B_i^t P_i x + x P_i B_i u$$

are negative for all $i = 1, \dots, M$. Choosing P_i as the solution of (5), the derivative of V_i will be negative if and only if (3) is satisfied. □

Example. We apply our results to the stabilization of the short-period longitudinal modes of the McDonnell Douglas F4–E aircraft. This is a three-state model. The states are normal acceleration, pitch rate, and elevator angle. We consider two operating points representing (1) Mach 0.5, altitude 5000 ft., and (2) Mach 9, and altitude 35,000 ft. The system matrices corresponding to these flight conditions are

$$A_1 = \begin{bmatrix} -.9896 & 17.41 & 96.15 \\ .2648 & -.8512 & -11.39 \\ 0.0 & 0.0 & -30.0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -.6607 & 18.11 & 84.34 \\ .08201 & -.6587 & -10.81 \\ 0.0 & 0.0 & -30.0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -.97.78 \\ 0.0 \\ 30.0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -272.2 \\ 0.0 \\ 30.0 \end{bmatrix},$$

and

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The output-feedback controller

$$K = [-4.141 \qquad -2.923]\,,$$

found using (3) – (6), stabilizes both operating points. The eigenvalues of operating point (1) are $-433.02 \quad -1.86 \pm 7.48j$ and operating point (2) has eigenvalues -135.3, -1.8, and -20.3

3. CONCLUSIONS

We utilized the state and control weighting matrices of the LQR output-feedback controller for solving the problem of stabilizing more than one system by a fixed controller. The stabilization of F4–E aircraft at two different operating points is presented.

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