

Quasi-Copulas with Quadratic Sections in One Variable

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Abstract: We introduce and characterize the class of multivariate quasi-copulas with quadratic sections in one variable. We also present and analyze examples to illustrate our results.

Keywords: 1-Lipschitz condition; copula; quasi-copula; quadratic sections;

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