

Symmetries of Random Discrete Copulas

Arturo Erdely; José M. González-Barrios; Roger B. Nelsen

Abstract: In this paper we analyze some properties of the discrete copulas in terms of permutations. We observe the connection between discrete copulas and the empirical copulas, and then we analyze a statistic that indicates when the discrete copula is symmetric and obtain its main statistical properties under independence. The results obtained are useful in designing a nonparametric test for symmetry of copulas.

Keywords: discrete copulas; r-symmetric permutations; independence;

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References

- [1] I. Aguiló, J. Suñer, and J. Torrens: Matrix representation of discrete quasi-copulas. *Fuzzy Sets and Systems* 159 (2008), 1658–1672.
- [2] C. Alsina, M. J. Frank, and B. Schweizer: *Associative Functions: Triangular Norms and Copulas*. World Scientific Publishing Co., Singapore 2006.
- [3] P. Deheuvels: La fonction de dépendance empirique et ses propriétés. Un test non paramétrique d'indépendance. *Acad. Roy. Belg. Bull. Cl. Sci.* 65 (1979), 5, 274–292.
- [4] E. P. Klement, R. Mesiar, and E. Pap: *Triangular Norms*. Kluwer Academic Publishers, Dordrecht 2000.
- [5] E. P. Klement and R. Mesiar: *Logical, Algebraic, Analytic, and Probabilistic Aspects of Triangular Norms*. Elsevier, Amsterdam 2005.
- [6] A. Kolesárová, R. Mesiar, J. Mordelová, and C. Sempi: Discrete copulas. *IEEE Trans. Fuzzy Systems* 14 (2006), 698–705.
- [7] A. Kolesárová and J. Mordelová: Quasi-copulas and copulas on a discrete scale. *Soft Computing* 10 (2006), 495–501.
- [8] G. Mayor, J. Suñer, and J. Torrens: Copula-like operations on finite settings. *IEEE Trans. Fuzzy Systems* 13 (2005), 468–477.
- [9] G. Mayor, J. Suñer, and J. Torrens: Sklar's Theorem in finite settings. *IEEE Trans. Fuzzy Systems* 15 (2007), 410–416.

- [10] R. Mesiar: Discrete copulas – what they are. In: Joint EUSFLAT-LFA 2005, Conference Proceedings (E. Montseny and P. Sobrevilla, eds.) Universitat Politècnica de Catalunya, Barcelona 2005, pp. 927–930.
- [11] W. Miller: The maximum order of an element of a finite symmetric group. Amer. Math. Monthly 94 (1987), 6, 497–506.
- [12] R.B. Nelsen: An Introduction to Copulas. Second edition. Springer, New York 2006.
- [13] B. Schweizer and A. Sklar: Probabilistic Metric Spaces. North-Holland, New York 1983.
- [14] S. Skiena: The cycle structure of permutations. In: Implementing Discrete Mathematics: Combinatorial and Graph Theory with Mathematica. Addison-Wesley, Reading, MA 1990, pp. 20–24.