EXPLICIT CONDITIONS
FOR RIPPLE–FREE DEAD–BEAT CONTROL

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A generalized and systematic approach to the problem of dead-beat response to polynomial time-domain inputs is presented. Necessary and sufficient conditions for the impulse response coefficients of the system, in the form of a set of linear equations, are derived. For the case of minimum prototype systems, a formula for the explicit computation of the overall pulse transfer function of the system is deduced and their properties are further studied, together with their ability to track complex inputs. It is also presented an analytical study of the system's control sequence, which plays an important role for the inter sample activity of the plant. By eliminating possible oscillations that may be present in this sequence, necessary and sufficient conditions for ripple-free dead-beat response are derived.

1. INTRODUCTION

Discrete-time control systems may exhibit the unique feature of settling the system error to zero in a finite number of control steps. This is called dead-beat (DB) response [1, 12, 13, 15, 24], and the design of DB control systems has long been one of the fundamental problems of control theory. The reference inputs commonly considered are prototype time-domain signals of the form t^m, where m is a non-negative integer. The conventional approach to the design of a SISO discrete-time control system that is desired to exhibit DB response to such an input, can be summarized in the following relations. First, the closed-loop pulse transfer function $F(z)$ must be expressed as a finite polynomial in terms of powers in z^{-1}

$$F(z) = \sum_{i=1}^{N} f_i z^{-i}. \tag{1}$$

Yet, the finite impulse response (FIR) nature of the system is not sufficient for DB response, and additional relations between the coefficients of the system are needed. Observing that the z-transform of the input signal considered is of the form

$$R(z) = \frac{A(z)}{(1 - z^{-1})^{m+1}} \tag{2}$$

where $A(z)$ is a polynomial in z^{-1} with no zeros at the point $z = 1$, and the z-transform of the error sequence, for the unity-feedback scheme presented in Figure 1, is written as

$$E(z) = R(z)\left[1 - F(z)\right] \tag{3}$$

then, in order the error sequence to be finite, $1 - F(z)$ must be expressed as another finite polynomial in the form

$$1 - F(z) = \left(1 - z^{-1}\right)^{m+1}\left(1 + \sum_{i=1}^{N-m-1} g_i z^{-i}\right). \tag{4}$$

Fig. 1. Digital control system.

The relations (1) and (4) are well known since the early contributions to the field of the sampled-data control systems [6, 7, 21]. Although these relations appear in a rather coupled configuration, since Eq. (1) is associated with the impulse response of the system, while Eq. (4) with the error sequence of the system due to the specific prototype input, they are still used in the design of SISO control systems [8]. On the other hand, explicit conditions for DB response which include the impulse response coefficients only, would be very useful in the design of minimum and non-minimum prototype DB control systems.[1] In this respect, Bergen and Ragazzini [6] have pointed out the way in which one can derive conditions on the impulse response only, by equating to zero the right-hand side of Eq. (4) and the successive derivatives of $F(z)$ with respect to z, up to the order of m, all evaluated at the point $z = 1$; nevertheless, there is no reference to explicit conditions for DB response to general polynomial signals.

Although the DB response has been defined irrespectively of the inter sample response of the system, the DB control of certain classes of continuous-time plants may present problems, because of the remaining inter sample activity, which is undesirable. The term ripple-free dead-beat (RFDB) response refers to the elimination of this inter sample activity, that is the accomplishment of zero error response for all the continuous time after the finite settling-time [1, 15, 22, 24]. During the last years, there has been an extensive study of ripple-free digital control systems

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[1] A minimum prototype system is characterized by the minimum system-order for dead-beat response to a specified prototype input.

$[1, 4, 5, 9-11, 18-20, 22, 24]$, mainly because of the development of the micropro- cessors technology that permits the application of such modern techniques to the control of a wide variety of plants.

According to Urikura and Nagata [22], ripple can appear in the following two cases: (a) when certain pole-zero cancellations occur, and (b) when modes that cannot be unobservable in the error sequence remain. Case (a) can be easily reduced by identifying the pole-zero cancellations that lead to ripples, and then avoiding them, while case (b) is associated with the structure of the control system, and calls for the existence of a minimum signal model of the reference in the plant (internal model principle). Zafiriou and Morari [23] have proposed a method for designing digital controllers for RFDB response of strictly stable SISO plants, with the minimum settling-time. They identified one special case of pole-zero cancellation that leads to ripple, that is of the cancellation of negative real zeros of the plant inside the unit circle of the z-plane. The oscillation of the control signal is another more general factor that implies the appearance of ripple at the output of system [15], but again there is no reference to explicit conditions restricting the impulse response coefficients of the system for RFDB response to polynomial inputs.

In the first part of this work, it is presented a generalized and systematic approach to the problem of DB response to polynomial time-domain inputs. The analysis is realized in the time-domain using only the overall pulse transfer function of the system, without taking into consideration subjects relating to the internal structure of the system, or the design of the digital controller. Thus, the results obtained are valid for every SISO DB response system, irrespectively of the existence of unstable zeros and/or poles of the plant. A set of necessary and sufficient conditions for the impulse response coefficients of the system are derived. For the case of minimum prototype systems, a formula for the computation of the impulse response coefficients is deduced, their properties are further studied, and their ability to track a complex input is demonstrated.

In the second part of this work, it is presented an analytical study of the control sequence of a DB control system. The existence of inter sample ripple at the output of the plant is closely related to the form of the control signal. As it has been stated before, the oscillation of the control signal, after the finite settling-time, has as an effect the appearance of ripple at the output of the plant, and consequently, something that must be achieved for RFDB response is exactly the elimination of this oscillation. The study of the control sequence that is presented, results to necessary and sufficient conditions, in such a way that this sequence is restricted to constant or monotonously varying values, after the finite settling-time. These conditions are at the same time necessary and sufficient conditions for the rejection of the ripple at the output of the plant.

2. CONDITIONS FOR DB CONTROL

For DB response, the system is required to exhibit a finite settling-time with zero steady-state error, which entails that it must exhibit a finite-duration impulse re- sponse. Hence, the system's pulse transfer function must be in the form of a poly-

nomial in terms of powers in z^{-1}

$$F(z) = \sum_{i=0}^{N} f_i z^{-i} \tag{5}$$

where the coefficients f_i, $i = 0, 1, 2, \ldots, N$, constitute the impulse-response sequence of the system. Since the system cannot respond instantaneously when an input signal of finite magnitude is applied, that is, the response comes at a delay of at least one sampling period [15], the lowest-power term of this series expansion of $F(z)$ in z^{-1} must be zero, or f_0 must be equal to 0, which means that the expansion has to be of the form of Eq. (1).

It is assumed that the system is forced by time-domain polynomial inputs in the form

$$r(t) = \sum_{j=1}^{M} A_j t^j. \tag{6}$$

For a linear system, that responds to every term of Eq. (6) independently of the existence of the other terms, it is sufficient to examine the response of the system to a prototype input signal of degree m with respect to t, or equivalently to kT, that is

$$r(kT) = (kT)^m. \tag{7}$$

The set of necessary and sufficient conditions for the impulse response coefficients of the system are given in the following theorem.

Theorem 1. An Nth order discrete-time system described by the FIR pulse transfer function (1), exhibits DB response to polynomial time-domain inputs of degree m, if and only if the impulse response coefficients f_i, $i = 1, 2, \ldots, N$, satisfy the following set of $m + 1$ linear algebraic equations

$$\sum_{i=1}^{N} f_i i^j = \delta_j \quad \text{for} \quad j = 0, 1, 2, \ldots, m \tag{8}$$

where δ_j is the unit impulse sequence.[2]

P r o o f. The output sequence $c(kT)$ of the system is given by the convolution sum of the impulse response sequence f_i, $i = 1, 2, \ldots, N$, and the input sequence $r(kT)$ given from Eq. (7), as

$$c(kT) = \sum_{i=1}^{k} f_i (k - i)^m T^m. \tag{9}$$

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[2] The unit impulse sequence is defined by the relation: $\delta_0 = 1$, $\delta_{j \neq 0} = 0$.

For $k \geq N$, for the output to be identical to the input, it must be

$$\sum_{i=1}^{k} f_i(k-i)^m\, T^m \equiv (kT)^m. \tag{10}$$

By use of the binomial theorem, the term $(k-i)^m$ can be expanded in terms of powers of k and i, that is

$$(k-i)^m = \sum_{j=0}^{m}(-1)^j \binom{m}{j} k^{m-j}\, i^j. \tag{11}$$

and substituting the right-hand side of this into Eq. (10), we obtain

$$\sum_{i=1}^{k} f_i \left(\sum_{j=0}^{m}(-1)^j \binom{m}{j} k^{m-j} i^j \right) T^m \equiv (kT)^m. \tag{12}$$

Elimination of the term $(kT)^m$ from both sides of this, leads to the identity

$$\sum_{i=1}^{k} f_i \left(\sum_{j=0}^{m}(-1)^j \binom{m}{j} k^{-j} i^j \right) \equiv 1 \tag{13}$$

The left-hand side of this equation is a polynomial in terms of powers in k, and it must be equal by identity to 1, for every value of k. Interchanging the order of summation, it results to

$$\sum_{j=0}^{m}(-1)^j\, k^{-j} \binom{m}{j} \left(\sum_{i=1}^{k} f_i\, i^j \right) \equiv 1 \tag{14}$$

from which follows Eq. (8). □

Some general remarks that originate from the previous theorem follow. The explicit conditions expressed by Eq. (8) do not contain the sampling period T, which means that the shape of the DB response to prototype inputs is independent of the selection of the sampling period, although the control signal tends to get higher values, as the sampling becomes faster [15].

Another remark is that the conditions for DB response to a time-domain input of degree m contain the conditions for DB response to time-domain inputs of lower degree, and it is concluded that a system which exhibits DB response to a time-domain input of degree m, will exhibit DB response to every time-domain input of lower degree.

According to the relationship between the order of the system N and the degree of the prototype input m, the following cases exist:

(a) If $m + 1 > N$, then there are more equations than unknowns in Eq. (8) and there is no solution; that is the system cannot exhibit DB response to a time-domain input of degree m. This fact indicates that the minimum system-order for DB response to a polynomial input of degree m, is $m + 1$.

(b) If $m + 1 = N$, then there are as many equations as unknowns in Eq. (8), and a unique solution set of f_i's can be determined. In this case, there is the minimum number of coefficients f_i, and the system constitutes a *minimum prototype system (MPS)*. An MPS for a polynomial input of degree m exhibits DB response to every polynomial input of degree less or equal to m, but it may provide unsatisfactory response to these inputs. MPSs are further studied in the following section.

(c) If $m + 1 < N$, then we can either select arbitrary values for the $N - m - 1$ coefficients, or compute the coefficients through an optimization process, in order to obtain better transient response to specific time-domain inputs, based to a convenient performance criterion. The conditions expressed by Eq. (8), in combination with any other subsidiary constraints imposed on the transient behavior of the system, such as the maximum overshoot, or the settling-time, or even the minimization of the sum of the squared errors at the sampling instants, enable the calculation of the coefficients f_i, $i = 1, 2, \ldots, N$, and consequently the computation of $F(z)$. Such optimization problems, for the transient response shaping, have been extensively studied in the past [1–3, 7, 14, 16, 17].

2.1. Minimum prototype systems

As it has been stated in the previous section, an MPS is defined for $N = m + 1$, that is, it is a system with a polynomial pulse transfer function in terms of powers in z^{-1} of order N that exhibits DB response to polynomial inputs of degree $N - 1$. In this case, Eq. (8) is written

$$\sum_{i=1}^{N} f_i \, i^j = \delta_j \quad \text{for} \quad j = 0, 1, 2, \ldots, N - 1. \tag{15}$$

Expanding the last set of equations and using matrix notation, we obtain

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 3 & \cdots & N \\ 1 & 2^2 & 3^2 & \cdots & N^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2^{N-1} & 3^{N-1} & \cdots & N^{N-1} \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_N \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{16}$$

The square matrix in the last equation is a Vandermonde matrix of order N, and the solution of Eq. (16) is achieved by the computation of the elements of the first column of the inverse of this matrix, namely

$$f_i = (-1)^{i-1} \binom{N}{i} \quad \text{for} \quad i = 1, 2, \ldots, N \tag{17}$$

thus obtaining the impulse response coefficients of an Nth order MPS. These coefficients are the same with those obtained by the method suggested by Kuo [13], that

is by expanding the right-hand side binomial of

$$1 - F(z) = \left(1 - z^{-1}\right)^{m+1}.$$ (18)

The step response error sequence coefficients for an Nth order MPS are given from

$$a_k \doteq 1 - \sum_{i=1}^{k} f_i = \sum_{i=0}^{k} (-1)^i \binom{N}{i}$$ (19)

and the first sample overshoot of the step response is linearly increasing with the order of the system, that is $-a_1 = N - 1$, while the maximum absolute error of the step response is easily shown to be

$$\max_k |a_k| = \binom{N-1}{\left\lfloor \frac{N-1}{2} \right\rfloor}.$$ (20)

It is apparent that as the order of an MPS increases, its step response becomes even more unsatisfactory (e. g. from Figure 2 the maximum absolute error equals to 600% for $N = 5$), as regarding the application of DB control to practical systems.

Fig. 2. Maximum absolute error of the step response for a minimum prototype system.

In Figures $3-7$ is presented the response of some MPSs to a rather complex input signal. As such a signal, it has been selected the Bessel function of zero order $J_0(kT)$, where the sampling period T is taken equal to 0.4. The output of the system is calculated by the convolution sum of Eq. (9). As it is expected, higher-order DB systems can track in a better way the input signal, but they present large deviations at the initial transient of the response, because of the step character of the input in this region. For the above reasons, non-minimum prototype DB systems are usually utilized, in which the step response overshoot can be reduced by extending the settling-time.

Fig. 3. Response to a complex input signal: Minimum prototype system for $N = 1$.

Fig. 4. Response to a complex input signal: Minimum prototype system for $N = 2$.

Fig. 5. Response to a complex input signal: Minimum prototype system for $N = 3$.

Fig. 6. Response to a complex input signal: Minimum prototype system for $N = 4$.

Fig. 7. Response to a complex input signal: Minimum prototype system for $N = 5$.

3. CONDITIONS FOR RFDB CONTROL

From Figure 1, the z-transform of the control sequence is given by the product of the digital controller pulse transfer function, and the z-transform of the error sequence, that is

$$U(z) = D(z) E(z). \tag{21}$$

The digital controller pulse transfer function can be written [13, 15]

$$D(z) = \frac{1}{G_h(z)} \cdot \frac{F(z)}{1 - F(z)} \tag{22}$$

and the z-transform of the error sequence is given by Eq. (3). Substituting Eqs. (22) and (3) into Eq. (21), we get

$$U(z) = \frac{F(z) R(z)}{G_h(z)} \tag{23}$$

from which, by applying the inversion integral method [15], the control sequence is expressed by the relation

$$u_k = \frac{1}{2\pi j} \oint_\Gamma \frac{F(z)\,R(z)}{G_h(z)}\, z^{k-1}\, \mathrm{d}z \qquad (24)$$

where Γ is any simple closed curve that encloses all the isolated singular points (poles) of the integrand. Taking into account that the closed-loop pulse transfer function takes the form of a finite polynomial in z^{-1} that dictates Eq. (1), and that the z-transform of the input sequence takes the form that dictates Eq. (2), then Eq. (24) is written

$$u_k = \frac{1}{2\pi j} \oint_\Gamma I_m(z)\, \mathrm{d}z \qquad (25)$$

where we have defined that

$$I_m(z) \doteq \frac{A(z)}{G_h(z)} \left(\sum_{i=1}^{N} f_i\, z^{N-i} \right) \frac{z^{k-(N-m)}}{(z-1)^{m+1}}. \qquad (26)$$

The former integral can be computed by applying the residue theorem, according to which it is equal to $2\pi j$ times the sum of the residues of the integrand at the isolated singular points, that is the poles of the integrand, inside the curve Γ.

It is readily seen that the poles of the integrand consist of the $m+1$ poles of the z-transform of the input signal $R(z)$ at the point $z = 1$, from Eq. (2), the poles that are due to the zeros of the plant pulse transfer function $G_h(z)$, and the poles at the point $z = 0$ arising from the factor $z^{k-(N-m)}$ for $k < N - m$, and the polynomial $A(z)$, as well.

Since for the rejection of the ripple we are interested in the behavior of the control sequence after the end of the transient response, that is for $k \geq N$, it is apparent that oscillation of the control sequence results only from zeros of the plant transfer function that are taking on negative values, as it is seen from the existence of the factor z^k in the former integrand. This is in full agreement with the observation made by Zafiriou and Morari [23] for the stable negative real zeros of the plant; simply the approach adopted here expands the application area of this observation to RFDB systems with non-minimum settling-time. The conditions that are derived in this way are useful in the design of optimum RFDB systems [1, 4].

3.1. Conditions for step input

The step input corresponds to $m = 0$, and Eq. (25) is written

$$u_k = \frac{1}{2\pi j} \oint_\Gamma I_0(z)\, \mathrm{d}z \qquad (27)$$

with

$$I_0(z) \doteq \frac{1}{G_h(z)} \left(\sum_{i=1}^{N} f_i\, z^{N-i} \right) \frac{z^{k-N}}{z-1} \qquad (28)$$

and for $k \geq N$, it leads to

$$u_{k \geq N} = \sum_{\text{at the zeros of } G_h(z)} \text{Res}\{I_0(z)\} + \underset{\text{at } z=1}{\text{Res}}\{I_0(z)\}. \tag{29}$$

Taking into account the condition for the impulse response coefficients that results from Eq. (8) for $m = 0$, that is, their sum is equal to unit, then the last residue appearing in Eq. (29) is further simplified to

$$u_{k \geq N} = \sum_{\text{at the zeros of } G_h(z)} \text{Res}\{I_0(z)\} + \frac{1}{G_h(1)}. \tag{30}$$

It is readily seen that the existence of the factor z^k in Eq. (28) leads to the exponential variation of the control signal for $k \geq N$, and its further oscillation in the case that a zero of the plant takes on negative values. Thus, for the control signal to be constant, after the end of the transient response, it is required that the following relation holds

$$\underset{\text{at the zeros of } G_h(z)}{\text{Res}} \left\{ \frac{1}{G_h(z)} \left(\sum_{i=1}^{N} f_i z^{N-i} \right) \frac{z^{k-N}}{z-1} \right\} = 0 \tag{31}$$

which, if all the zeros of the plant transfer function $G_h(z)$ are simple, it can take the simpler linear form

$$\sum_{i=1}^{N} f_i z^{N-i} \Bigg|_{\text{at simple zeros of } G_h(z)} = 0. \tag{32}$$

Example 1. In the following, there is a study of a plant that its discrete-time model has a simple zero at the point $z = a$, that can reside everywhere on the z-plane, except from the point $z = 1$, since in this case it is eliminated from a pole of the input signal. Let's define that

$$G_h(z) \doteq \frac{z-a}{G_1(z)} \tag{33}$$

where the function $G_1(z)$ has no poles or zeros at the point $z = a$. In this case, Eq. (30) is written

$$\begin{aligned}
u_{k \geq N} &= \underset{\text{at } z=a}{\text{Res}} \left\{ \frac{G_1(z)}{z-a} \left(\sum_{i=1}^{N} f_i z^{N-i} \right) \frac{z^{k-N}}{z-1} \right\} + \frac{G_1(1)}{1-a} \\
&= \frac{G_1(a)}{a-1} \left(\sum_{i=1}^{N} f_i a^{N-i} \right) a^{k-N} + \frac{G_1(1)}{1-a}.
\end{aligned} \tag{34}$$

The variation of the control signal is due to the existence of the factor a^k at the first part of the right hand side of Eq. (34), and it is eliminated by taking

$$\sum_{i=1}^{N} f_i a^{N-i} = 0 \tag{35}$$

which is a linear condition for the closed-loop impulse response coefficients.

3.2. Conditions for ramp input

The ramp input corresponds to $m = 1$, and Eq. (25) is written

$$u_k = \frac{1}{2\pi j} \oint_\Gamma I_1(z)\,\mathrm{d}z \tag{36}$$

with

$$I_1(z) \doteq \frac{T}{G_h(z)} \left(\sum_{i=1}^{N} f_i\, z^{N-i} \right) \frac{z^{k-N}}{(z-1)^2} \tag{37}$$

and for $k \geq N$, it leads to

$$u_{k \geq N} = \sum_{\text{at the zeros of } G_h(z)} \mathrm{Res}\,\{I_1(z)\} + \underset{\text{at } z=1}{\mathrm{Res}}\,\{I_1(z)\}. \tag{38}$$

Taking into account the condition for the impulse response coefficients that results from Eq. (8) for $m = 1$, that is, their sum is equal to unit and their first moment is zero, then the last residue appearing in Eq. (38) is further simplified to

$$u_{k \geq N} = \sum_{\text{at the zeros of } G_h(z)} \mathrm{Res}\,\{I_1(z)\} + \frac{T}{G_h(1)} \left(k - \frac{G_h'(1)}{G_h(1)} \right). \tag{39}$$

It is apparent that the existence of the factor z^k in Eq. (37) leads to the exponential variation of the control signal for $k \geq N$, and its further oscillation in the case that a zero of the plant takes on negative values. Thus, for the control signal to be monotonously varying according to the last term of Eq. (39), after the end of the transient response, it is required that the following relation holds

$$\underset{\text{at the zeros of } G_h(z)}{\mathrm{Res}} \left\{ \frac{1}{G_h(z)} \left(\sum_{i=1}^{N} f_i\, z^{N-i} \right) \frac{z^{k-N}}{(z-1)^2} \right\} = 0. \tag{40}$$

Example 2. Next follows a study of the plant that was defined with Eq. (33). In this case, Eq. (39) is written

$$
\begin{aligned}
u_{k \geq N} &= \underset{\text{at } z=a}{\mathrm{Res}} \left\{ T\frac{G_1(z)}{z-a} \left(\sum_{i=1}^{N} f_i\, z^{N-i} \right) \frac{z^{k-N}}{(z-1)^2} \right\} + \frac{T}{G_h(1)} \left(k - \frac{G_h'(1)}{G_h(1)} \right) \\
&= T\frac{G_1(a)}{(a-1)^2} \left(\sum_{i=1}^{N} f_i\, a^{N-i} \right) a^{k-N} + T\frac{G_1(1)}{1-a} \left(k - \frac{1}{1-a} + \frac{G_1'(1)}{G_1(1)} \right).
\end{aligned}
\tag{41}
$$

The variation of the control signal is due to the existence of the factor a^k at the first part of the right hand side of Eq. (41), and it is eliminated by taking

$$\sum_{i=1}^{N} f_i\, a^{N-i} = 0 \tag{42}$$

which is the same linear condition for the closed-loop impulse response coefficients, as that expressed with Eq. (35).

4. CONCLUSIONS

In this work, a generalized and systematic approach to the problem of DB response of SISO systems to polynomial time-domain inputs was introduced. This approach led to the derivation of a set of necessary and sufficient conditions for the impulse response coefficients of the system, from which general characteristics of these systems became readily apparent. The aforementioned conditions, for the case of minimum prototype systems, allowed the reduction of a formula for the explicit computation of the impulse response coefficients and consequently the overall pulse transfer function of the system.

The existence of a number of equations expressing the conditions for DB response to a specified prototype input places constraints in the choice of the system's parameters for desired time response to other inputs. This is especially emphatic when we intend to design an MPS in order to obtain the minimum settling-time for zero error. MPSs present large overshoots to step inputs, that may not be acceptable in real systems. But, it is not necessary to endeavor a minimum prototype response, especially when the computer implementation of the digital controller can provide a sampling period that is much smaller than the time constants of the system. Thus, a system which is restricted to be DB only in step and ramp inputs can easily track complex inputs, provided that the sampling frequency is high enough.

There was also presented an analytical study of the control sequence of a DB control system. By restricting this sequence to be constant or monotonously varying, after the finite settling-time, necessary and sufficient conditions for the rejection of the inter sample ripple at the output of the plant were derived, for the case that only step and/or ramp inputs are considered.

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