# Detecting nonlinear oscillations in broadband signals

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## Abstract

A framework for detecting nonlinear oscillatory activity in broadband time series is presented. First, a narrow-band oscillatory mode is extracted from a broadband background. Second, it is tested whether the extracted mode is significantly different from linearly filtered noise, modelled as a linear stochastic process possibly passed through a static nonlinear transformation. If a nonlinear oscillatory mode is positively detected, it can be further analyzed using nonlinear approaches such as the phase synchronization analysis. For linear processes standard approaches, such as the coherence analysis, are more appropriate. The method is illustrated in a numerical example and applied to analyze experimentally obtained human EEG time series from a sleeping subject.

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**Many recent scientific efforts focus on the importance of oscillatory activity in biological and physical systems especially in the context of phase dynamics and phase synchronization. For instance, oscillations in various frequency bands of the electroencephalogram have become important in understanding the function of the central nervous system. Typical approaches to analyzing data involve applying Fourier decomposition, wavelet transform or the newer empirical mode decomposition to a time series to extract a set of *modes* or time series confined within a certain frequency band. These methods themselves, however, do not address the question of what character is the content of the extracted mode. In order to satisfy further data processing based on nonlinear approaches such as phase synchronization analysis one should provide sufficient evidence that the obtained signal can be interpreted as oscillatory activity of a self-sustained, nonlinear dynamical system.**

## I. INTRODUCTION

The search for repetitive patterns in erratic, seemingly random dynamical behavior is an important way how to understand, model and predict complex phenomena. Cyclic, oscillatory phenomena are sought in complex dynamics observed in diverse fields from physics and technology, through meteorology and climatology to neurophysiology. In cortical networks oscillatory phenomena are observed which span five orders of magnitude in frequency [1]. These oscillations are phylogenetically preserved, suggesting that they are functionally relevant. Among the well-known neural oscillatory phenomena the $\delta$, $\theta$, $\alpha$, $\beta$ and $\gamma-$waves can be observed in the scalp EEG. The electroencephalogram, EEG, is a record of the oscillations of brain electric potentials registered from electrodes attached to the human scalp, revealing synaptic action that is moderately to strongly correlated with brain states. Oscillatory phenomena in the brain electrical activity and their synchronization are related to cognitive processes [2] and their dynamical and synchronization properties change under cognitive disorders such as Alzheimer disease, schizophrenia, bipolar disorder or attention-deficit hyperactivity disorder [3]. It is understandable that the detection and characterization of oscillatory phenomena in the brain activity are subjects of intensive research.

Besides the Fourier spectral analysis, typical approaches to study brain waves involve applying wavelet decomposition or, more recently, empirical mode decomposition (EMD) and other filtering techniques to a time series to extract a set of *modes* or time series which contain a part of the original signal confined within a certain frequency band. The extracted narrow-band modes are frequently further analyzed using modern nonlinear methods, such as the synchronization analysis, in order to infer possible cooperative behavior of distant parts of the human brain. For such synchronization [4] or directionality (causality) analyses [5] the extracted oscillatory modes are used to compute so-called instantaneous phase, a characteristic variable of self-sustained, nonlinear oscillatory dynamical systems. Thus it is desirable to provide arguments that the observed oscillatory phenomena come from self-sustained nonlinear dynamical systems; in order to avoid applications of nonlinear approaches to linearly filtered noise. The above mentioned filtration methods themselves do not address the question of what character is the content of the extracted mode. More intricate procedures such as singular spectrum analysis (SSA), used especially in the field of climatology and meteorology [6], perform a principal components analysis in the time domain and test whether variance of each candidate mode is significantly higher than noise background. In a sophisticated procedure known as the Monte Carlo SSA [7], the existence of oscillatory modes is tested by means of computing the variance (energy content) of each mode and verifying if it is outside the expected range for a particular background process, such as a red noise process. Recently, the method has been modified to test the dynamics of the candidate mode [8]. While the standard Monte Carlo SSA [7] discerns any kind of oscillatory activity different from a filtered red noise considering the process covariance structure, Paluš and Novotná [8] identify the modes with dynamics which is more regular and better predictable than a narrow-band oscillations obtained as linearly filtered noise. The latter approach has been successfully applied to identify oscillatory phenomena in long-term climate records and in geomagnetic activity [9]. In neurophysiology, oscillatory activity is often extracted from time series using bandpass filtering, wavelet analysis or empirical mode decomposition (EMD). A spectral peak is usually all the evidence considered for the existence of oscillatory activity. However, sufficiently narrow-band time series extracted from a broadband process have an oscillatory waveform which is strongly constrained by the properties of the filter or the extraction method. Even a clear spectral peak may not be sufficient to assume the existence of limit cycle dynamics of a self-sustained dynamical system as a generator of the oscillatory activity. Phase derived from a time series has a clear physical meaning only if the generating system follows a trajectory with stable amplitude. It follows that the generating system must be nonlinear to achieve limit cycle dynamics with a stable amplitude [4]. In this work a method is proposed which is able to detect weak oscillatory signals with dynamics different from that of filtered noise.

The paper continues with a description of the detection methods in Sec. II, with results obtained in numerical experiments and on actual data in Sec. III and finishes with a discussion and conclusion in Sec. IV.

## II. METHODS

Important parts of the proposed method are the procedure to generate the surrogate time series, the mathematical objects that statistically capture required properties of analyzed time series and the function which quantifies the difference between the dynamical structures of two time series, namely the original and the surrogate one. Here the surrogate time series are constructed so that their linear structure (autocorrelation structure) matches that of the analyzed data. If at the same time the nonlinear structure of the data significantly deviates from that of the surrogate time series, then it is inferred that a nonlinear process is involved in the generation of the data.

The data is first preprocessed by an amplitude adjustment procedure the purpose of which is to make the sample distribution of the analyzed data segment Gaussian. The samples in the time series are ranked and an equally sized normally distributed set of samples is created. The time series samples are replaced with samples of equal rank from the normally distributed set. This step ensures that the influence of any bijective nonlinear measurement function is excluded from the test for nonlinear structure. The original unadjusted time series data is not used henceforth and any reference to original data refers to the amplitude-adjusted version. The surrogate data set is generated by repeated runs of an autoregressive model that has been fit to the time series. An autoregressive model to the time series is powerful enough to represent any type of filtered noise. An AR model of order $K$ is specified as

$$x(t) = \sum_{i=1}^{K} a_i x(t-i) + \mu + \sigma\xi(t), \tag{1}$$

where $a_i$ are the coefficients of the model, $\mu$ is the mean of the generated time series and $\sigma$ is the standard deviation of the uncorrelated Gaussian noise term $\xi(t)$. The optimal order of the autoregressive model is unknown and a model selection method must be employed. Here the Bayesian Information Criterion (BIC) [10] is used. The BIC is given by

$$\mathrm{BIC}(K) = N \log\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon(i)^2\right) + (K+2)\log N, \tag{2}$$

where $\epsilon(i)$ are the residuals of the best model fit to the original time series and $N$ is the number of points in the time series. The number of free parameters of the estimation is $K+2$ as besides the $K$ model coefficients also the mean value and the standard deviation of the input noise is estimated from the same dataset. A maximum admissible order is specified before the fitting procedure begins and models of all smaller orders are fit using least-squares to the time series. The BIC is computed for each fitted model and the model with the smallest BIC value is selected. Surrogate data are generated by randomly shuffling the residuals of the fit and feeding them back into the identified model as the source noise. Using this procedure an arbitrary amount of surrogate time series can be generated.

The linear structure of a time series is characterized by linear regularity or predictability measure which is based on the linear version of time delayed mutual information ("linear redundancy" [11])

$$I_{\mathrm{lin}}(X; X_\tau) = -\frac{1}{2}\log(1-\rho_\tau^2), \tag{3}$$

where $\rho_\tau$ is the correlation coefficient of the time series of process $X$ and a version of itself shifted by $\tau$ samples $X_\tau$. If the autoregressive model replicates the linear properties of the analyzed data accurately then the sequence $I_{\mathrm{lin}}(X; X_\tau)$ for $\tau \in \{1, 2, ..., \tau_{\max}\}$ will agree with the corresponding linear regularity from the surrogates. Clearly the maximum lag $\tau_{\max}$ for which the shapes of the linear redundancy sequence coincide with the surrogates must be limited as the autoregressive model is only an approximation of the underlying generating system. For further analysis a $\tau_{\max}$ should be selected such that linear regularity is well matched between the data and the surrogate set. This must be confirmed by visual analysis of the sequences. A quantitative test of the agreement of the sequences is also a part of the method. However this is a supplementary test and is not a substitute for visual examination.

The nonlinear structure is captured by nonlinear regularity [11] which is defined analogically to linear regularity. Nonlinear regularity is the mutual information between a time series and its shifted version. In this work equiquantal binning is first applied to assign the time series samples to a discrete set of bins $\xi \in \Xi$. Mutual information may then be estimated as

$$I(X;X_\tau) = \sum_{\xi_1,\xi_2\in\Xi} p(\xi_1,\xi_2)\log\frac{p(\xi_1,\xi_2)}{p(\xi_1)p(\xi_2)}, \tag{4}$$

where $p(\xi_1)$ is the probability with which the symbol $\xi_1$ appears in the time series and $p(\xi_1,\xi_2)$ is the estimated probability that symbols $\xi_1$ and $\xi_2$ occur at the same point in the original and shifted version of the time series.

A function that estimates the similarity of two sequences is necessary to compare the linear and nonlinear structures of time series. A signed version of the $l_2$ metric

$$l_2^\pm(x(n),y(n)) = \frac{1}{\tau_{\max}}\sum_{i=1}^{\tau_{\max}} \mathrm{sgn}(x(i)-y(i))(x(i)-y(i))^2, \tag{5}$$

where $\tau_{\max}$ is the maximum lag, is used to quantitatively estimate how much two sequences match. The sign of $l_2^\pm(\cdot,\cdot)$ is positive if the first sequence lies mainly above the second sequence and negative if the opposite is true. If the points of the sequences are close together then the absolute value of the function is close to zero. If the second sequence $y(n)$ is fixed, then the function has only one free parameter $x(n)$ and computes how close the given sequence is to the reference sequence $y(n)$.

The method proceeds by performing two hypothesis tests. The first test checks if the linear structure of the surrogates matches that of the data and the second does the same for the nonlinear structure. Each test is prepared in an identical fashion: the regularities are computed for lags $\tau \in \{1,2,...,\tau_{\max}\}$ for the data time series and for a chosen number of surrogate time series. A reference sequence $m(n)$ is constructed by averaging all the regularity sequences from the surrogate time series. This reference sequence is set as the second argument of (5). A set of indices may now be computed using the function $l_2^\pm(\cdot,m(n))$ for each regularity sequence of the surrogate time series and for the data. Note that the above is done separately for linear and nonlinear regularities.

In the test for the match of linear structures, a two sided hypothesis test is constructed which will indicate if the index $l_2^\pm(x(n),m(n))$ computed on the data significantly deviates from the distribution of the same index on the surrogates. For the test at a nominal significance level $\alpha$ it is checked if

$$\begin{aligned} &l_2^\pm(x(n),m(n)) < q_{\frac{\alpha}{2}} \qquad \text{or} \\ &l_2^\pm(x(n),m(n)) > q_{1-\frac{\alpha}{2}}, \end{aligned} \tag{6}$$

where $x(n)$ is the linear regularity sequence of the original data and $q_\beta$ is the $\beta$ quantile of the distribution of $l_2^\pm(\cdot, m(n))$ estimated from the the surrogate linear regularity sequences. A two-sided test ensures that the linear regularity of the data does not significantly deviate in either direction, above or below, from the mean of the linear regularity sequence of the surrogate time series.

The purpose of the nonlinear structure test is to verify whether the nonlinear regularity sequence computed from the original data is significantly greater than the mean nonlinear regularity sequence computed from the surrogate time series. The test statistic is again the $l_2^\pm(\cdot, m(n))$, where the reference sequence $m(n)$ is the mean of the nonlinear regularity sequences from the surrogate time series. The test can be denoted as

$$l_2^\pm(x(n), m(n)) > q_{1-\alpha}, \tag{7}$$

where $x(n)$ is the nonlinear regularity sequence of the original data and $q_\beta$ is the $\beta$ quantile of the distribution of $l_2^\pm(\cdot, m(n))$ estimated from the the surrogate nonlinear regularity sequences. The test is one sided as only those time series the regularity of which is higher than that of the surrogates are of interest. These time series exhibit a higher amount of regularity than filtered noise.

In case of a broadband signal no constraint is placed on the extraction procedure of a candidate narrow-band mode. Simple bandpass filtering (Butterworth 4th order zero phase shift filtering) is used in the numerical example and in the analysis of experimentally obtained EEG data. Wavelet extraction, empirical mode decomposition (EMD) or singular-system analysis (SSA)-based decomposition can be applied equally well.

## III. RESULTS

In this section the method is first tested on a synthetic dataset and then the method is applied to sleep EEG from an entire night. The results are compared to the changes in relative power in the analyzed frequency band.

### A. Numerical example

In the numerical example it is shown how nonlinear oscillatory dynamics of the Lorenz system (which does not produce any peak in the power spectrum) is detected in a mix-

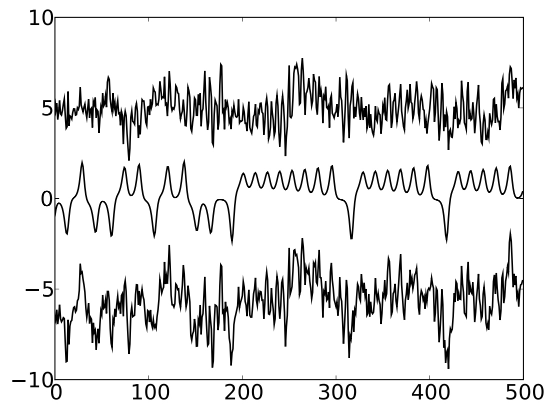


FIG. 1: A sample of the analyzed time series. The curves from top to bottom: the AR(5) process, the x-coordinate of the Lorenz oscillator and the mixed signal. Clearly the Lorenz signal is not introducing any clear oscillatory activity into the linear autoregressive process.

ture with the 5th order autoregressive process. The Lorenz system is a chaotic nonlinear dynamical system exhibiting complex behavior and given by the equations

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= x(\rho - z) - y\,, \\ \dot{z} &= xy - \beta z \end{aligned} \tag{8}$$

where $\sigma = 10$ is the Prandtl number, $\rho = 28$ is the Rayleigh number and $\beta = 8/3$. The differential equations were integrated with the 4th order Runge-Kutta scheme with a timestep of $\mathrm{dt} = 0.005$ and subsampled by a factor of 10. The resulting time series was normalized to zero mean and unit variance and added to a similarly normalized series of the AR(5) process

$$\begin{aligned} x(t) &= 0.4x(t-1) - 0.05x(t-2) - 0.1x(t-3) \\ &- 0.01x(t-4) + 0.6x(t-5) + 0.6\xi(t), \end{aligned} \tag{9}$$

where $\xi(t)$ is a white normally distributed noise input. A sample of the analyzed time series is shown in Fig. 1. The Lorenz system has not introduced a clear oscillatory activity into the autoregressive process. The spectrum of the time series estimated using the Welch periodogram method is shown in Fig. 2. Its examination confirms that no clear oscillatory peaks have arisen through the mixing process although significant power has been added by the Lorenz oscillator to lower frequencies. In the following analysis the low frequency region $(0.05 - 1.05)$ is tested and the dominant peak at frequency $1.2 - 2.2$ is tested as a control.

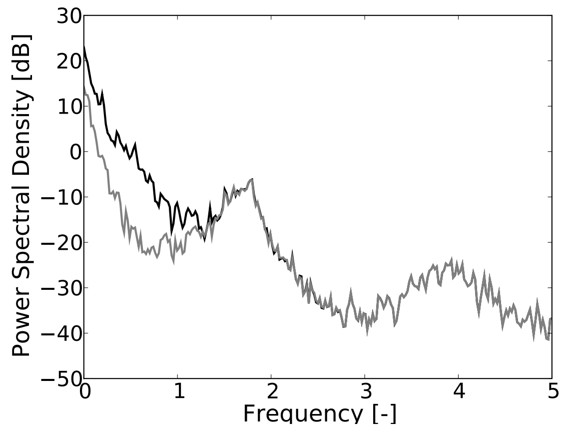


FIG. 2: Spectrum of AR(5) process (grey line) and of the AR(5) process with the Lorenz oscillator activity added (black line). The curves agree beyond frequency $\approx 1.5$. Although the Lorenz oscillator adds broadband power in the lower frequencies, no clear oscillatory peak can be identified in the spectrum.

The mode extraction was accomplished using simple bandpass filtering with a 2nd order Butterworth FIR filter (forward/backward strategy equivalent to a 4th order filter with zero phase shift). The surrogate dataset consisted of 200 realizations of the identified AR(5) process constructed by shuffling the residuals. For each maximum lag 200 repetitions of the experiment were performed and the number of experiment realizations in which the test positively detects the presence of a nonlinear component was expressed as the "detection rate" (the ratio of significant to total number of realizations).

The results for the detection of the Lorenz activity in the band $(0.05 - 1.05)$ are summarized in Fig. 3 which shows how the detection statistics vary with the number of points per period of the central frequency (0.55). The plot results indicate that a sampling rate of 5 points per period of the central frequency does not facilitate a sensitive detection. Better results are obtained for 10 or 20 points per periods.

As a control, the strong peak at frequency around 1.7 was also tested with the same bandwidth as the Lorenz activity. The edge frequencies of the Butterworth filters were set to $(1.2 - 2.2)$. Positive detections lie always under 1% for both linear and nonlinear redundancy up to a maximum lag of 60 (results not shown).

Although there is a strong peak in the power spectrum of the signal at the frequency 1.7 and none where the Lorenz activity is concentrated, the proposed method has been able to

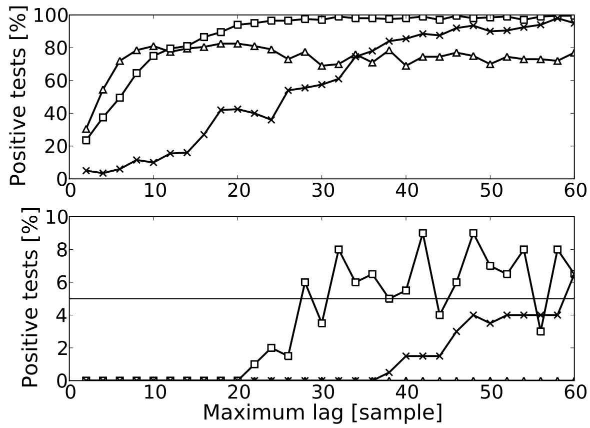


FIG. 3: Detection rates of nonlinear Lorenz activity in an AR(5) process (cf. Figs. 1, 2) obtained from 200 realizations for different sampling rates: triangles represent detections for 5 points per period, squares for 10 points per period and crosses for 20 points per period. In the bottom plot showing positive detections from the linear redundancy statistic indicates at which maximum lag the surrogates definitely cannot be used. Positive detections using nonlinear redundancy (top) show the true positives detected. At 5 points per period, the detection rate does not reach above 80%, for 10 points per period, the optimal lag seems to be 24 or 26 and for 20 points per period, maximum lags 50-58 seem to offer the highest sensitivity.

discriminate between these two cases perfectly by confirming the nonlinear activity in the band $0.05 - 1.05$, and rejecting nonlinear oscillations in the band tailored to the spectral peak $1.2 - 2.2$.

### B. Experimental data analysis

Two nights of sleep EEG were analyzed from one healthy subject to show how the method works on an experimentally obtained dataset. The EEG was measured within the framework of the European Commission funded Siesta program. The sampling frequency was 256 Hz and the measured signal was filtered by a high-pass filter with frequency 0.1Hz and a low pass filter with frequency of 75Hz. The recording was split into 30s segments which have been classified into sleep stages according to the standard Rechtschaffen & Kales [12] criteria.

In this work sigma band and alpha band activity were analyzed in the EEG obtained from the electrode C3 with a contralateral reference on the right mastoid. Both activities

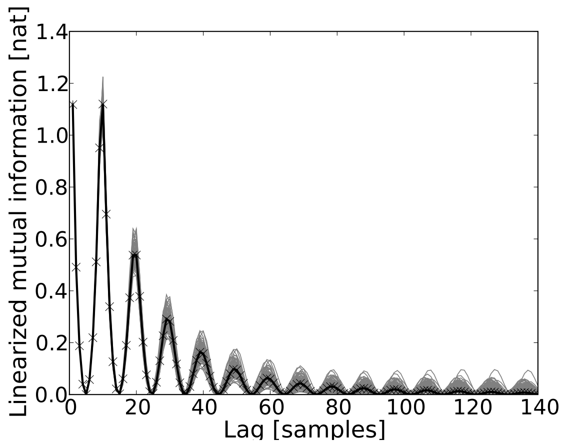


FIG. 4: Linear redundancy curves for data (thick black line) and for 50surrogates. The redundancy curve for the data segment matches the shape of the surrogate curves up to maximum lag of about 60 in this example.

were extracted using bandpass filtering in the same manner as in the numerical example: using a backward/forward filtering strategy with a 2nd order Butterworth filter to nullify the phase shift. The filter edge frequencies were set to 11Hz and 17Hz for the sigma band and to 8Hz and 12Hz for the alpha band.

The time series was subsampled so that $\approx 9$ points per period(at the center frequency 14Hz) were available for the analysis of sigma band activity. Fig. 4 shows the linear redundancies for a sample 30s segment of the EEG filtered in the sigma band. The surrogates replicate the linear structure accurately at least until lag 61 in the given example, which was selected as the maximum lag for the data analysis in this work. Upon the examination of the linear redundancy curves of several random segments it was ascertained that maximum lags between 60 and 100 were adequate. The following analysis has been repeated for maximum lags of 100 and the results were practically identical.

The detection results (number of segments with positively detected nonlinear oscillatory activity relative to the total number of segments of a particular sleep stage) for the first night are shown in Figs. 5 and 6. The nonlinear oscillatory activity in the sigma band was detected only in the 2nd sleep stage using the proposed method. The relative power of the sigma band seems to agree well with these results. In the alpha band, the proposed detection method has not indicated any nonlinear oscillation, whereas the relative power statistic clearly supports the claim that strong alpha band activity exists in the second sleep

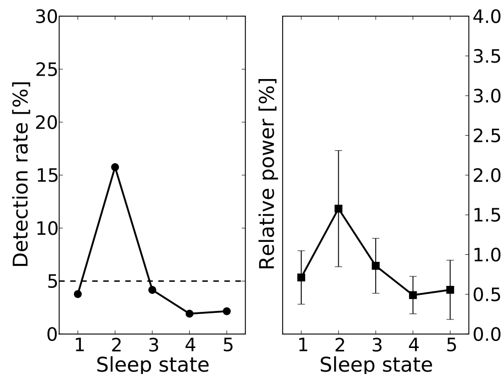


FIG. 5: Detection of nonlinear oscillations in the sigma band during the first recording night (left). The proposed method indicates the existence of nonlinear oscillations in the sigma band in the second sleep stage exclusively. Relative power of the sigma band (right) suggests a similar relationship in this example.

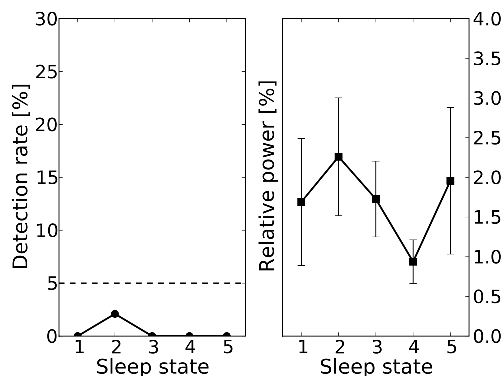


FIG. 6: Detection of nonlinear oscillations in the alpha band during the first recording night (left). The proposed method does not suggest any nonlinear oscillation in the alpha band exists during any of the sleep cycles. The relative power (right) however suggests that alpha activity is increased in sleep stage 2 and in REM sleep.

stage and in REM sleep (stage 5). Figs. 7 and 8 show the results for the second analyzed night of the same person. The proposed method has identified clear nonlinear oscillations in the sigma band in the second sleep stage. Additionally in the first sleep stage, some segments have been marked as containing nonlinear sigma band activity. The portion of significant windows is very low ($\approx 7.5\%$) compared to the nominal significance level of the test (5%). The detected segments could be a result of a statistical fluctuation or could indicate that

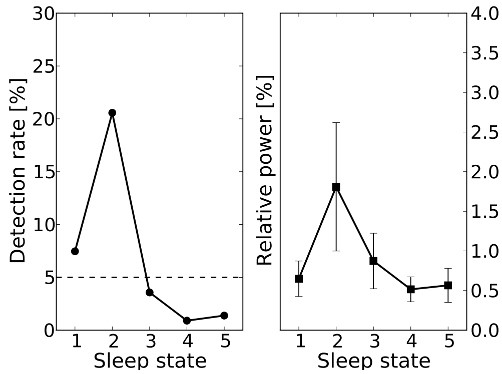


FIG. 7: Detection of nonlinear oscillations in the sigma band during the second recording night (left) and related relative power in the sigma band (right).

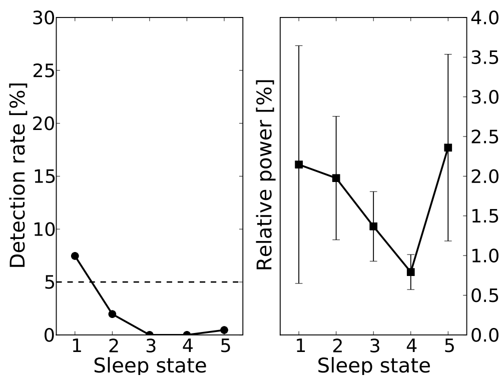


FIG. 8: Detection of nonlinear oscillations in the alpha band during the second recording night (left); and related relative power in the alpha band (right).

a small amount of segments contain low amplitude sigma spindles not visually perceptible in the broadband signal. The relative power plot seems to indicate that sigma activity should be expected in the second sleep stage. For the alpha band the previous situation is reiterated: the proposed method does not give any indication of nonlinear oscillations in the alpha band in any sleep stage but relative power statistics indicate a proliferation of alpha band activity in multiple stages (sleep stage 1, 2 and REM sleep).

## IV. DISCUSSION AND CONCLUSION

The proposed method attempts to identify consistent nonlinear oscillatory activity inside a part of a broadband signal. Frequently parts of broadband signal are extracted using several available methods such as bandpass filtering, wavelet convolution, EMD or SSA decomposition. The focus of this work is a method to statistically test if such an extracted mode can be assumed to have been generated by a nonlinear process. This is important because even filtered noise seems to have an oscillatory character and it is often misleading to suppose that the narrow-band signal is a result of an oscillatory activity generated by some underlying dynamics. The test is constructed using the method of surrogate time series which are generated using an autoregressive model fit to the data. A visual examination and a linear redundancy index are employed to verify whether the surrogate time series match the linear structure of the original data sufficiently well and under which conditions such as sampling rate (points per period) and maximum lag. This information is then used to construct a test for nonlinearity for a particular mode. If the nonlinear redundancy index can be used to reject the hypothesis that the generating system is linear, then it is inferred that a nonlinear process was involved in the generation of the analyzed activity and the activity is consistent inside some analyzed time segment.

The motivation and purpose of the presented method is fundamentally different from previously introduced method to detect particular activity types. The method introduced by Olbrich and Achermann [13, 14] analyzes the shape of the broadband EEG signal (without narrow-band filtering) and identifies short-lived activity by fitting an autoregressive model to a short window and analyzing the model properties (frequency and damping). This is an accurate determination of the existence of oscillatory activity. However the type of oscillatory activity is not the main issue. Additionally short-lived activity such as clear sleep spindles can be detected by the method.

Another approach advocated by Chavez et al. [15] is aimed at testing whether the instantaneous phase extracted from a mode satisfies the conditions that are assumed to hold for the phase. The authors use thresholds to determine whether the variations of amplitude are slow enough with respect to the change of the phase. This method examines the inherent variability in the phase and amplitude and is thus another approach different from both that of Olbrich et al. and of the proposed method.

The method suggested in this work focuses on activity that has a longer duration but may be difficult to detect without prior extraction but can be nevertheless attributed to a source with nonlinear dynamics. A positive detection using our method supports further analysis using phase dynamics. A negative statement can help identify signals where it might be futile to attempt to detect synchronization or directional influence using the phase dynamics approach for lack of acceptable nonlinear oscillatory activity. In such cases the standard linear coherence analysis is preferred.

The method has been shown to work on a numerical example which mixed the x-component of the Lorenz oscillator with a 5th order autoregressive process. The method has detected the nonlinear signal in the low frequency range. On the other hand, a clear peak in the spectrum of the signal which arose by a filtering of white noise (by the autoregressive process itself) has not been identified as having nonlinear content. This experiment has shown that rather than being sensitive to the shape of the signal, the method is sensitive to the type of dynamics that generated the signal.

Experimental data in the form of EEG from a sleeping subject is analyzed and the findings are shown to conform to the expected results based on the Rechtschaffen & Kales [12] criteria for sleep stage classification. The sleep stage 2, characterized by the sigma spindles activity has been correctly identified as containing nonlinear oscillatory components. Relative power also indicates a similar tendency for the sigma activity. In the alpha band, the proposed method gives results consistent with the Rechtschaffen and Kales criteria stating no oscillatory (spindle) phenomena in the alpha band during sleep, although the relative power indicates alpha band activity in three of the five sleep stages.

The proposed method is promising for identification of nonlinear oscillatory processes embedded or hidden in a broadband noisy background. Such problems frequently arise in neurophysiology when analyzing signals recorded on various levels of organization of brain tissues, as well as in other fields when possibly interacting and synchronizing oscillations emerge in complex dynamical processes.

## V. ACKNOWLEDGMENTS

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