Windsurfer, Fully Probabilistic Control Design and the Jobcontrol Package

Ladislav Jirsa & Ludvík Tesař
Institute of Information Theory and Automation,
Academy of Sciences of the Czech Republic,
E-mail: jirsa@utia.cas.cz

Fully probabilistic control design (FPD)

Fully probabilistic control design is a control design based on a description of a control closed loop behaviour using probabilistic models.

Formal conventions

These conventions are used:

- u_t — vector of system inputs in time t,
- y_t — vector of system outputs in time t,
- d_t — data vector, $d_t = (u_t, y_t)$,
- $d(t)$ — data vector including the past, i.e. $d(t) = (d_t, d_{t-1}, ..., d_{t-\partial})$, where ∂ is some time horizon.

Control scheme

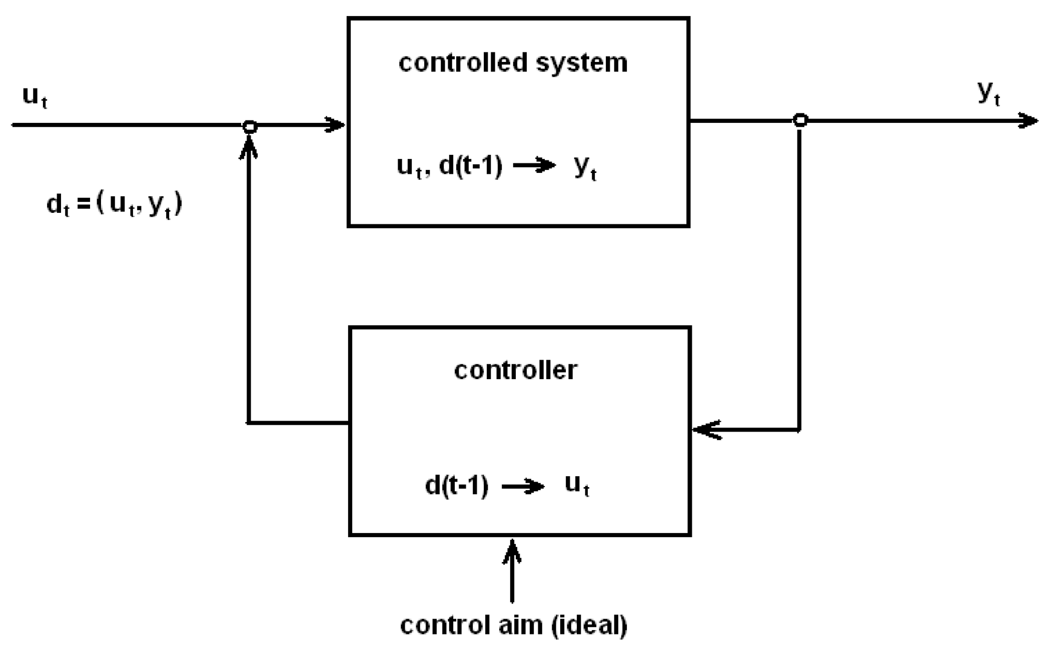

Figure 1: Control scheme

Probabilistic description of the system

Joint probability density function (pdf) of the system and the controller is

$$f(d(T)) \equiv f(u(T), y(T)) = \prod_{t=1}^{T} \underbrace{f(d_t|d(t-1))}_{\text{closed loop model}}$$

$$= \prod_{t=1}^{T} \underbrace{f(y_t|u_t, d(t-1))}_{\text{system model}} \underbrace{f(u_t|d(t-1))}_{\text{controller}}$$

Optimal control

Assume the system to have a finite-dimensional observable state ϕ_{t-1} and define $\psi_t \equiv [u_t', \phi_{t-1}']$

$$f(y_t|u_t, d(t-1)) = f(y_t|\psi_t).$$

The optimal controller $\left\{{}^Of(u_t|d(t-1))\right\}_{t=1}^{T}$ is obtained as a minimizer of *Kullback-Leibler divergence* $\mathcal{D}$ of actual closed loop behaviour $f(d(T))$ and *ideal* closed loop behaviour ${}^If(d(T))$ over all controllers

$$ {}^Of(u_t|\phi_{t-1}) = \arg \min_{\{f(u_t|\phi_{t-1})\}_{t=1}^{T}} \mathcal{D}(f(d(T))||{}^If(d(T))) \qquad (1)$$

where

$$\mathcal{D}(f(x)||g(x)) = \int f(x) \, \ln \frac{f(x)}{g(x)} \, dx.$$

The optimal controller is then expressed as

$$ {}^Of(u_t|\phi_{t-1}) = \frac{{}^If(u_t|\phi_{t-1} \, \exp[-\omega(\psi_t)]}{\underbrace{\int {}^If(u_t|\phi_{t-1} \, \exp[-\omega(\psi_t)] du_t}_{\gamma\phi_{t-1}}},$$

$$\omega(\psi_t) = \int f(y_t|\psi_t) \, \ln \left(\frac{f(y_t|\psi_t)}{\gamma(\phi_t) \, {}^If(y_t|\psi_t)} \right)$$

The question is

How to obtain the ideal ${}^I\mathbf{f}(\mathbf{d}(\mathbf{T}))$?

Windsurfer's approach to elicitation of control aim (ideal)

The *windsurfer's approach* suggests a conservative construction of the ideal pdf.

What the windsurfer-beginner is doing?

windsurfer's ideal: to sail across a lake

his aims:

1. to climb on a surf and to keep there
2. to pull up the sail (and not to fall)
3. to learn manipulation with the sail
4. to move forward, do stop
5. etc.

He has *realistic* aims according to his *knowledge* and *observations* and modifies them.

How is this approach reflected in theory?

The conservative construction of the ideal pdf is based on

- given setpoints and input/output constraints,
- modelling of the observed closed loop behaviour by a "rough" model ${}^Rf$,
- gradual modifications of the rough model according to the closed loop behaviour in order to express better the control aims.

Theory

Generally, inputs and outputs can be separated in two parts:

- interesting part ${}^iu_t$, ${}^iy_t$ included in the probabilistic description and FPD,
- non-interesting part ${}^nu_t$, ${}^ny_t$ distributions of which are not considered.

The latter part can be potentially void.

Verbal description of the control aim

- keep interesting outputs ${}^iy_t$ as close as possible to the given setpoints ${}^sy_t$,
- keep all inputs u_t within the given intervals (constraints) with a high probability.

Formal inference

Main idea is an approximation of the ideal ${}^If(d(T))$ by a *rough* model ${}^Rf(d(T))$ and its gradual refinement. The rough model describes the closed loop behaviours in the neighbourhood of the actual one.

1. Choose a set of ideals ${}^If(d(T)) = \prod_{t \in t^*} {}^If(d_t|\phi_{t-1})$, where the factors ${}^If(d_t|\phi_{t-1})$ are time-invariant.
2. Using a chain rule, decompose the factor ${}^If(d_t|\phi_{t-1})$ in time t:
$$ {}^If(d_t|\phi_{t-1}) = {}^If(u_t|y_t, \phi_{t-1}) \, {}^If({}^ny_t|{}^iy_t, \phi_{t-1}) \, {}^If({}^iy_t|\phi_{t-1}).$$
3. The term ${}^If({}^ny_t|{}^iy_t, \phi_{t-1})$ concerns non-interesting outputs, therefore it is conservatively substituted by its rough model counterpart,
$$ {}^If(d_t|\phi_{t-1}) = {}^If(u_t|y_t, \phi_{t-1}) \, {}^Rf({}^ny_t|{}^iy_t, \phi_{t-1}) \, {}^If({}^iy_t|\phi_{t-1}).$$
4. Put the maximum of the rough factor ${}^Rf({}^iy_t|\phi_{t-1})$ on the setpoints ${}^sy_t$ by finding the value of "optimal" input ${}^Ou_t$, i.e.
$$ {}^Rf({}^sy_t|{}^Ou_t, \phi_{t-1}) \geq {}^Rf({}^iy_t|{}^Ou_t, \phi_{t-1}), \quad \forall {}^iy_t \in {}^iy_t^*.$$
Such a factor is called *optimistic* ${}^{OI}f({}^iy_t|\phi_{t-1}) \equiv {}^Rf({}^iy_t|{}^Ou_t, \phi_{t-1})$.
5. Substitute the term ${}^I({}^iy_t|\phi_{t-1})$ by the optimistic factor, i.e.
$$ {}^If(d_t|\phi_{t-1}) = {}^If(u_t|y_t, \phi_{t-1}) \, {}^Rf({}^ny_t|{}^iy_t, \phi_{t-1}) \, {}^{OI}f({}^iy_t|\phi_{t-1}). \qquad (2)$$
6. Minimize $\mathcal{D}({}^Rf(d_t|\phi_{t-1})||{}^If(d_t|\phi_{t-1}))$, where ${}^If(d_t|\phi_{t-1})$ is from (??), over the first factor ${}^If(u_t|y_t, \phi_{t-1})$, to get the *optimistic ideal*
$$ {}^{OI}f(d_t|\phi_{t-1}) = {}^Rf(u_t|y_t, \phi_{t-1}) \, {}^Rf({}^ny_t|{}^iy_t, \phi_{t-1}) \, {}^{OI}f({}^iy_t|\phi_{t-1}). \qquad (3)$$

The optimistic ideal

The decomposition (??) can be expressed as

$$ {}^{OI}f(d_t|\phi_{t-1}) = {}^Rf(d_t|\phi_{t-1}) \, \frac{{}^{OI}f({}^iy_t|\phi_{t-1})}{{}^Rf({}^iy_t|\phi_{t-1})}. \qquad (4)$$

Verbally, opimistic ideal (K-L optimized approximation of unknown ideal) is obtained by replacing the rough factor ${}^Rf({}^iy_t|\phi_{t-1})$ by the optimistic factor ${}^{OI}f({}^iy_t|\phi_{t-1})$.

Algorithm

1. Initial mode
   - initialize the control loop, estimation of the system model, estimation of the rough closed loop model,
   - select the type of adaptive control strategy.
2. Iterative mode
   - evaluate the rough approximation to the ideal closed loop model
3. Recursive mode
   - collect data and update estimates of $f(d_t|\phi_{t-1})$ and ${}^Rf(d_t|\phi_{t-1})$,
   - construct FPD for the current $f(d_t|\phi_{t-1})$ and ${}^{OI}f(d_t|\phi_{t-1})$,
   - generate and apply the system input,
   - after some iterations, go to beginning of the Iterative mode.

Experiments

A system with transfer function

$$\begin{bmatrix} \frac{1}{s+0.8} & \frac{0.5}{s+1} \\ \frac{0.5}{s+1.2} & \frac{1}{s+1.4} \end{bmatrix} \text{ was mapped on } S(z) = \begin{bmatrix} \frac{0.0961}{z-0.9231} & \frac{0.04758}{z-0.9048} \\ \frac{0.04712}{z-0.8869} & \frac{0.09332}{z-0.8694} \end{bmatrix}$$

and sampled with a period 0.1 s, added white normal noise with variance 0.001. Inputs are limited in the interval $[-5, 5]$ with probability 0.9. Outputs are controlled to zero. The second order AR rough model was used. Initial learning data were obtained in loop closed by the proportional controller $u_t = -y_{t-1}$. After 300 iterations, a new rough model was evaluated.

WS iterations	channel	mean	variance	min	max
1	y_1	0.0603	**0.4998**	-0.7612	0.8825
	y_2	-0.0490	**0.2499**	-0.4584	0.3604
	u_1	-0.0314	0.3109	-0.1234	0.1075
	u_2	0.0075	0.1870	-0.3000	0.3151
2	y_1	-0.2548	0.4014	-0.9150	0.4054
	y_2	0.0943	0.3387	-0.4628	0.6514
	u_1	0.1814	0.2340	-0.2034	0.5663
	u_2	-0.1073	0.2538	-0.5247	0.3102
4	y_1	-0.3184	0.4586	-1.0729	0.4360
	y_2	-0.0401	0.3638	-0.6385	0.5582
	u_1	0.2033	0.2708	-0.2422	0.6487
	u_2	-0.0072	0.2781	-0.4647	0.4502
6	y_1	-0.0630	**0.3805**	-0.6888	0.5629
	y_2	0.1433	**0.1707**	-0.1375	0.4241
	u_1	0.0775	0.2206	-0.2854	0.4405
	u_2	-0.1419	0.1333	-0.3612	0.0775

Results for off-line FPD using Designer package:

channel	mean	variance	min	max
y_1	-0.1218	0.5639	-1.5906	0.9855
y_2	0.0113	0.0705	-0.2021	0.2558
u_1	0.0246	0.1661	-0.5823	0.4713
u_2	0.0280	0.1913	-0.5164	0.5173

Jobcontrol package

The purpose of software package Jobcontrol is to integrate algorithms present in Mixtools and Designer Matlab packages to simplify its use for end user.

Jobcontrol package splits the task of model identification and process control into logical steps. It starts by data filtration step. It is followed by four modelling steps, which are "prior information" step, model structure identification step, model parameters estimation step, and model validation step. After it, control design is performed in these steps: user ideal construction, design, and controller verification step.

Applications

The Jobcontrol system was applied in several real applications: caoline production line identification and control, control of the biotechnology process for biodegradable polymers production, diagnostics in lymphoscintigraphy and application for stock market futures trading.

Acknowledgements

The work on jobcontrol package was supported by DAR 1M0572.