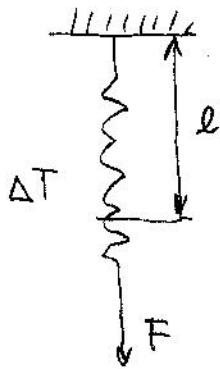


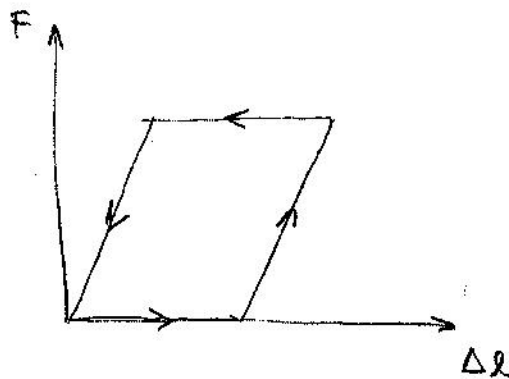
Př: Hösche



α = součinitel tepelné roztažnosti

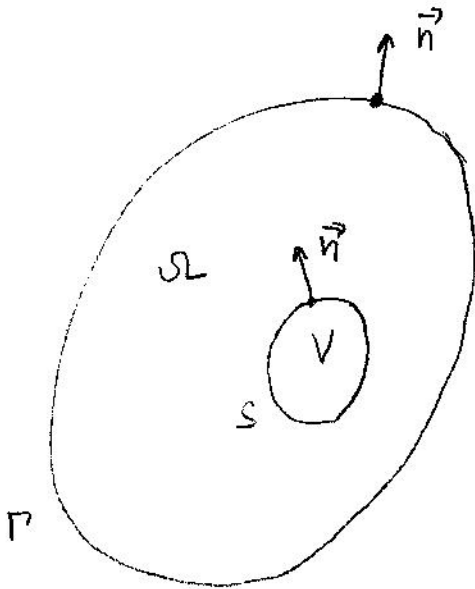
C = součinitel tepelné kapacity

k = tuhost



$$\Delta W = -F \Delta l \Delta T \quad \text{tepelná práce}$$

Perpetuum mobile?



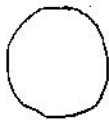
$\Omega(V)$: okružná smíšená oblast

$\Gamma(S)$: Lipschitz

$\bar{\Omega} = \Omega \cup \Gamma$, $\bar{V} = V \cup S$

\vec{n} mají normálu $\|\vec{n}\| = 1$

Pr



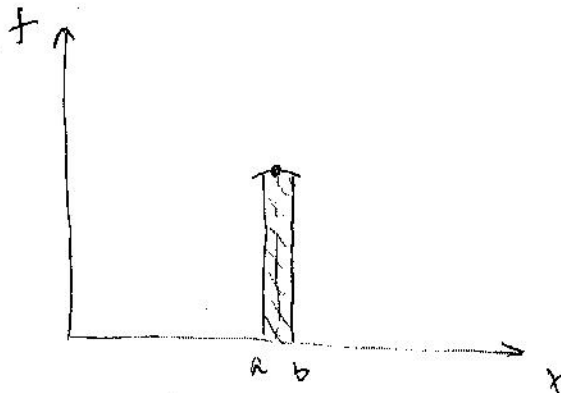
trhlinka (nemí L.)

Pr

$\int_0^{2\pi} \sin x \, dx = 0$

$\forall a, L \in \mathbb{R}, f \in C^0(a, b)$

$\int_a^b f(x) \, dx = 0 \iff f(x) \equiv 0$



Platí i pro 3D oblasti.

Green's wile

$$u \in C^1(\bar{V}) \quad \int_V \frac{\partial u}{\partial x_i} dV = \int_S u n_i dS$$

Gauss's wile (divergence theorem)

$$\begin{aligned} \int_V \operatorname{div} \vec{N} dV &= \int_V \frac{\partial N_i}{\partial x_i} dV = \int_V \frac{\partial N_1}{\partial x_1} dV + \int_V \frac{\partial N_2}{\partial x_2} dV + \int_V \frac{\partial N_3}{\partial x_3} dV = \\ &= \int_S N_1 n_1 dS + \int_S N_2 n_2 dS + \int_S N_3 n_3 dS = \int_S N_i n_i dS = \int_S \vec{N} \cdot \vec{n} dS \end{aligned}$$

Definition leslet

unitic' energie $U(V) = \int_V u dV$ u [J/m³]

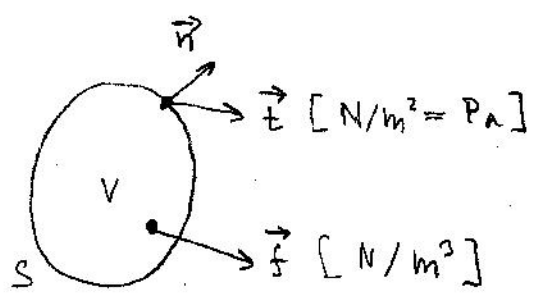
Helmholtz $\Lambda(V) = \int_V \psi dV$ ψ [J/m³]

entropije $S(V) = \int_V \eta dV$ η [J/m³K]

lj'kon $\dot{W}(V) = \int_V \dot{w} dV$ \dot{w} [W/m³]

lj'pelenj' lj'kon $\dot{Q}(V) = \int_V \dot{q} dV$ \dot{q} [W/m³]

Vy'lovu vnitřního tlaku



$$t = \sigma n \quad t_i = \sigma_{ij} n_j$$

$$\sigma_{ij,i} + f_i = 0$$

$$\sigma^T = \sigma \quad \sigma_{ij} = \sigma_{ji}$$

$$\dot{W}(V) \stackrel{def}{=} \int_V \vec{f} \cdot \vec{n} \, dV + \int_S \vec{t} \cdot \vec{n} \, dS$$

$$\dot{W}(V) = \int_V f_i \dot{u}_i \, dV + \int_S t_i \dot{u}_i \, dS$$

$$\int_S t_i \dot{u}_i \, dS = \int_S \sigma_{ij} n_j \dot{u}_i \, dS = \int_V (\sigma_{ij} \dot{u}_i)_{,j} \, dV =$$

$$= \int_V \sigma_{i,jj} \dot{u}_i \, dV + \int_V \sigma_{ij} \dot{u}_{i,j} \, dV$$

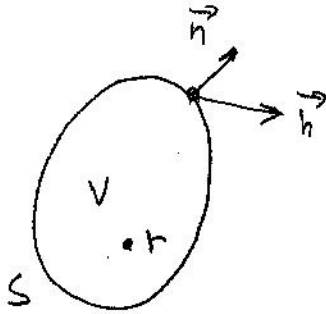
$$\dot{W}(V) = \int_V \underbrace{(f_i + \sigma_{i,jj})}_{0} \dot{u}_i \, dV + \int_V \sigma_{ij} \dot{u}_{i,j} \, dV$$

$$\sigma_{ij} \dot{u}_{i,j} = \frac{1}{2} \sigma_{ij} \dot{u}_{i,j} + \frac{1}{2} \sigma_{ji} \dot{u}_{i,j} = \frac{1}{2} \sigma_{ij} \dot{u}_{i,j} + \frac{1}{2} \sigma_{ij} \dot{u}_{j,i} =$$

$$= \sigma_{ij} \frac{1}{2} (\dot{u}_{i,j} + \dot{u}_{j,i}) = \sigma_{ij} \frac{1}{2} (\dot{u}_{i,j} + \dot{u}_{j,i}) = \sigma_{ij} \dot{\epsilon}_{ij}$$

$$\dot{W}(V) = \int_V \sigma_{ij} \dot{\epsilon}_{ij} \, dV = \int_V \dot{w} \, dV$$

$$\dot{w} = \sigma_{ij} \dot{\epsilon}_{ij} \quad [W/m^3]$$

Thermally v' Law

$$\vec{h} \text{ [W/m}^2\text{]}$$

$$r \text{ [W/m}^3\text{]}$$

$$\dot{Q}(V) \stackrel{\text{def}}{=} \int_V r dV - \int_S \vec{h} \cdot \vec{n} dS$$

$$\int_S \vec{h} \cdot \vec{n} dS = \int_V \text{div } \vec{h} dV$$

$$\dot{Q}(V) = \int_V (r - \text{div } \vec{h}) dV = \int_V \dot{q} dV$$

$$\dot{q} = r - \text{div } \vec{h} \text{ [W/m}^3\text{]}$$

1. Zellenherkunft

⑥

$$V: \Delta Q + \Delta W = u_2 - u_1$$

$$\dot{Q} + \dot{W} = \dot{U}$$

Libbmanni produkti V:

$$\int_V (\dot{q} + \dot{w} - \dot{u}) dV = 0$$

$$\boxed{\dot{q} + \dot{w} = \dot{u}}$$

2. zákon termodinamiky

(7)

$\exists \eta$ [J/m^3K] entropie = stavová veličina

$$S(V) = \int_V \eta dV$$

Clausius - Duhem

$$\dot{S}(V) \geq \int_V \frac{\dot{r}}{T} dV - \int_S \frac{\vec{h} \cdot \vec{n}}{T} dS$$

Průběh:

Funkce η je určena nepochybně pro konkrétní mat. model.

Kalkulace pro CD - nerovnost

$$\int_V \dot{\eta} dV \geq \int_V \frac{\dot{r}}{T} dV - \int_V \operatorname{div} \left(\frac{\vec{h}}{T} \right) dV$$

$$\dot{\eta} \geq \frac{\dot{r}}{T} - \operatorname{div} \left(\frac{\vec{h}}{T} \right)$$

$$\operatorname{div} \left(\frac{\vec{h}}{T} \right) = \left(\frac{h_i}{T} \right)_{,i} = \frac{h_{i,i} T - h_i T_{,i}}{T^2} = \frac{\operatorname{div} \vec{h}}{T} - \frac{\vec{h} \cdot \operatorname{grad} T}{T^2}$$

$$\dot{\eta} \geq \frac{\dot{r}}{T} - \frac{\operatorname{div} \vec{h}}{T} + \frac{\vec{h} \cdot \operatorname{grad} T}{T^2}$$

$$\dot{\eta} \geq \frac{\dot{q}}{T} + \frac{1}{T^2} \vec{h} \cdot \operatorname{grad} T$$

Dissipatívny nerovnosť

8

Idea: q je makroskopická miera a i

CD + 1. z. $T\dot{q} \geq i - \dot{w} + \frac{1}{T} \vec{h} \cdot \text{grad } T$

$$\dot{q}T = (\eta T)' - \eta \dot{T} \quad (\text{stručne' veličiny})$$

$$-\eta \dot{T} + \dot{w} - \frac{1}{T} \vec{h} \cdot \text{grad } T = i - (\eta T)' = \underbrace{(w - \eta T)'}_{\Psi \text{ [J/m}^2\text{]}}$$

Legendre: $\Psi \stackrel{\text{def}}{=} w - \eta T$ Helmholtz (vlastná energia)

$$-\eta \dot{T} + \dot{w} - \frac{1}{T} \vec{h} \cdot \text{grad } T \geq \dot{\Psi}$$

Prípad zotavenia: $\dot{T} = 0, \text{ grad } T = 0 \Rightarrow \dot{w} \geq \dot{\Psi}$

$$\left. \begin{array}{l} \text{zahrievanie: } \Delta W_z \geq \Psi_2 - \Psi_1 \\ \text{ochladenie: } \Delta W_o \geq \Psi_1 - \Psi_2 \end{array} \right\} \Delta W_z \geq \Psi_2 - \Psi_1 \geq -\Delta W_o$$

předpoklad stability mechaniky: $\Delta W_z > 0, \Delta W_o < 0$

$$|\Delta W_z| \geq \Psi_2 - \Psi_1 \geq |\Delta W_o| \Rightarrow \text{def. energie}$$

První: $\Delta q + \Psi_2 - \Psi_1 = u_2 - u_1, \Delta q \approx 10 \Delta W$ (Pozn: $\Delta q + \Delta W = \Delta u$)

První: $-\eta \dot{T}$ zvyšuje bezpečnosť chaos. Nový grad T je zdrojom významného katalyzátora.

Termodinamika

9

$$\varepsilon_{ij}, T: \sigma_{ij}(\varepsilon_{ij}, T), \psi(\varepsilon_{ij}, T), \eta(\varepsilon_{ij}, T), \mu(\varepsilon_{ij}, T)$$

d'iss. nevast:

$$-\eta \dot{T} + \sigma_{ij} \dot{\varepsilon}_{ij} - \frac{1}{T} \vec{h} \cdot \text{grad} T \geq \frac{\partial \psi}{\partial \varepsilon_{ij}} \dot{\varepsilon}_{ij} + \frac{\partial \psi}{\partial T} \dot{T}$$

$$-\left(\eta + \frac{\partial \psi}{\partial T}\right) \dot{T} + \left(\sigma_{ij} - \frac{\partial \psi}{\partial \varepsilon_{ij}}\right) \dot{\varepsilon}_{ij} - \frac{1}{T} \vec{h} \cdot \text{grad} T \geq 0$$

$\dot{T} = 0, \text{grad} T = 0$ (pomalé taléžón'ni)

$$\underbrace{\left(\sigma_{ij} - \frac{\partial \psi}{\partial \varepsilon_{ij}}\right)}_0 \dot{\varepsilon}_{ij} \geq 0$$

Dúhosa $A_{ij} \dot{\varepsilon}_{ij} \geq 0, \quad \dot{\varepsilon}_{ij} = -\alpha A_{ij}, \quad \text{kele } \alpha > 0$

$$-\alpha A_{ij} A_{ij} \geq 0 \Rightarrow A_{ij} = 0$$

$$\boxed{\sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}}}$$

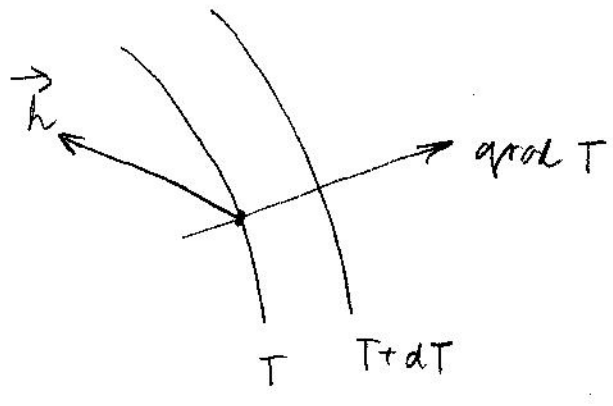
$$-\left(\eta + \frac{\partial \psi}{\partial T}\right) \dot{T} - \frac{1}{T} \vec{h} \cdot \text{grad} T \geq 0$$

$\text{grad} T = 0$ (spuvelikeé odličnó'ni válece)

$$-\left(\eta + \frac{\partial \psi}{\partial T}\right) \dot{T} \geq 0 \Rightarrow \boxed{\eta = -\frac{\partial \psi}{\partial T}}$$

$$-\frac{1}{T} \vec{h} \cdot \text{grad } T \geq 0 \Rightarrow \boxed{\vec{h} \cdot \text{grad } T \leq 0}$$

Fourierova rovnost:



Pr $\vec{q} = -\lambda \text{grad } T$, $\lambda [W/mK]$, $\lambda > 0$

λ může být i + def. matice (anisotropní mat.)

unitární rovnice

$$\dot{u} = (\psi + \eta T)' = \underbrace{\frac{\partial \psi}{\partial \epsilon_{ij}} \dot{\epsilon}_{ij}}_{\sigma_{ij}} + \underbrace{\frac{\partial \psi}{\partial T} \dot{T}}_{-\eta} + \dot{\eta} T + \underline{\eta \dot{T}}$$

$$\left. \begin{aligned} \dot{u} &= \sigma_{ij} \dot{\epsilon}_{ij} + \dot{\eta} T = \dot{w} + \dot{\eta} T \\ \text{t.j. } \dot{u} &= \dot{w} + \dot{q} \end{aligned} \right\} \Rightarrow \dot{q} = \dot{\eta} T$$

$$\boxed{\dot{\eta} = \frac{\dot{q}}{T}}$$

definice entropie pro "vratné" ději

Pr $\dot{\eta} = \frac{\dot{q}}{T} \wedge \frac{1}{T^2} \vec{h} \cdot \text{grad } T \leq 0 \Rightarrow \dot{\eta} \geq \frac{\dot{q}}{T} + \frac{1}{T^2} \vec{h} \cdot \text{grad } T$

CD - rovnost

$$\exists \Psi(\epsilon_{ij}, T)$$

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \epsilon_{ij}}$$

$$p = - \frac{\partial \Psi}{\partial T}$$

$$\dot{m} = \frac{\dot{q}}{T}$$

$$\dot{q} = v - \text{div} \vec{h}$$

$$\vec{h} \cdot \text{grad} T \leq 0$$

Videmi' kopla

Fourierov zakon $\vec{h} = - \lambda \text{grad} T$

$$\dot{q} = v - \text{div} \vec{h} = v + \text{div}(\lambda \text{grad} T)$$

$$\dot{q} = T \dot{m} = T \frac{\partial \Psi}{\partial \epsilon_{ij}} \dot{\epsilon}_{ij} + T \frac{\partial \Psi}{\partial T} \dot{T}$$

$$C_E \stackrel{\text{def}}{=} T \frac{\partial \Psi}{\partial T}, \quad C_E = C_V \approx C_P \approx C \quad (\text{može se eksperimentalno})$$

$$v + \text{div}(\lambda \text{grad} T) = C_V \dot{T} + T \frac{\partial \Psi}{\partial \epsilon_{ij}} \dot{\epsilon}_{ij}$$

↑
termoelastični vektor

$$\lambda \text{ je funkcija } x, y, z \Rightarrow \text{div}(\lambda \text{grad} T) = \lambda \Delta T$$

$$\vec{f} = v - T \frac{\partial \Psi}{\partial \epsilon_{ij}} \dot{\epsilon}_{ij}$$

$$\lambda \Delta T = C_V \dot{T} - \vec{f}$$

$$\varepsilon = \frac{\sigma}{E} + \alpha \Delta T \quad \alpha [1/K], \quad \Delta T = T - T_0$$

$$\sigma = E(\varepsilon - \alpha \Delta T) \quad \text{DN - Zakon}$$

$$\alpha, E = \text{konst.}$$

$$G = \frac{\partial \Psi}{\partial \varepsilon} \Rightarrow \Psi = \frac{1}{2} E \varepsilon^2 - E \alpha \Delta T \varepsilon + f(T)$$

$$\eta = - \frac{\partial \Psi}{\partial T} = E \alpha \varepsilon - f'(T), \quad f' = \frac{df}{dT}$$

$$\begin{aligned} \eta &= \alpha E (\varepsilon - \alpha \Delta T) + \alpha^2 E \Delta T - f'(T) = \\ &= \alpha \sigma + q(T) \end{aligned}$$

$$\dot{q} = T \dot{\eta} = \alpha T \dot{\sigma} + T q'(T) \dot{T}, \quad q' = \frac{dq}{dT}$$

soniční tepelná kapacita $C_\sigma \stackrel{\text{def.}}{=} T q'(T)$ při konstantním úhyně. Lze doložit $C_\sigma = C_p$.

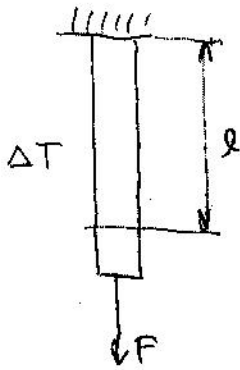
$$\dot{q} = \alpha T \dot{\sigma} + \dot{T}$$

Př: $T = \text{konst.} \quad \Delta q = \alpha T \Delta \sigma$

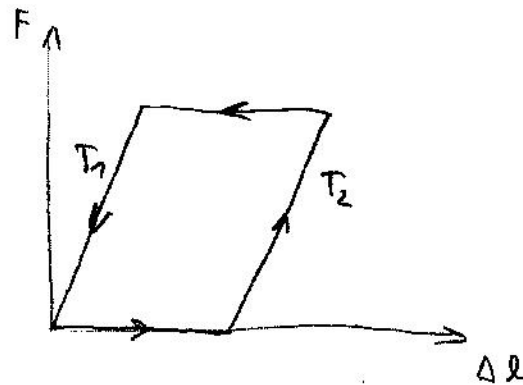
$$\Delta q \approx 10^{-5} \times 300 \times 10^8 = 3 \times 10^5 \text{ [J/m}^2\text{]}$$

$$\Delta W \approx \frac{1}{2} \frac{(10^8)^2}{2 \times 10^{11}} = 0.25 \times 10^5 \text{ [J/m}^2\text{]}$$

PF



Dáta: $E, \alpha, C_{\sigma} = C, A = \text{poširka}$



- 1) $\Delta W_1 = 0$ $\Delta Q_1 = C \Delta T A l$
- 2) $\Delta W_2 = \frac{1}{2} \frac{F^2 l}{EA}$ $\Delta Q_2 = \alpha T_2 \frac{F}{A} A l$
- 3) $\Delta W_3 = -F l \alpha \Delta T$ $\Delta Q_3 = -C \Delta T A l$
- 4) $\Delta W_4 = -\frac{1}{2} \frac{F^2 l}{EA}$ $\Delta Q_4 = -\alpha T_1 \frac{F}{A} A l$

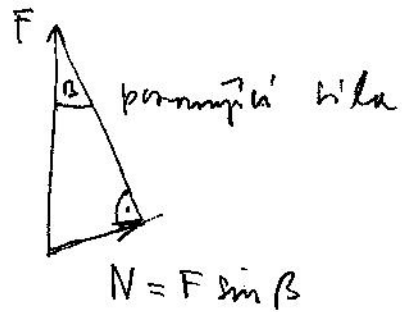
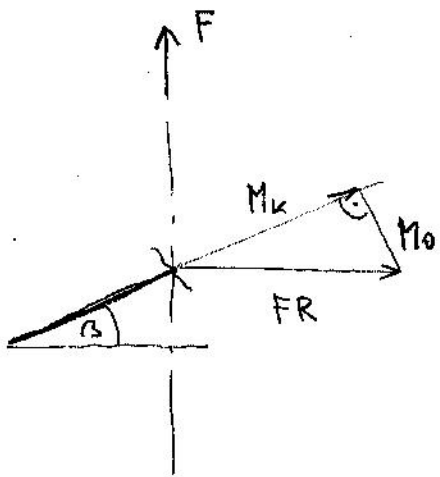
$$\Delta W = -F l \alpha \Delta T \quad \Delta Q = F l \alpha (T_2 - T_1)$$

$$\eta_{\text{Carnot}} = \frac{|\Delta W|}{\Delta Q_2} = \frac{F l \alpha \Delta T}{\alpha T_2 F l} = \frac{\Delta T}{T_2} = 1 - \frac{T_1}{T_2} \quad (\text{Carnot})$$

$$1 - \frac{T_1}{T_2} \approx 1 - \frac{300}{500} = 40\%$$

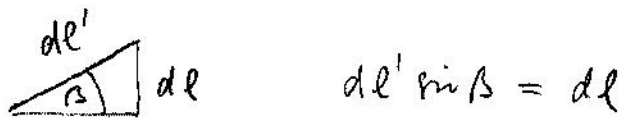
Prům. elektřiny, libeří motorů

Vincula' pinnata



$$\Delta Q = \alpha T \frac{F \sin \beta}{A} A l' = \alpha T F l' \sin \beta$$

l' = della dra'tu l = u'ita pinnata



integrare $\Delta Q = \alpha T F l$

Invariants ϵ

$X_{ii}, \epsilon_{ij}, \epsilon$

$X'_{ii}, \epsilon'_{ij}, \epsilon'$

def. invariance: $f(\epsilon_{ii}) = f(\epsilon'_{ii})$

Př $\text{tr}(\epsilon) = \epsilon_{11} + \epsilon_{22} + \epsilon_{33} = \epsilon_{ii} = \epsilon'_{ii} = \text{tr}(\epsilon')$

Definice $\epsilon^2 = \epsilon\epsilon$

$\epsilon^3 = \epsilon\epsilon\epsilon$

Pozn Matricemi matice matricem tenzorů charakter

$$\left. \begin{aligned}
 I_1 &= \text{tr}(\epsilon) \\
 I_2 &= \frac{1}{2} \text{tr}(\epsilon^2) \\
 I_3 &= \frac{1}{3} \text{tr}(\epsilon^3)
 \end{aligned} \right\} I_n = \frac{1}{n} \text{tr}(\epsilon^n)$$

Pozn: je 3 invariantů je 3 maximální

úloh čísla je invariantů

$\det|\epsilon - \lambda I| = 0$

$\det|\epsilon' - \lambda I| = 0$

$\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$

$\epsilon_1, \epsilon_2, \epsilon_3$ hlavní deformace

Charakteristická rovnice

$$\lambda^3 - \bar{I}_1 \lambda^2 + \bar{I}_2 \lambda - \bar{I}_3 = 0$$

$$\bar{I}_1 = I_1$$

$$\bar{I}_2 = \frac{1}{2} I_1^2 - I_2$$

$$\bar{I}_3 = I_3 - I_1 I_2 + \frac{1}{6} I_1^3$$

Prům: $I_n \rightarrow \bar{I}_n \rightarrow \varepsilon_n$

Zvolíme $x_i' = \text{slavný osy } \varepsilon$

$$f(\varepsilon_{ij}) = f(\varepsilon_{ij}') = f(\varepsilon_1, \varepsilon_2, \varepsilon_3) = f(I_1, I_2, I_3)$$

Indexy tr

$$I_1 = \text{tr}(\varepsilon) = \varepsilon_{ii}$$

$$A = \varepsilon^2 = \varepsilon \varepsilon$$

$$I_2 = \frac{1}{2} \text{tr}(A)$$

$$A_{ik} = \varepsilon_{ij} \varepsilon_{jk}$$

$$I_2 = \frac{1}{2} A_{ii} = \frac{1}{2} \varepsilon_{ij} \varepsilon_{ji}$$

$$I_3 = \frac{1}{3} \varepsilon_{ij} \varepsilon_{jk} \varepsilon_{ki}$$

Derivace invariantů

(17)

$$\frac{\partial I_1}{\partial \varepsilon_{11}} = \frac{\partial}{\partial \varepsilon_{11}} (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) = 1 \quad \text{podobně pro } \varepsilon_{22}, \varepsilon_{33}$$

$$\frac{\partial I_1}{\partial \varepsilon_{ij}} = \begin{cases} 1 & \text{pro } i=j \\ 0 & \text{pro } i \neq j \end{cases} \delta_{ij} \text{ Kronecker}$$

$$\frac{\partial I_2}{\partial \varepsilon_{ij}} = \frac{1}{2} \frac{\partial}{\partial \varepsilon_{ij}} (\varepsilon_{kk} \varepsilon_{kk}) = \frac{1}{2} \frac{\partial \varepsilon_{kk}}{\partial \varepsilon_{ij}} \varepsilon_{kk} + \frac{1}{2} \varepsilon_{kk} \frac{\partial \varepsilon_{kk}}{\partial \varepsilon_{ij}} =$$

$$= \frac{1}{2} \delta_{ik} \delta_{jl} \varepsilon_{kk} + \frac{1}{2} \varepsilon_{kk} \delta_{il} \delta_{jk} =$$

$$= \frac{1}{2} \varepsilon_{ji} + \frac{1}{2} \varepsilon_{ji} = \varepsilon_{ji} = \varepsilon_{ij} \quad (\text{symetrie})$$

$$\frac{\partial I_3}{\partial \varepsilon_{ij}} \text{ k zprávi podobně}$$

$$\frac{\partial I_1}{\partial \varepsilon_{ij}} = \delta_{ij}$$

$$\frac{\partial I_2}{\partial \varepsilon_{ij}} = \varepsilon_{ij}$$

$$\frac{\partial I_3}{\partial \varepsilon_{ij}} = \varepsilon_{ik} \varepsilon_{kj}$$

Elasticita

T = konst. $\exists \Psi(\epsilon_{ij})$ $\sigma_{ij} = \frac{\partial \Psi}{\partial \epsilon_{ij}}$

$$d\sigma_{ij} = \frac{\partial \sigma_{ij}}{\partial \epsilon_{kl}} d\epsilon_{kl} = \frac{\partial^2 \Psi}{\partial \epsilon_{ij} \partial \epsilon_{kl}} d\epsilon_{kl}$$

$$d\sigma_{ij} = C_{ijkl} d\epsilon_{kl} \quad C_{ijkl} = \frac{\partial^2 \Psi}{\partial \epsilon_{ij} \partial \epsilon_{kl}}$$

(dim) ^{řád} = $3^4 = 81$ $i \leftrightarrow j, k \leftrightarrow l, ij \leftrightarrow kl$
 21 koeficientů

Posun: symetrie C v MKP.

Isotropní materiál

$$\Psi(\epsilon_{ij}) = \Psi(\epsilon_1, \epsilon_2, \epsilon_3) = \Psi(I_1, I_2, I_3)$$

$$\begin{aligned} \sigma_{ij} &= \frac{\partial \Psi}{\partial I_1} \frac{\partial I_1}{\partial \epsilon_{ij}} + \frac{\partial \Psi}{\partial I_2} \frac{\partial I_2}{\partial \epsilon_{ij}} + \frac{\partial \Psi}{\partial I_3} \frac{\partial I_3}{\partial \epsilon_{ij}} = \\ &= \frac{\partial \Psi}{\partial I_1} \delta_{ij} + \frac{\partial \Psi}{\partial I_2} \epsilon_{ij} + \frac{\partial \Psi}{\partial I_3} \epsilon_{ik} \epsilon_{kj} \end{aligned}$$

lineární + isotropní

$$\sigma_{ij} = \lambda I_1 \delta_{ij} + 2\mu \epsilon_{ij} \quad \text{Hooke (Voigt 1910)}$$

λ, μ = Lame'sho konstanty

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad \mu = G = \frac{E}{2(1+\nu)}$$

Deforacii entropie

$$\frac{\partial \Psi}{\partial I_1} = \lambda I_1 \quad \frac{\partial \Psi}{\partial I_2} = 2\mu \quad \frac{\partial \Psi}{\partial I_3} = 0$$

$$\Psi = \frac{1}{2} \lambda I_1^2 + 2\mu I_2$$

$$I_1 = \epsilon_1 + \epsilon_2 + \epsilon_3 \quad I_2 = \frac{1}{2} (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)$$

$$\begin{aligned} \Psi &= \frac{1}{2} \lambda (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + 2\epsilon_1\epsilon_2 + 2\epsilon_2\epsilon_3 + 2\epsilon_3\epsilon_1) + \mu (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) = \\ &= \left(\frac{1}{2} \lambda + \mu\right) (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2) + \lambda (\epsilon_1\epsilon_2 + \epsilon_2\epsilon_3 + \epsilon_3\epsilon_1) \end{aligned}$$

Predp. stabilitaty $\Psi \geq 0$

$\Psi =$ kvadraticka forma, Hessiana: $H_{ij} = \frac{\partial^2 \Psi}{\partial \epsilon_i \partial \epsilon_j} = + \text{def.}$

$$\frac{\partial^2 \Psi}{\partial \epsilon_1^2} = \lambda + 2\mu \quad \frac{\partial^2 \Psi}{\partial \epsilon_1 \partial \epsilon_2} = \lambda$$

$$H = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda \\ \lambda & \lambda + 2\mu & \lambda \\ \lambda & \lambda & \lambda + 2\mu \end{bmatrix}$$

uprikladzime pozitivnu definitnost

$$H \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3\lambda + 2\mu \\ 3\lambda + 2\mu \\ 3\lambda + 2\mu \end{bmatrix} = (3\lambda + 2\mu) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{hydrostat. mod}$$

$$H \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2\mu \\ 0 \\ -2\mu \end{bmatrix} = 2\mu \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \text{deform. mod}$$

$$H \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2\mu \\ -2\mu \\ 0 \end{bmatrix} = 2\mu \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \text{deform. mod}$$

práčižim $3\lambda + 2\mu = 3K > 0$, $2\mu = 2G > 0$

modul objemoví pružnosti modul pružnosti ve křivce

$$K = \frac{E}{3(1-2\nu)}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\underline{E < 0}$$

$$1 + \nu < 0 \quad \wedge \quad 1 - 2\nu < 0$$

$$\nu < -1 \quad \wedge \quad \nu > \frac{1}{2}$$

$$\underline{E > 0}$$

$$\nu > -1 \quad \wedge \quad \nu < \frac{1}{2}$$

$$E, K, G > 0 \quad \nu \in (-1, \frac{1}{2})$$