Pattern Recognition Letters, vol. 15, pp. 433-436, 1994

Affine moment invariants: A new tool for character recognition

Jan Flusser and Tomáš Suk

Institute of Information Theory and Automation
Academy of Sciences of the Czech Republic
Pod vodárenskou věží 4, 182 08 Prague 8, Czech Republic
E-mail: flusser@utia.cas.cz

Abstract

Affine moment invariants (AMIs) have been derived recently by Flusser and Suk (1992). In this paper, the AMIs are used as the features for recognition of handwritten characters independent on their size, slant and other variations. A comparison with classical moment invariants is also given.

Keywords: Character recognition, handwritten characters, invariant features, affine moment invariants, Hu invariants.

1. Introduction

Feature-based recognition of handwritten characters independent on their position, size, orientation, slant and other variations has been the goal of ongoing research. Finding efficient invariant features is the key to solving this problem. There have been several kinds of features used for recognition. The following three groups of features are used very frequently:

- *Fourier shape descriptors.* In this scheme, characters are thresholded and their borders are extracted. The border of each character can be represented by its Fourier transform. Fourier coefficients with significant values are called Fourier descriptors and can be used as the features for character recognition (Arbter et al., 1990).
- *Stroke vectors.* In this method the characters are described in terms of a set of geometrical features such as strokes at different orientation (Kohan and Pavlidis, 1987). The disadvantages of using this technique are that the features are not invariant under character slant and that the thinning of the characters is required before feature extraction in order to represent the characters by pixel thick lines only.
- *Moment invariants.* Moment invariants are features based on statistical moments of characters. They are traditional and widely-used tool for character recognition (e.g. El-Khaly and Sid-Ahmed, 1990; Belkasim et al., 1991). Classical moment invariants were introduced by Hu (1962) and they were successfully used in numerous applications not only for character recognition. Hu invariants are invariant under translation, rotation and scaling. Maitra (1979) extended Hu invariants to be invariant under image contrast. Recently, Flusser and Suk (1992) have derived a new set of moment invariants, that are invariant under general affine transformation. These affine moment invariants (AMIs) represent a significant contribution to the progress in the field of invariant pattern recognition. Because affine transformation can describe variations in character slant, AMIs are supposed to be effective features for recognition of handwritten characters.

The aim of the paper. The main goal of our paper is to test the possibility of using the AMIs for recognition of handwritten characters and to compare them with classical Hu invariants. We will consider isolated characters. In the case of connected characters, any algorithm for character separation has to be applied before the moment computation.

2. Hu moment invariants

Hu (1962) derived seven moment invariants that are invariant under translation, rotation and scaling of the object. We have used only first four of them for numerical simplicity:

$$\begin{aligned}
\Phi_1 &= \nu_{20} + \nu_{02} \\
\Phi_2 &= (\nu_{20} - \nu_{02})^2 + 4\nu_{11}^2 \\
\Phi_3 &= (\nu_{30} - 3\nu_{12})^2 + (3\nu_{21} - \nu_{03})^2 \\
\Phi_4 &= (\nu_{30} + \nu_{12})^2 + (\nu_{21} + \nu_{03})^2
\end{aligned}$$

where ν_{pq} is the normalized central moment of order $(p+q)$

$$\nu_{pq} = \frac{\mu_{pq}}{\mu_{00}^w}, \qquad w = \frac{p+q}{2} + 1$$

and μ_{pq} is the central moment of order $(p+q)$. For binary object G it is defined as

$$\mu_{pq} = \int\int_G (x - x_t)^p (y - y_t)^q dxdy,$$

where (x_t, y_t) are coordinates of the center of gravity of object G.

3. Affine moment invariants

The AMIs were derived by means of the theory of algebraic invariants. Full derivation and comprehensive discussion on the properties of invariants can be found in the paper by Flusser and Suk (1992).

The AMIs are invariant under *general affine transformation*

$$\begin{aligned}
u &= a_0 + a_1 x + a_2 y \\
v &= b_0 + b_1 x + b_2 y,
\end{aligned}$$

where (x,y) and (u,v) are coordinates in the image plane before and after the transformation, respectively.

Four simplest AMIs that we have used for character recognition are listed below:

$$\begin{aligned}
I_1 &= \frac{1}{\mu_{00}^4}(\mu_{20}\mu_{02} - \mu_{11}^2) \\
I_2 &= \frac{1}{\mu_{00}^{10}}(\mu_{30}^2\mu_{03}^2 - 6\mu_{30}\mu_{21}\mu_{12}\mu_{03} + 4\mu_{30}\mu_{12}^3 + 4\mu_{03}\mu_{21}^3 - 3\mu_{21}^2\mu_{12}^2) \\
I_3 &= \frac{1}{\mu_{00}^7}(\mu_{20}(\mu_{21}\mu_{03} - \mu_{12}^2) - \mu_{11}(\mu_{30}\mu_{03} - \mu_{21}\mu_{12}) + \mu_{02}(\mu_{30}\mu_{12} - \mu_{21}^2))
\end{aligned}$$

$$\begin{aligned}
I_4 = \frac{1}{\mu_{00}^{11}}(&\mu_{20}^3\mu_{03}^2 - 6\mu_{20}^2\mu_{11}\mu_{12}\mu_{03} - 6\mu_{20}^2\mu_{02}\mu_{21}\mu_{03} + 9\mu_{20}^2\mu_{02}\mu_{12}^2 + 12\mu_{20}\mu_{11}^2\mu_{21}\mu_{03} + \\
&+6\mu_{20}\mu_{11}\mu_{02}\mu_{30}\mu_{03} - 18\mu_{20}\mu_{11}\mu_{02}\mu_{21}\mu_{12} - 8\mu_{11}^3\mu_{30}\mu_{03} - 6\mu_{20}\mu_{02}^2\mu_{30}\mu_{12} + \\
&+9\mu_{20}\mu_{02}^2\mu_{21}^2 + 12\mu_{11}^2\mu_{02}\mu_{30}\mu_{12} - 6\mu_{11}\mu_{02}^2\mu_{30}\mu_{21} + \mu_{02}^3\mu_{30}^2).
\end{aligned}$$

4. Numerical experiment

In order to demonstrate the performance of character recognition by AMIs, the following experiment was carried out.

- *Data*

  Handwritten capital letters **A, B, C, D** and **E** were used in the experiment (see Fig. 1). The letters in the first row of Fig. 1 are the templates, the other letters are unknown characters which we want to recognize. The letters in the second, third and fourth rows of Fig. 1 meet the presumption of affine deformation (however, approximately, not exactly), the letters in the last row have non-linear distortions. The size of the image in Fig. 1 is 512×512 pixels.

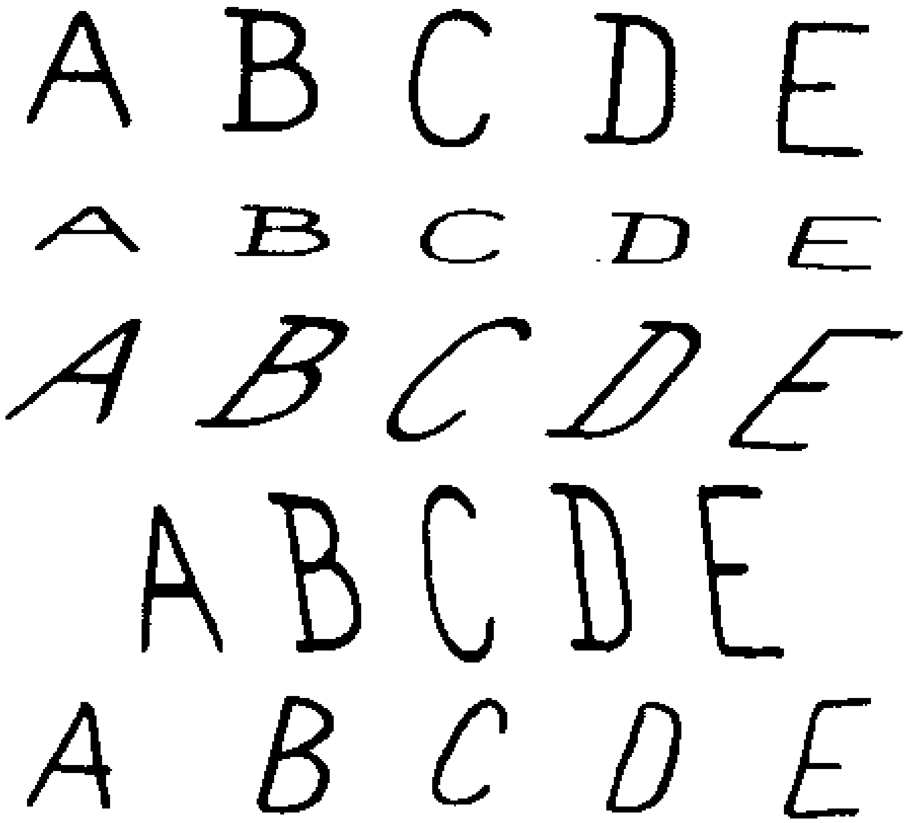

Figure 1: The letters used in the experiment; 1st row: the templates, 2nd – 5th rows: unknown characters.

- *Feature extraction*

  First, the letters were thresholded in order to obtain binary image. No additional preprocessing technique such as thinning was applied. Then four Hu invariants and four AMIs were computed for each letter.

- *Classification scheme*

  Classification of the letters was performed by minimum distance classifier in 4-D Euclidean feature space. Each letter was classified twice – by Hu invariants and by AMIs. If we have to recognize a larger set of characters, it is possible to use the classifier with supervised or unsupervised learning.

- *Results*

  Experimental results are summarized in the tables. Table 1 shows the values of invariants Φ_1 and I_1, respectively, for all tested letters. It can be seen that I_1 values do not change significantly for different rows

Row	A	B	C	D	E
1	576	593	1399	793	1119
2	810	617	1145	701	880
3	741	765	1918	1102	1587
4	679	815	2027	1099	1553
5	566	567	1118	814	1259
1	82	72	311	115	200
2	82	73	297	105	166
3	81	70	287	112	189
4	80	72	299	109	176
5	74	58	173	116	210

Table 1: The values of the first Hu invariant Φ_1 of all letters (top part) and the values of the first affine moment invariant I_1 (bottom part). All values are multiplied by 10^3.

Row	A	B	C	D	E
2	D	B	E	B	D
3	D	D	C	E	C
4	D	D	C	E	C
5	A	B	E	D	C
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	E	D	E

Table 2: Letter classification by the first four Hu invariants (top part) and by the first four affine moment invariants (bottom part).

of Fig. 1 (except the last one). On the other hand, Φ_1 values change very much. Similar situation is in the case of the other AMIs and Hu invariants.

Table 2 shows the results of classification of each letter by Hu invariants and AMIs, respectively. It proves that Hu invariants cannot be used as the features for recognition of characters with slant: Only 30% of characters were recognized correctly. On the other hand, it demonstrates the advantageous properties of the AMIs : 95% of characters were recognized successfully. The only error occurs in the last row: **C** was classified as **E**. It was caused by its non-linear deformation under which the AMIs are not invariant.

We have done a number of numerical experiments like this one. There have been achieved very similar results to those described above. The recognition rate for Hu invariants was always lower than 40% and for the AMIs it was higher than 90%.

5. Conclusion

The paper has dealt with feature-based recognition of handwritten characters. Main goal of the paper has been to prove that the affine moment invariants recently developed by the authors can be used as the size- and slant-invariant features.

In numerical experiments, we have used only first four Hu invariants and first four AMIs. The application of more invariants increases the computing complexity of the task but does not produce significantly different results.

The experiments have confirmed the following claims :

- If the deformation between the templates and the unknown characters is approximately affine, the classification based on AMIs yields high reliable results.
- If the character deformation is more complex (non-linear), some errors may occur in the classification. The number of errors can be reduced by composing of the training set from letters of various fonts.
- In comparison with classical Hu invariants, the AMIs yield much more accurate results.

In the near future, we want to focuse our attention to finding features invariant under non-linear variations in handwriting.

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