THE CAPILLARY FLOW OF A YIELD-STRESS FLUID

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The wicking of a Newtonian liquid in a capillary tube is described by the well-known Washburn equation [1]. In the absence of gravity effects (horizontal tube), the motion is entirely determined by a force balance between capillarity and the frictional force on the wall, which becomes increasingly larger as the liquid advances:

$$\tau_w \pi d x = \pi d \sigma \qquad (1)$$

This force balance does not depend on the fluid rheology, so that it is true also for non-Newtonian fluids. Here, we want to investigate the behaviour of yield-stress fluids, i.e. those fluids which respond like elastic solids for applied stresses lower than a certain threshold value τ_0 (the yield stress), and flow only when the yield stress is overcome.

When a yield stress fluid flows in a horizontal tube under the action of the capillary force only, according to Eq. (1) there is a critical distance from the tube inlet for which the wall shear stress equals the yield value:

$$\bar{x} = \sigma / \tau_0 \qquad (2)$$

Since the shear stress is maximum at the wall, this means that $\tau < \tau_0$ everywhere inside the capillary tube, so that the fluid cannot flow anymore [2].

This work brings experimental evidence that the theory outlined above cannot describe the behaviour of a model yield-stress fluid (a commercial gel). In particular, the maximum penetration length of the fluid in the capillary is one order of magnitude larger than the prediction of Eq. (2), and is strongly dependent on the tube diameter.

Experiments were carried out using borosilicate glass tubes 125 mm long and with diameters ranging from 0.46 to 1.5 mm; the position of the advancing fluid was measured by means of a CCD camera placed above the tubes (a typical image is shown in Fig. 1).

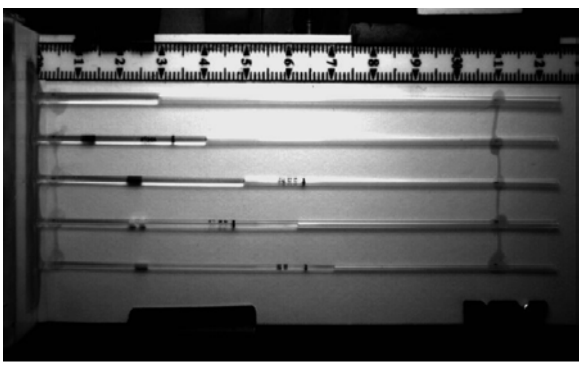

Figure 1. Experimental arrangement.

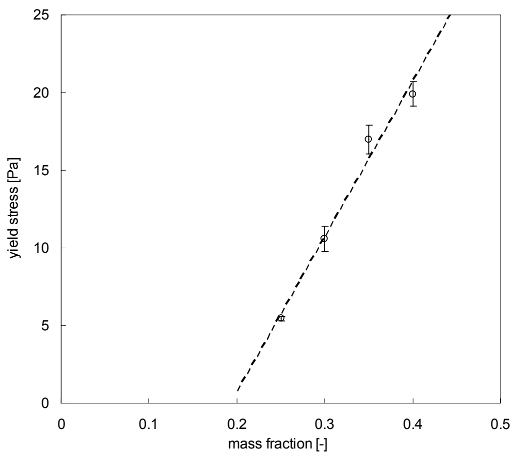

Figure 2. Yield stress vs. mass fraction of gel in the solution

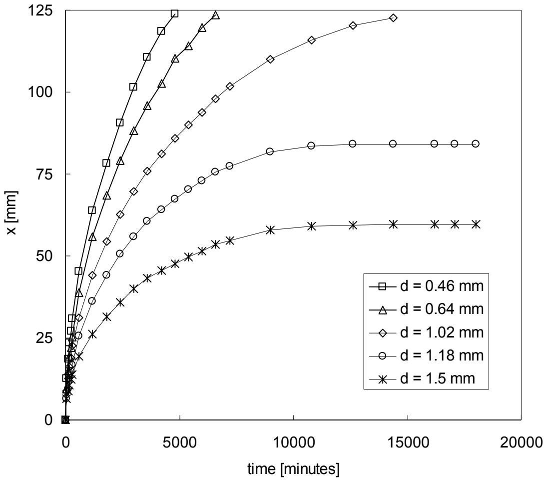

Figure 3. Fluid penetration vs diameter in different tubes ($\tau_0 = 10.7$ Pa).

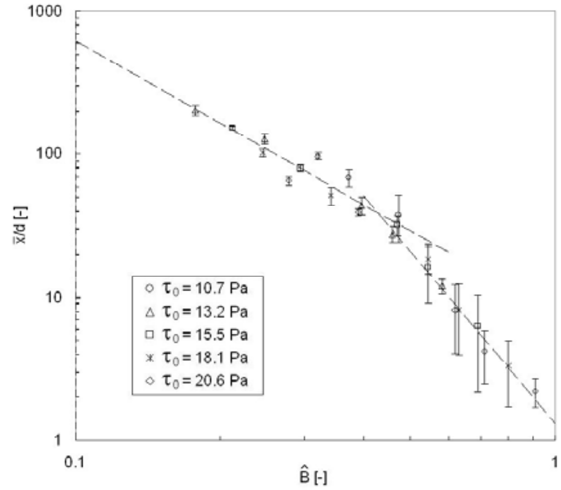

Figure 4. Dimensionless maximum penetration length as a function of the Bingham-capillary number.

The surface tension of solutions (measured by a De Nouy tensiometer) was 34 mN/m, while the yield stress (measured by a rotational rheometer with plane-plane geometry) could be varied from about 5 to 20 Pa by diluting the gel in water at different concentrations, as shown in Fig. 2. Such values yield maximum penetration distances of a few mm.

A typical result obtained for a given yield stress is reported in Fig. 3, which shows that the fluid does not stop at the distance predicted by Eq. (2) and that the maximum penetration length is strongly dependent on the tube diameter.

Because Eq. (1) is always true, the simplest explanation for such experimental results is that the fluid does not remain homogeneous under shear flow in the vicinity of the wall, causing a substantial reduction of the effective yield stress. In particular, when a fluid consists of a particle suspension (including colloids), which is the case of most yield-stress fluids, this phenomenon is caused by a depletion of particles near the wall and is known as "apparent wall slip" [3].

In the present system, wall slip seems to be related to the competition between the yield stress and the capillary pressure, which can be expressed by the dimensionless number $\hat{B} = \tau_0 d / \sigma$ (the product of the Bingham and the Capillary numbers). If one plot the dimensionless maximum penetration length of the fluid in the capillary with respect to this parameter, all data collapse on a single curve, as shown in Fig. 4. The best-fit of experimental data leads to a scaling law of the form:

$$\bar{x}/d \sim \hat{B}^n \tag{3}$$

where $n = -2$ for $\hat{B} < 0.5$ and $n = -4$ for $\hat{B} > 0.5$.

References

[1] E.W. Washburn, Phys. Rev. 273-283 (1921).
[2] A.H. Skelland, Non-Newtonian Flow and Heat Transfer, Wiley, New York, 1967.
[3] H.A. Barnes, J. Non-Newtonian Fluid Mech. 56, 221–251 (1995). Z.D> Jastrzebski, Ind. Eng. Chem. Fundam. 6, 445–453 (1967).