NONLINEAR LIFTING LINE METHOD FOR AIRPLANE WINGS

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A simple method of calculating nonlinear aerodynamic characteristic of a wing with moderate hardware requirements was developed. The method uses nonlinear characteristic of several airfoil sections along the span of the wing obtained separately before the computation of wing characteristics. Then the local lift coefficients and the local loads and local induced angles on the wing are calculated by iterative way. The results in the form of wing nonlinear lift curve, induced drag polar curve, load distribution and induced angle of attack distribution are available.
Keywords: lifting line, nonlinear, lift curve, wing

Introduction

In certain phase of an aircraft design, the designer needs to know basic aerodynamic characteristic, lift curve, drag polar curve, wing load, with high accuracy including nonlinear components, mainly the nonlinearities related to high angles of attack. A three-dimensional circumference of wing is a complex case characterized by nonlinear phenomena especially at high angles of attack. The computation of such flow with using of complex CFD methods is not easy and concurrently is hardware demanding and time consuming. Much simpler linearised methods for example panel methods, can not give the requested results in the nonlinear areas. A simple nonlinear method was developed to propose a partial solution. The method separates acquisition of the two dimensional airfoil section aerodynamic characteristics and the computation of the three-dimensional wing characteristics. The airfoil nonlinear lift curves are acquired separately and independently on the wing computation, for example by 2D computation or by windtunnel testing, and then are used as input for the computation of wing characteristics.

Basic idea

The basic idea consists of the separation of two-dimensional and three-dimensional cases. First, the two-dimensional nonlinear aerodynamic characteristic of several airfoil sections are acquired. Either as result of computation (relatively simpler for 2D case than for extremely complex 3D case) or as result of windtunnel testing or as a result of literature survey. The 2D airfoil section lift curves are then used as input data for 3D computation of wing aerodynamic characteristics, much less complex, less hardware demanding and less time consuming than 3D CFD computation. Thus many data necessary for the decisions of designer in advanced stages of wing design could be acquired in relatively simple manner.

Wing model

The wing can be of arbitrary planform. The wing twist as well as deflected high-lift devices or control surfaces can be considered. The different local aerodynamic qualities are included in local airfoil section lift curves characteristics. For example the geometric twist is simply included by local changes in the geometric angle of attack, the flap deflection is included by the use of lift curve of the airfoil with deflected flap etc.

The computation method

The flowfield around the wing is modelled by a linear superposition of N horseshoe vortices with bounded segments on the quarter-chord line of the wing, as shown in Illustration 1. The unknown strengths (circulations) of these vortices are determined by a set of N nonlinear equations. To set up these equations, we use the Biot-Savart law to calculate induced velocities in the centers of bounded segments. Using these velocities, we calculate the lift on each segment in two ways: interpolating the lift curves of the given nonlinear airfoil characteristics, and using the vector form of Kutta-Joukowski law. Since the circulations are unknown, we require the two computed lifts on each segment to be (approximately) equal, and thus we obtain a set of N nonlinear equations for the unknown circulations:

$$\boldsymbol{F}(\boldsymbol{\Gamma}, \alpha) = \mathbf{0} \quad (1),$$

where $\mathbf{\Gamma}$ is the vector of circulations and α is the angle of attack. The computation proceeds by continuously tracking $\mathbf{\Gamma}(\alpha)$ using a predictor-corrector paradigm. At any angle of attack α, we choose a step $\Delta\alpha$, and use approximate differentiation of the previous equation to obtain the *predictor* in the form:

$$\nabla \boldsymbol{F}(\boldsymbol{\Gamma}, \alpha) \cdot \Delta \boldsymbol{\Gamma} = -\boldsymbol{F}(\boldsymbol{\Gamma}, \alpha + \Delta\alpha) \quad (2)$$

with $\Delta\mathbf{\Gamma}$ unknown. Afterwards, we replace $\boldsymbol{\Gamma} + \Delta\boldsymbol{\Gamma} \rightarrow \boldsymbol{\Gamma},\ \alpha + \Delta\alpha \rightarrow \alpha$ and use the new value of $\mathbf{\Gamma}$ as a starting point for the Levenberg-Marquardt method (see [1]) applied to the system of nonlinear equations (1). This is the *corrector* step. We employed a modification of the freely available MINPACK code ([2]). The step $\Delta\alpha$ needs to be chosen adaptively and some control against blow-up of the predictor solution is necessary. A full automation of this process will be a subject of further research. It is important to note that the nonlinear system (1) usually gets ill-conditioned when local lift maxima are reached and, moreover, the residual of the optimal solution may increase; therefore, a robust method like Levenberg-Marquardt is needed rather than, e.g. a simple Newton method.

Example results

In Illustrations 2-4, we compare the method presented here against a linear method on a simple trapezoidal wing with different root and tip airfoils.
Illustration 2 compares the spanwise lift distribution computed at three different angles of attack:

5,12 and 16 degrees. Illustration 3 compares the lift curves, and Illustration 4 compares the polars. It can be seen that for the smallest angle of attack the results roughly agree, but they significantly divert as the angle of attack increases.
In Illustration 5, we show the lift distribution computed for a real wing with deflected flaps.

Discussion

It is obvious from the principle of the method that the accuracy and thus usefulness of the 3D results depend on the quality of airfoil section characteristics used as the input data and on the density and distribution of locations where the 2D airfoil characteristics are available. It is evident that it is necessary to have sufficient input data in the locations of brusque alterations, like lateral edges of deployed high-lift devices where it is necessary to have input data from the both sides, from the side of deflected side as well from the side of basic wing.

Conclusions

A simple and effective method of computation of nonlinear wing characteristics was developed. The method can be useful for designer as it is capable to provide nonlinear lift curves, induced drag polar curves, moment curves, nonlinear wing loads, all including influence of deflections of high lift devices and control surfaces and also to provide the location of initial flow separation, all with accuracy sufficient for many stages of design. It seems to be quite convenient for light aircraft design as the output data can be sufficient also for certification process and the necessary hardware equipment is affordable also for small companies. Because the computation is very fast (on a typical hardware) compared to CFD computations, it can also be used as a low-fidelity-quick-response model in design optimization.

[1] Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P.: Numerical Recipes in FORTRAN: the art of scientific computing (2nd ed.), Cambridge University Press, New York, USA. ISBN 0-521-43064-X

[2] MINPACK: www.netlib.org/minpack

[3] XFOIL, http://web.mit.edu/drela/Public/web/xfoil/

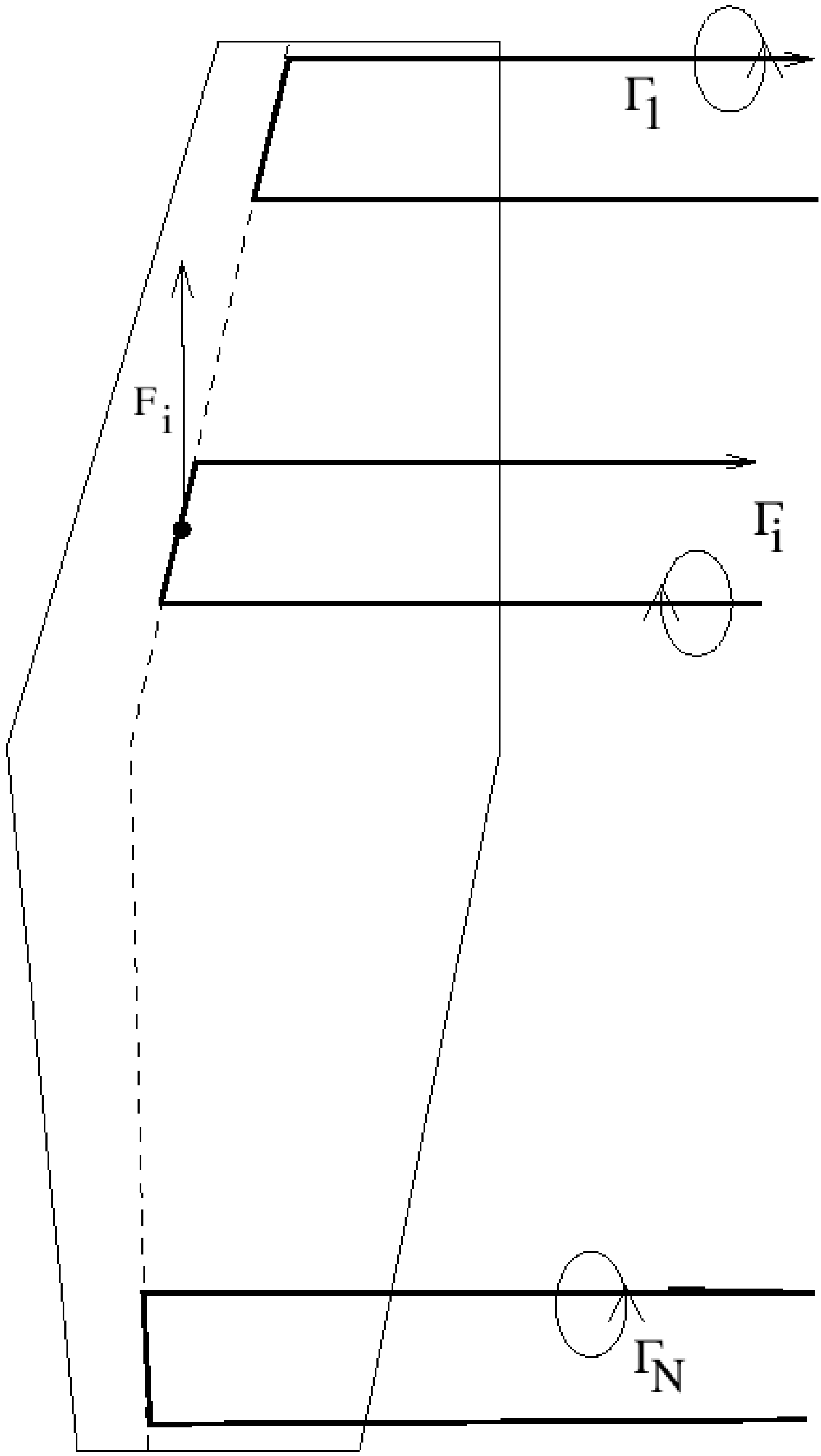

Illustration 1: The vortex model of the wing

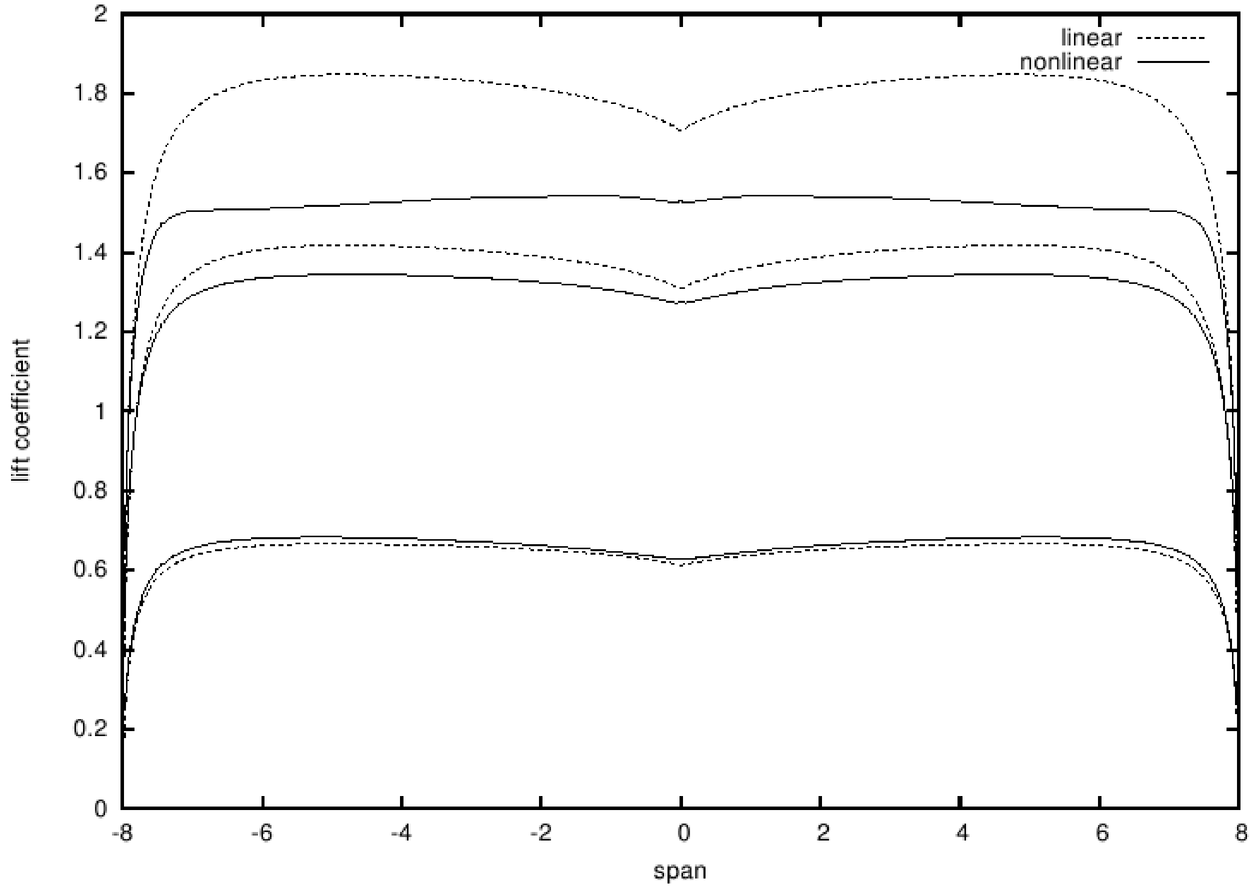

Illustration 2: spanwise lift distribution comparison

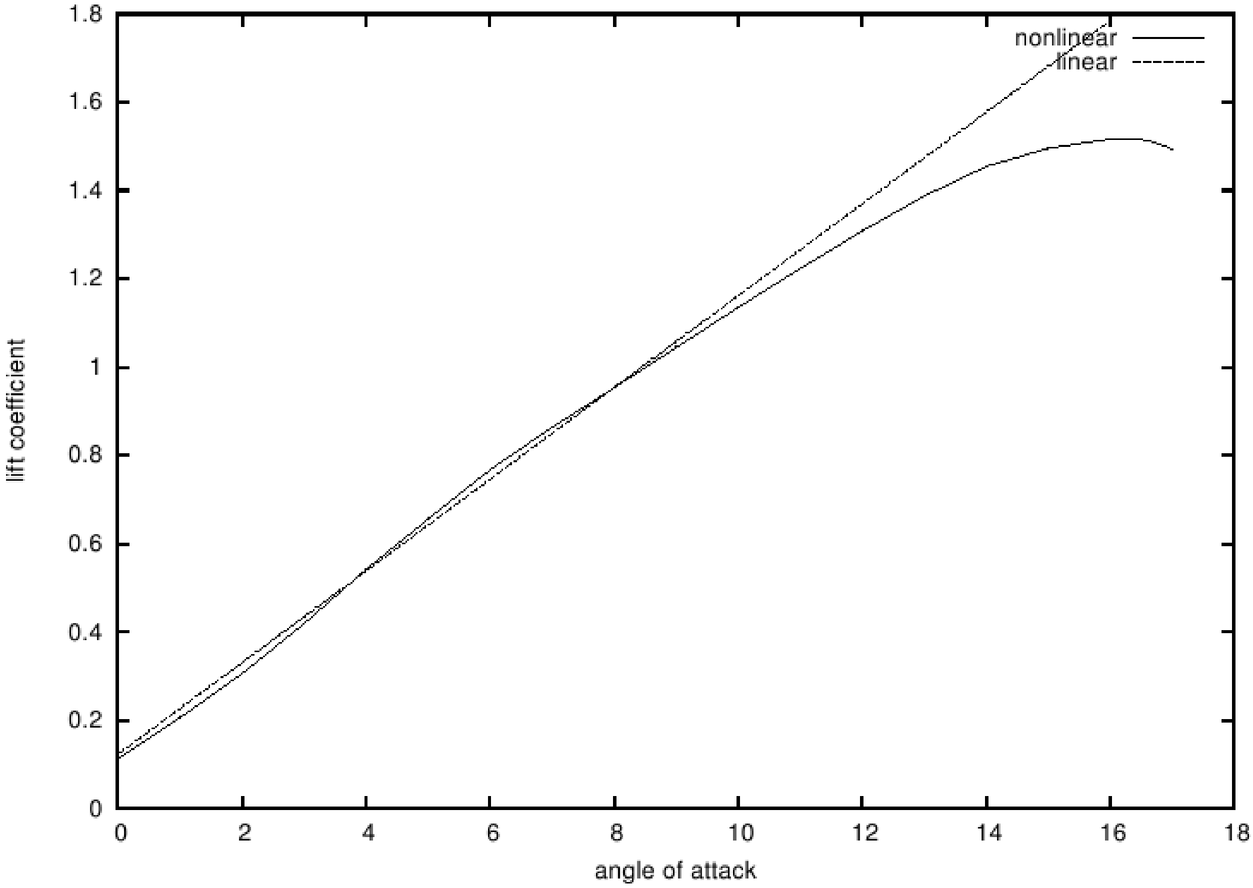

Illustration 3: lift curve comparison

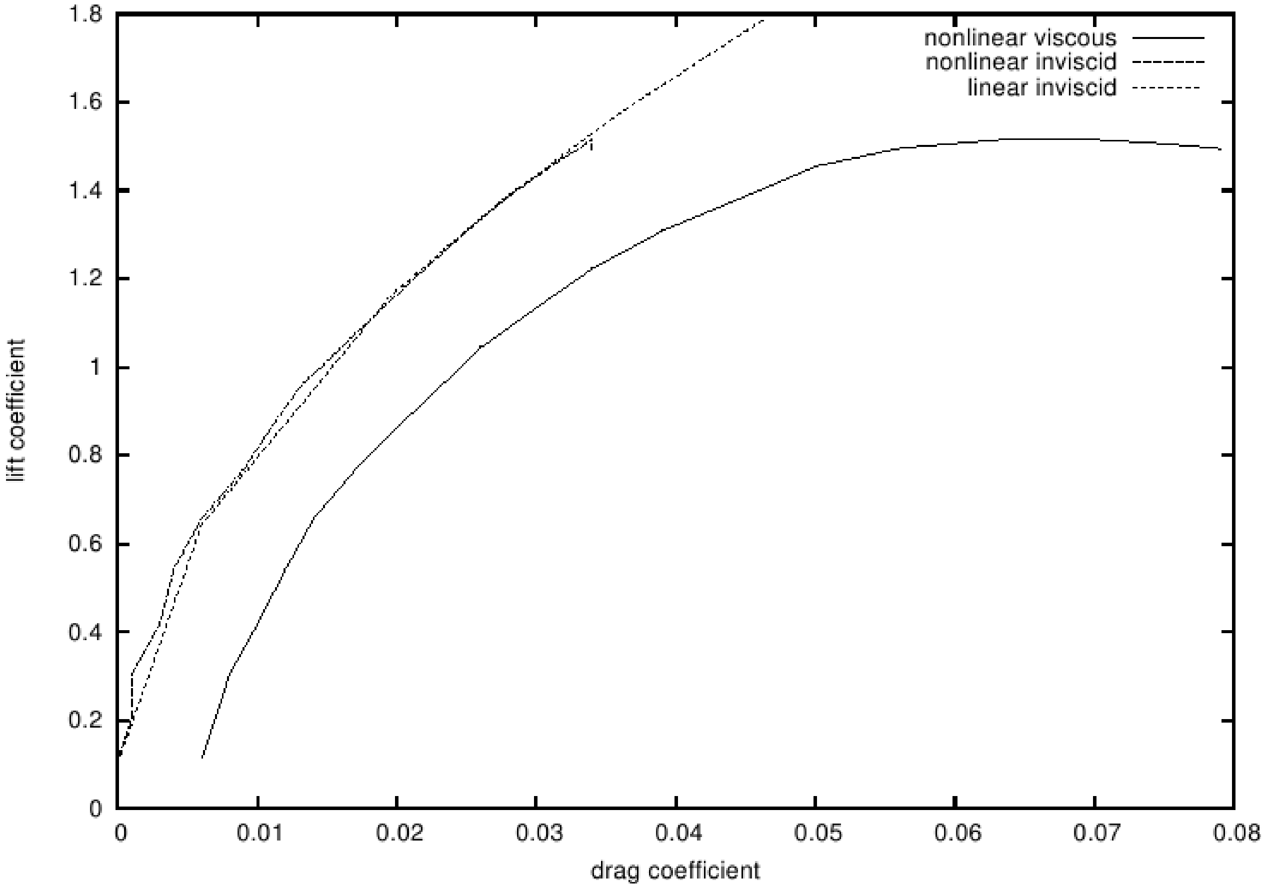

Illustration 4: polar curve comparison

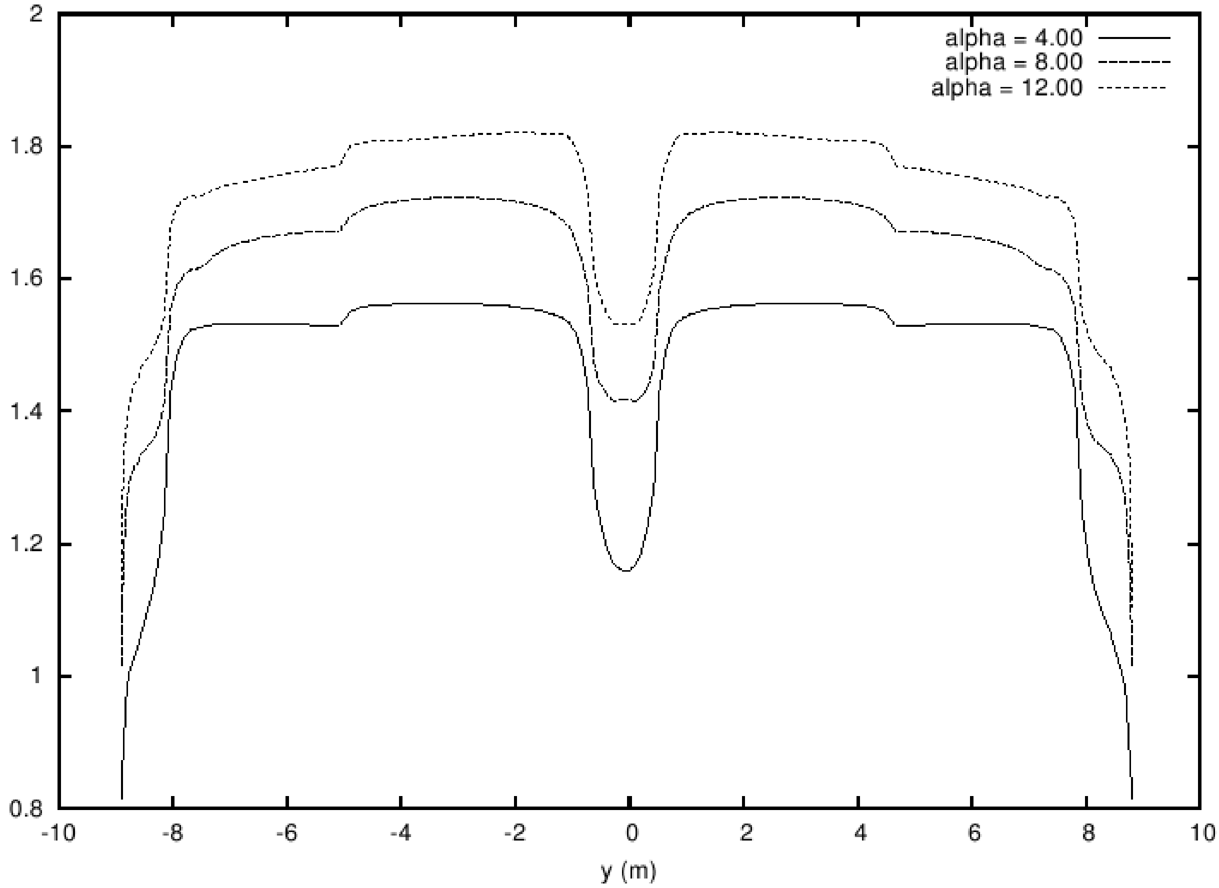

Illustration 5: spanwise lift distribution on a real wing with deflected flaps