NUMERICAL SOLUTION OF COMPRESSIBLE FLOW IN A CHANNEL WITH MOVING WALLS

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Introduction

This work is considered with the numerical solution of inviscid compressible fluid flow through a channel with moving walls. The governing Euler equations written in the ALE (Arbitrary Lagrangian-Eulerian) form are discretized by the discontinuous Galerkin method. We apply a semiimplicit linearization with respect to time. Currently, the movement of the wall must be prescribed by a given formula.

Formulation of the problem

We consider the flow in a bounded 2D domain Ω_t depending on time t with boundary $\partial\Omega_t = \Gamma_I \cup \Gamma_O \cup \Gamma_{W_t}$, where Γ_I and Γ_O represent the inlet and outlet and Γ_{W_t} represents moving impermeable walls.

The dependence of the domain on time is taken into account with the aid of a regular ALE mapping $\mathcal{A}_t : \overline{\Omega}_0 \to \overline{\Omega}_t$, i.e. $\boldsymbol{X} \mapsto \boldsymbol{x} = \boldsymbol{x}(\boldsymbol{X}, t)$. Further, we define the ALE velocity: $\tilde{\boldsymbol{z}}(\boldsymbol{X}, t) = \frac{\partial}{\partial t}\boldsymbol{x}(\boldsymbol{X}, t) = \frac{\partial}{\partial t}\mathcal{A}_t(\boldsymbol{X})$, $\boldsymbol{z}(\boldsymbol{x}, t) = \tilde{\boldsymbol{z}}(\mathcal{A}_t^{-1}(\boldsymbol{x}), t),\ t \in [0, T],\ \boldsymbol{x} \in \overline{\Omega}_t$ and the ALE derivative of a function $f = f(\boldsymbol{x}, t)$: $\frac{D^A}{Dt} f(\boldsymbol{x}, t) = \frac{\partial \tilde{f}}{\partial t}(\boldsymbol{X}, t)|_{X=\mathcal{A}_t^{-1}(x)}$, where $\tilde{f}(\boldsymbol{X}, t) = f(\mathcal{A}_t(\boldsymbol{X}), t),\ \boldsymbol{X} \in \Omega_0$.

It is possible to show that

$$\frac{D^A f}{Dt} = \frac{\partial f}{\partial t} + \mathbf{z} \cdot \nabla f = \frac{\partial f}{\partial t} + \mathrm{div}(\mathbf{z} f) - f \mathrm{div}\mathbf{z}. \tag{1}$$

This leads to two different formulations of the Euler equations in ALE form:

$$\begin{aligned} &1) \quad \frac{D^A \mathbf{w}}{Dt} + \sum_{s=1}^{2} \frac{\partial \boldsymbol{f}_s(\mathbf{w})}{\partial x_s} - \mathbf{z} \cdot \nabla \mathbf{w} = 0, \\ &2) \quad \frac{D^A \mathbf{w}}{Dt} + \sum_{s=1}^{2} \frac{\partial \mathbf{g}_s(\mathbf{w})}{\partial x_s} + \mathbf{w}\mathrm{div}\mathbf{z} = 0 \end{aligned} \tag{2}$$

where $\boldsymbol{f}_s, s = 1, 2$, are the *inviscid fluxes* and

$$\begin{aligned} \mathbf{w} &= (\rho, \rho v_1, \rho v_2, e)^{\mathrm{T}} \in \mathbb{R}^4, \\ \boldsymbol{f}_i(w) &= (f_{i1}(\mathbf{w}), \ldots, f_{i4}(\mathbf{w}))^{\mathrm{T}} = (\rho v_i, \rho v_1 v_i + \delta_{1i} p, \rho v_2 v_i + \delta_{2i} p, (e + p) v_i)^{\mathrm{T}} \end{aligned} \tag{3}$$

and $\mathrm{g}_s, s = 1, 2$, are modified inviscid fluxes

$$\mathbf{g}_s(w) := \boldsymbol{f}_s(w) - \mathbf{z}_s \mathbf{w}. \tag{4}$$

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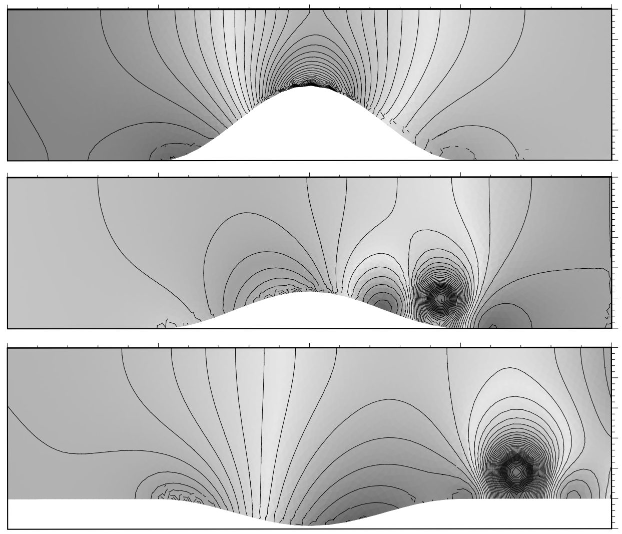

Figure 1: Pressure isolines.

This system is equiped with standard inlet and outlet boundary conditions. On the moving wall we impose the impermeability condition $\mathbf{v} \cdot \mathbf{n} = \mathbf{z} \cdot \mathbf{n}$, where $\mathbf{n}$ is the unit outer normal to Γ_{W_t}. The discretization of equations 1) and 2) in (2) is carried out by the discontinuous Galerkin finite element method, which uses piecewise polynomial function spaces without the assumption of interelement continuity, e.g. [2].

Discretization in time is carried out using the semi-implicit linearization of the backward Euler method used in [1]. Additional terms which are a result of the ALE formulation of (2) do not require special treatment, since they are linear with respect to the unknown state vector $\mathbf{w}$ and therefore can be taken implicitly.

Now we describe the construction of the ALE mapping. We assume that the inlet and outlet are straight segments given by the conditions $X_1 = -2$ and $X_1 = 2$, respectively. In our first results we assume that the upper wall is given by the condition $X_2 = 1$ and that the ALE mapping is equal to the identity in the sets $[-2, -1] \times [0, 1]$ and $[1, 2] \times [0, 1]$. Otherwise we construct the ALE mapping so that lower wall is represented at time t by the graph of the smooth function.

$$sin(0.5 * t) * (cos(\pi * X_1) + 1)/4.$$

This movement is interpolated to the rest of the domain.

The computed solution is periodic and although the flow is inviscid, a vortex forms after the lower wall starts to descend. This vortex is then convected out of the domain through the outlet. Figure 1 shows the pressure isolines at three different time instants: before the vortex is formed, after the formation and the vortex convection.

References

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[2] Feistauer M., Felcman J., Straškraba I.: *Mathematical and Computational Methods for Compressible Flow*, Oxford University Press, Oxford, (2003).