# On The Validation 2D–Flow Study Over an ERCOFTAC Hill

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## Abstract

The paper deals with a validation flow study performed on the mathematical/numerical model of atmospheric boundary layer flow. The mathematical model is based on the system of RANS equations closed by the two-equation $k-\varepsilon$ turbulence model together with wall functions. The finite volume method and the explicit Runge–Kutta time integration method are utilized for the numerics. The test–case is related to a neutral boundary layer 2D-flow over an isolated hill with a rough wall.

## 1 Mathematical formulation

The flow itself is assumed to be a turbulent, viscous, incompressible, stationary and indifferently stratified as well. The mathematical model is based on the RANS approach and the governing equations modified according to the method of artificial compressibility can be re-casted in the conservative and vector form

$$\vec{W}_t + \begin{pmatrix} u \\ u^2+\frac{p}{\varrho} \\ uv \\ uw \end{pmatrix}_x + \begin{pmatrix} v \\ vu \\ v^2+\frac{p}{\varrho} \\ vw \end{pmatrix}_y + \begin{pmatrix} w \\ wu \\ wv \\ w^2+\frac{p}{\varrho} \end{pmatrix}_z = \begin{pmatrix} 0 \\ Ku_x \\ Kv_x \\ Kw_x \end{pmatrix}_x + \begin{pmatrix} 0 \\ Ku_y \\ Kv_y \\ Kw_y \end{pmatrix}_y + \begin{pmatrix} 0 \\ Ku_z \\ Kv_z \\ Kw_z \end{pmatrix}_z \tag{1}$$

where $\vec{W} = (p/\beta^2, u, v, w)^T$ stands for the vector of unknown variables: the pressure $p$, the velocity vector $\vec{V} = (u, v, w)^T$ and the parameters $K$ refers to the turbulent diffusion coefficients, see equation (4) and $\beta$ is related to the artificial sound speed.

## 2 Turbulence model

Closure of the system (1) is performed by a standard high-Re $k-\varepsilon$ turbulence model. Two additional transport equations are added to the system (1) for the turbulent kinetic energy abbreviated by $k$ and for the rate of dissipation of turbulent kinetic energy denoted by $\varepsilon$

$$(ku)_x + (kv)_y + (kw)_z = (K^{(k)} k_x)_x + (K^{(k)} k_y)_y + (K^{(k)} k_z)_z + P - \varepsilon, \tag{2}$$

$$(\varepsilon u)_x + (\varepsilon v)_y + (\varepsilon w)_z = (K^{(\varepsilon)} \varepsilon_x)_x + (K^{(\varepsilon)} \varepsilon_y)_y + (K^{(\varepsilon)} \varepsilon_z)_z + C_{\varepsilon 1}\frac{\varepsilon}{k} P - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \tag{3}$$

where $P$ denotes the turbulent production term $P = \tau_{ij}\frac{\partial v_i}{\partial x_j}$ for the Reynolds stress written as $\tau_{ij} = -\frac{2}{3} k\,\delta_{ij} + \nu_T\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ and the terms $K^{(k)}$, $K^{(\varepsilon)}$, $\nu_T$ stand for the diffusion coefficients and the turbulence viscosity

$$K^{(k)} = \nu + \frac{\nu_T}{\sigma_k}, \quad K^{(\varepsilon)} = \nu + \frac{\nu_T}{\sigma_\varepsilon}, \quad \nu_T = C_\mu \frac{k^2}{\varepsilon}. \tag{4}$$

The model closure coefficients are described in Castro (1996) [3].

## 3 Boundary conditions

The system (1)+(2)+(3) is solved with the following boundary conditions, Castro (1981) [3]
Inlet: $u = \frac{u^*}{\kappa} \ln\left(\frac{z}{z_0}\right)$, $v = 0$, $w = 0$, $k = \frac{u^{*2}}{\sqrt{C_\mu}} \left(1 - \frac{z}{D}\right)^2$, $\varepsilon = \frac{C_\mu^{3/4} \cdot k^{3/2}}{\kappa \cdot z}$; Outlet: homogeneous Neumann conditions for all quantities; Top: $u = U_0$, $v = 0$, $\frac{\partial w}{\partial z} = 0$, $\frac{\partial k}{\partial z} = 0$, $\frac{\partial \varepsilon}{\partial z} = 0$; Wall: standard wall functions are applied; where $u^*$ is the friction velocity, $\kappa = 0.40$ denotes the von Karman constant, $z_0$ represents the roughness parameter.

## 4 Validation

The reference experimental data due to Khurshudyan (1981) [1] and corrected by Trombetti (1991) [2] are also available in the ERCOFTAC database. Moreover, Castro (1996) [3] performed flow and pollution dispersion reference numerical computations.
Computational domain is $9\,m$ long and $1.6\,m$ high. A hill with the highest slope has been tested for the free-stream air velocity $U_0 = 4\,m/s$ and boundary layer depth of $D = 1\,m$. The Reynolds number based on $U_0$ and hill height $H = 117\,mm$ is $Re \sim 3.1 \cdot 10^4$.

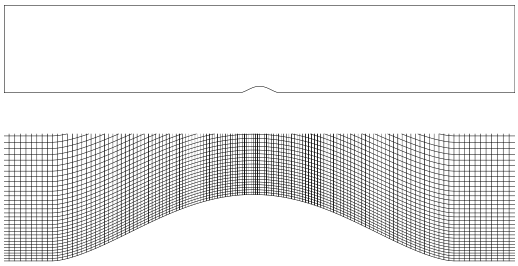

Fig 1: Computational domain 400x80 cells and zoom to non-uniform grid near hill.

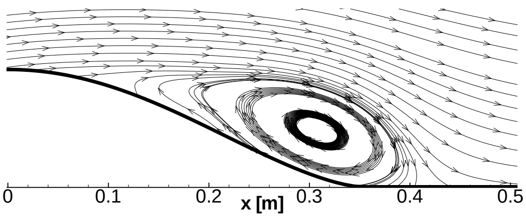


Fig 2: Zoom of separation zone behind hill.

## 5 Conclusion

The flow reattachment point is $x_r = 4.0H$ measured from the hill summit while the value due to Castro is $x_r = 4.1H$ for the standard $k - \varepsilon$ model and the experimentally determined value is $x_r = 6.5H$. The achieved results seems to be acceptable. The real–case 2D/3D numerical tests ($eg.$ in [4]) of the implemented turbulence model will follow.

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# References

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