Features Invariant Simultaneously to Convolution and Affine Transformation

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Abstract. The contribution is devoted to the recognition of objects and patterns deformed by imaging geometry as well as by unknown blurring. We introduce a new class of features invariant simultaneously to blurring with a centrosymmetric PSF and to affine transformation. As we prove in the contribution, they can be constructed by combining affine moment invariants and blur invariants derived earlier. Combined invariants allow to recognize objects in the degraded scene without any restoration.

1 Introduction

Recognition of objects and patterns that are deformed in various ways has been a goal of much recent research. The degradations (geometric as well as radiometric) are introduced during the image acquisition process by such factors as imaging geometry, illumination changes, wrong focus, lens aberration, systematic and random sensor errors, object occlusion, etc. Finding a set of invariant descriptors is a key step to recognizing degraded objects regardless of the particular deformations.

Many papers have been devoted to the invariants to transform of spatial coordinates, like rigid-body affine and projective transform (see [1], [2] for a survey and other references). Moment invariants [3], [4], Fourier-domain invariants [5], differential invariants [6] and point sets invariants [7], [8] belong to the most popular classes of geometric invariants.

Much less attention has been paid to invariants with respect to changes of the image intensity function (we call them radiometric invariants) and to combined radiometric-geometric invariants. Van Gool et al. introduced so-called affine-photometric invariants of graylevel [9] and color [10] images. These features are invariant to the affine transform and to the change of contrast and brightness of the image simultaneously. Some other authors used various local features (mostly derivatives of the image function) to find invariants to rigid-body transform and contrast brightness changes [11]. This technique has become popular in image retrieval, but it is far beyond the scope of this contribution. Numerous references can be found in [11].

An important class of radiometric degradations we are faced with often in practice is image blurring. Blurring can be caused by camera defocus, atmospheric turbulence, vibrations, and by sensor or/and scene motion, to name a few. If the scene is flat, blurring can be usually described by a convolution $g(x, y) = (f * h)(x, y)$, where f is an original (ideal) image, g is an acquired image and h is a point spread function (PSF) of the imaging system. Since in most practical tasks the PSF is unknown, having the invariants to convolution is of prime importance when recognizing objects in a blurred scene. A pioneer work on this field was done by Flusser and Suk [12] who derived invariants to convolution with an arbitrary centrosymmetric PSF. From the geometric point of view, their descriptors were invariant to translation only. Despite of this, the invariants have found successful applications in face recognition on defocused photographs [13], in normalizing blurred images into the canonical forms [14], in template-to-scene matching of satellite images [12], in blurred digit recognition [15], and in focus/defocus quantitative measurement [16]. Other sets of blur invariants (but still only shift- invariant) were derived for some particular kinds of PSF axisymmetric blur invariants [17] and motion blur invariants [18]. A significant improvement motivated by a problem of registration of blurred images was made by Flusser and Zitová. They introduced so-called combined blur-rotation invariants [19] and reported their successful usage in satellite image registration [20].

However, in a real world, the imaging geometry is projective rather than rigid-body. If the scene is flat and the camera far from the object in comparison to its size, the projective transform can be well approximated by an affine transform. Thus, having combined affine-blur invariants is in great demand.

In this contribution, we introduce combined invariants to affine transform and to convolution with an arbitrary centrosymmetric PSF, which are based on image moments. Since the affine transform does not preserve the shape of the objects, this cannot be done via straightforward generalization of previous work. In Section 2, we briefly recall the basic terms and earlier results of the theory of invariants that will be used later in the contribution. Section 3 performs the core of this work – we present there the central theorem on combined invariants. Section 4 demonstrates the numerical properties and recognition power of the invariants.

2 Recalling the Theory of the Invariants

In this Section we introduce some basic terms and briefly recall the theorems on the invariants which will be used later in this contribution.

2.1 Basic Terms

Definition 1: By *image function* (or *image*) we understand any real function $f(x, y) \in L_1$ having a bounded support and a nonzero integral.

Definition 2: *Central moment* $\mu_{pq}^{(f)}$ of order $(p+q)$ of the image $f(x,y)$ is defined as

$$\mu_{pq}^{(f)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-x_c)^p (y-y_c)^q f(x,y)dxdy, \tag{1}$$

where the coordinates (x_c, y_c) denote the centroid of $f(x,y)$.
Definition 3: *Affine transform* is a transform of spatial coordinates (x,y) into (u,v) defined by equations

$$\begin{aligned} u &= a_0 + a_1 x + a_2 y, \\ v &= b_0 + b_1 x + b_2 y. \end{aligned} \tag{2}$$

Proposition 1: Every PSF mentioned in this contribution is assumed to be *centrosymmetric* and *energy-preserving,* i.e.

$$h(x,y) = h(-x,-y),$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)dxdy = 1.$$

The assumption of centrosymmetry is not a significant limitation of practical utilization of the method. Most real sensors and imaging systems, both optical and non-optical ones, have the PSF with certain degree of symmetry. In many cases they have even higher symmetry than central one, such as axial or radial symmetry. Thus, the central symmetry is general enough to handle almost all practical situations.

Note that because of centrosymmetry all moments of odd orders of the PSF equal zero.

2.2 Blur Invariants

In our earlier paper [12], the following theorem of blur invariants was proven.
Theorem 1: Let $f(x,y)$ be an image function. Let us define the following function $C^{(f)} : \mathcal{N}_0 \times \mathcal{N}_0 \to \mathcal{R}$.
If $(p+q)$ is even then

$$C(p,q)^{(f)} = 0.$$

If $(p+q)$ is odd then

$$C(p,q)^{(f)} = \mu_{pq}^{(f)} - \frac{1}{\mu_{00}^{(f)}} \sum_{n=0}^{p} \sum_{\substack{m=0 \\ 0<n+m<p+q}}^{q} \binom{p}{n}\binom{q}{m} C(p-n, q-m)^{(f)} \cdot \mu_{nm}^{(f)}. \tag{3}$$

Then $C(p,q)$ is invariant to convolution with any centrosymmetric function $h(x,y)$, i.e.

$$C(p,q)^{(f)} = C(p,q)^{(f*h)}$$

for any p and q.

2.3 Affine Moment Invariants

Affine moment invariants (AMI's) were introduced independently by Reiss [21] and Flusser and Suk [22]. They originate from the classical theory of algebraic invariants [23], [24]. The fundamental theorem, which was used for the derivation of the explicit forms of the invariants, can be formulated as follows.
Theorem 2: If the binary form of order p has an algebraic invariant of weight w and order k

$$I(a'_{p,0}, \cdots, a'_{0,p}) = \triangle^w I(a_{p,0}, \cdots, a_{0,p})$$

($\triangle$ denotes the determinant of the respective affine transform) then the moments of order p have the same invariant but with the additional factor $|J|^k$:

$$I(\mu'_{p0}, \cdots, \mu'_{0p}) = \triangle^w |J|^k I(\mu_{p0}, \cdots, \mu_{0p}),$$

where $|J|$ is the absolute value of the Jacobian.

We refer to [22], [21] for the proof of Theorem 2 and for a deeper explanation of the theory of the AMI's.

3 Combined Blur and Affine Invariants

By combined blur-affine invariants (CBAI's) we understand any functional defined on the set of image functions whose value does not change if the image function is convolved with a centrosymmetric PSF and transformed by an affine transform. Note, that the convolution and the affine transform are commutative here. The following theorem not only guarantees the existence of the CBAI's but also provides an explicit algorithm how to construct them.
Theorem 3: Let $I(\mu_{00}, \cdots, \mu_{PQ})$ be an affine moment invariant. Then $I(C(0,0), \cdots, C(P,Q))$, where $C(0,0) = \mu_{00}$ and all other blur invariants $C(p,q)$ are defined by Theorem 1, is a combined blur-affine invariant.

The proof can be carried out in the following way. Since $I(C(0,0), \cdots, C(P,Q))$ is a function of blur invariants $C(p,q)$ only, it is also a blur invariant. To prove its invariance to affine transform, it is sufficient to prove that partial derivatives with respect to each parameter $a \in \{a_0, a_1, a_2, b_0, b_1, b_2\}$ of the affine transform equal zero. Since it holds for $I(\mu_{00}, \cdots, \mu_{PQ})$, it is sufficient to prove that this property is preserved when substituting $C(p,q)$ for μ_{pq}. To do that, the affine transform (2) can be decomposed into six one-parametric transformations: horizontal translation, vertical translation, uniform scaling, stretching, horizontal skewing and vertical skewing. It is sufficient to prove the invariance to each of these transformations separately, which is easier than to prove it directly to (2).

To express the simplest CBAI's in explicit forms, we take five affine moment invariants of the 3rd and 5th orders:

$$I_1 = (\mu_{30}^2\mu_{03}^2 - 6\mu_{30}\mu_{21}\mu_{12}\mu_{03} + 4\mu_{30}\mu_{12}^3 + 4\mu_{21}^3\mu_{03} - 3\mu_{21}^2\mu_{12}^2)/\mu_{00}^{10},$$

$$\begin{aligned}I_2 = (&\mu_{50}^2\mu_{05}^2 - 10\mu_{50}\mu_{41}\mu_{14}\mu_{05} + 4\mu_{50}\mu_{32}\mu_{23}\mu_{05} + 16\mu_{50}\mu_{32}\mu_{14}^2 - 12\mu_{50}\mu_{23}^2\mu_{14}\\&+16\mu_{41}^2\mu_{23}\mu_{05} + 9\mu_{41}^2\mu_{14}^2 - 12\mu_{41}\mu_{32}^2\mu_{05} - 76\mu_{41}\mu_{32}\mu_{23}\mu_{14} + 48\mu_{41}\mu_{23}^3\\&+48\mu_{32}^3\mu_{14} - 32\mu_{32}^2\mu_{23}^2)/\mu_{00}^{14},\end{aligned}$$

$$\begin{aligned}I_3 = (&\mu_{30}^2\mu_{12}\mu_{05} - \mu_{30}^2\mu_{03}\mu_{14} - \mu_{30}\mu_{21}^2\mu_{05} - 2\mu_{30}\mu_{21}\mu_{12}\mu_{14} + 4\mu_{30}\mu_{21}\mu_{03}\mu_{23}\\&+2\mu_{30}\mu_{12}^2\mu_{23} - 4\mu_{30}\mu_{12}\mu_{03}\mu_{32} + \mu_{30}\mu_{03}^2\mu_{41} + 3\mu_{21}^3\mu_{14} - 6\mu_{21}^2\mu_{12}\mu_{23}\\&-2\mu_{21}^2\mu_{03}\mu_{32} + 6\mu_{21}\mu_{12}^2\mu_{32} + 2\mu_{21}\mu_{12}\mu_{03}\mu_{41} - \mu_{21}\mu_{03}^2\mu_{50} - 3\mu_{12}^3\mu_{41}\\&+\mu_{12}^2\mu_{03}\mu_{50})/\mu_{00}^{11},\end{aligned}$$

$$\begin{aligned}I_4 = (&2\mu_{30}\mu_{12}\mu_{41}\mu_{05} - 8\mu_{30}\mu_{12}\mu_{32}\mu_{14} + 6\mu_{30}\mu_{12}\mu_{23}^2 - \mu_{30}\mu_{03}\mu_{50}\mu_{05}\\&+3\mu_{30}\mu_{03}\mu_{41}\mu_{14} - 2\mu_{30}\mu_{03}\mu_{32}\mu_{23} - 2\mu_{21}^2\mu_{41}\mu_{05} + 8\mu_{21}^2\mu_{32}\mu_{14} - 6\mu_{21}^2\mu_{23}^2\\&+\mu_{21}\mu_{12}\mu_{50}\mu_{05} - 3\mu_{21}\mu_{12}\mu_{41}\mu_{14} + 2\mu_{21}\mu_{12}\mu_{32}\mu_{23} + 2\mu_{21}\mu_{03}\mu_{50}\mu_{14}\\&-8\mu_{21}\mu_{03}\mu_{41}\mu_{23} + 6\mu_{21}\mu_{03}\mu_{32}^2 - 2\mu_{12}^2\mu_{50}\mu_{14} + 8\mu_{12}^2\mu_{41}\mu_{23}\\&-6\mu_{12}^2\mu_{32}^2)/\mu_{00}^{12},\end{aligned}$$

$$\begin{aligned}I_5 = (&\mu_{30}\mu_{41}\mu_{23}\mu_{05} - \mu_{30}\mu_{41}\mu_{14}^2 - \mu_{30}\mu_{32}^2\mu_{05} + 2\mu_{30}\mu_{32}\mu_{23}\mu_{14} - \mu_{30}\mu_{23}^3\\&-\mu_{21}\mu_{50}\mu_{23}\mu_{05} + \mu_{21}\mu_{50}\mu_{14}^2 + \mu_{21}\mu_{41}\mu_{32}\mu_{05} - \mu_{21}\mu_{41}\mu_{23}\mu_{14} - \mu_{21}\mu_{32}^2\mu_{14}\\&+\mu_{21}\mu_{32}\mu_{23}^2 + \mu_{12}\mu_{50}\mu_{32}\mu_{05} - \mu_{12}\mu_{50}\mu_{23}\mu_{14} - \mu_{12}\mu_{41}^2\mu_{05} + \mu_{12}\mu_{41}\mu_{32}\mu_{14}\\&+\mu_{12}\mu_{41}\mu_{23}^2 - \mu_{12}\mu_{32}^2\mu_{23} - \mu_{03}\mu_{50}\mu_{32}\mu_{14} + \mu_{03}\mu_{50}\mu_{23}^2 + \mu_{03}\mu_{41}^2\mu_{14}\\&-2\mu_{03}\mu_{41}\mu_{32}\mu_{23} + \mu_{03}\mu_{32}^3)/\mu_{00}^{13}.\end{aligned}$$

and substitute $C(p,q)$ for μ_{pq}. Since the even-order $C(p,q)$'s equal zero by definition, only those AMI's composed from odd-order moments are meaningful for this purpose. The explicit forms of $C(p,q)$'s can be found in [12] or derived directly from Theorem 1. Those needed for substitution into the above AMI's are listed below.

$$C(3,0) = \mu_{30}, \qquad C(2,1) = \mu_{21}, \qquad C(1,2) = \mu_{12}, \qquad C(0,3) = \mu_{03},$$

$$C(5,0) = \mu_{50} - \frac{10\mu_{30}\mu_{20}}{\mu_{00}},$$

$$C(4,1) = \mu_{41} - \frac{2}{\mu_{00}}(3\mu_{21}\mu_{20} + 2\mu_{30}\mu_{11}),$$

$$C(3,2) = \mu_{32} - \frac{1}{\mu_{00}}(3\mu_{12}\mu_{20} + \mu_{30}\mu_{02} + 6\mu_{21}\mu_{11}),$$

$$C(2,3) = \mu_{23} - \frac{1}{\mu_{00}}(3\mu_{21}\mu_{02} + \mu_{03}\mu_{20} + 6\mu_{12}\mu_{11}),$$

$$C(1,4) = \mu_{14} - \frac{2}{\mu_{00}}(3\mu_{12}\mu_{02} + 2\mu_{03}\mu_{11}),$$

$$C(0,5) = \mu_{05} - \frac{10\mu_{03}\mu_{02}}{\mu_{00}}.$$

4 Numerical Experiment

Two sequences of images (the comb and the picture of the Earth) of size 512×256 pixels were created to test the numerical properties of the new invariants. They were captured and affinely deformed by weak and strong affine transformations. Each image was then blurred by two square masks, 5×5 and 11×11. The values of five combined invariants from the previous section were computed for all combinations of blur and affine transformation (see Fig. 1). The values of the invariants are shown in Table 1.

We can see that the values of the invariants within one class are very close to each other. Slight deviations are caused by quantization and discretization errors. On the other hand, the difference between the invariant values of the comb and Earth pictures is much bigger than the intra-class deviations, what illustrates a good discrimination power of the invariants.

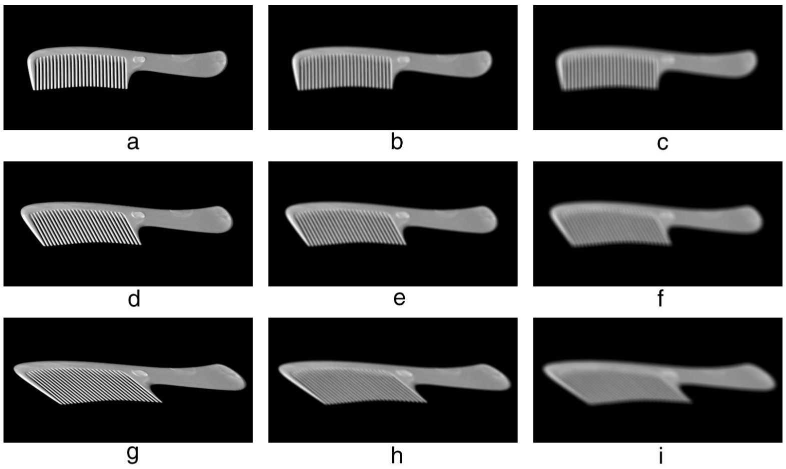

Fig. 1. The object "Comb", (a) original, (b) blurred by square mask 5×5, (c) blurred by square mask 11×11, (d) weakly affinely deformed, without blurring, (e) weakly affinely deformed and blurred by square mask 5×5, (f) weakly affinely deformed and blurred by square mask 11×11, (g) strongly affinely deformed, without blurring, (h) strongly affinely deformed and blurred by square mask 5×5 and (i) strongly affinely deformed and blurred by square mask 11×11.

5 Conclusion

The paper was devoted to the image features which are invariant simultaneously to blurring by a filter with centrally symmetric PSF and to the affine transformation. The major theoretical result of this work is Theorem 3, showing that

Table 1. The values of the combined affine and blur invariants

Object	Transform	Blur	$I_1[10^{-10}]$	$I_2[10^{-11}]$	$I_3[10^{-11}]$	$I_4[10^{-11}]$	$I_5[10^{-12}]$
Comb	-	-	-15810	-17820	2520	56040	-1641
Comb	weak	-	-15730	-17760	2514	55780	-1647
Comb	strong	-	-15730	-17770	2514	55780	-1642
Comb	-	5×5	-16150	-18450	2584	57650	-1692
Comb	weak	5×5	-16080	-18400	2578	57390	-1697
Comb	strong	5×5	-16080	-18410	2582	57400	-1695
Comb	-	11×11	-16190	-18510	2591	57810	-1699
Comb	weak	11×11	-16110	-18460	2582	57560	-1702
Comb	strong	11×11	-16110	-18470	2588	57550	-1.700
Earth	-	-	-2.095	6.952	3.758	5.844	3.067
Earth	weak	-	-2.074	6.852	3.593	5.791	3.082
Earth	strong	-	-2.045	6.387	3.712	5.903	3.224
Earth	-	5×5	-2.164	7.420	3.935	6.014	3.228
Earth	weak	5×5	-2.143	7.326	3.755	5.951	3.249
Earth	strong	5×5	-2.115	6.831	3.886	6.079	3.398
Earth	-	11×11	-2.171	7.462	3.945	6.016	3.242
Earth	weak	11×11	-2.143	7.383	3.767	5.920	3.258
Earth	strong	11×11	-2.122	6.891	3.903	6.069	3.415

the combined invariants can be constructed by substituting blur invariants into the affine moment invariants. The numerical experiment verified the theoretical results and also illustrated the discrimination power of the invariants.

Practical applications of the new invariants may be found in object recognition in blurred and noisy environment, in template matching, image registration, etc.

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