The Existence of States on Every Archimedean Atomic Lattice Effect Algebra with At Most Five Blocks

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Abstract: Effect algebras are very natural logical structures as carriers of probabilities and states. They were introduced for modeling of sets of propositions, properties, questions, or events with fuzziness, uncertainty or unsharpness. Nevertheless, there are effect algebras without any state, and questions about the existence (for non-modular) are still unanswered. We show that every Archimedean atomic lattice effect algebra with at most five blocks (maximal MV-subalgebras) has at least one state, which can be obtained by "State Smearing Theorem" from a state on its sharp elements.

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