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## Rational Semimodules Over the Max-plus Semiring and Geometric Approach to Discrete Event Systems

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Abstract: We introduce rational semimodules over semirings whose addition is idempotent, like the max-plus semiring, in order to extend the geometric approach of linear control to discrete event systems. We say that a subsemimodule of the free semimodule  $^n$  over a semiring is rational if it has a generating family that is a rational subset of  $^n$ ,  $^n$  being thought of as a monoid under the entrywise product. We show that for various semirings of max-plus type whose elements are integers, rational semimodules are stable under the natural algebraic operations (sum, product, direct and inverse image, intersection, projection, etc). We show that the reachable and observable spaces of max-plus linear dynamical systems are rational, and give various examples.

*Keywords:* invariant spaces; reachability; geometric control; rational sets; Presburger arithmetics; max-plus algebra; discrete event systems;

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