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NEW TRENDS IN DESIGN OF OBSERVERS
FOR TIME–DELAY SYSTEMS

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This paper presents some recent results about the design of observers for time-delay systems. It is focused on methods that can lead to design some useful observers in practical situations. First the links between observability properties and observers design is emphasized. Then some necessary and sufficient conditions and a method are provided to obtain unknown input observers for time-delay systems. Furthermore some H_∞ design using Lyapunov–Krasovskii and Lyapunov–Razumikhin theories are presented and compared. Finally a polynomial approach based on the parametrization of all observers is proposed that allows to design robust observers for systems including unstructured uncertainties.

1. INTRODUCTION

From the last ten years, control and analysis of systems with delays is subject to lots of studies and contributions. The main approaches are the state space approach based on Hilbert space (Manitius and Triggiani [29]) which today seems to be oldfashioned due to the difficulty of applying on potential industrial processes, the Lyapunov–Krasovskii and Lyapunov–Razumikhin stability theories (Dugard and Verriest [7], Niculescu [31]) which have proved their efficiency for stabilization purpose, in particular in H_∞ framework, and the ring theory (Conte and Perdon [4], Picard et al [35], Sename [40]) which has contributed to solve many control problems (decoupling, disturbance rejection...) in structural and geometric frameworks and which allows to use some polynomial approach in order to provide robust control laws for point and distributed time-delays.

In the same time, industrial applications of these themes rise up. Let us cite for instance the automotive industry where engine control is faced to delay in the control input application or in the measurements (for instance of the air-to-fuel ratio) [21, 38] or the electrical networks where transport delays may lead to unstability problems [20].

On the other hand, if many contributions concern control problems, much less are devoted to observer design which is of great importance in practice. Nevertheless some interesting recent results have been provided for this observation problem and it then seems interesting to take a photograph of these recent design methods.
This paper makes a survey of the observer design problem for time-delay systems.

The aim is to give to the reader the ideas of the main methods one can use to design some observers. If the new trends here presented do not include all contributions concerning this subject, these are focused on methods that can lead to design some useful observers in practical situations. Moreover, in order to be more synthetic, the area is restricted to continuous-time processes with point-time delays.

The outline of the paper is the following. The considered models (mainly a ring model) for time-delay systems are presented in Section 2. Section 3 is devoted to observability properties of time-delay systems. Section 4 emphasizes the main observer design families according to the observability properties. The case of unknown input observer is tackled in Section 5. Section 6 deals with H_∞ observers and Section 7 presents two main approaches of robust observer design: H_∞ one and the factorization one. Some future work directions are given in conclusion.

Notations:

$\mathbb{R}$ is the field of real numbers and $\mathbb{N}$ is the set of positive integer numbers,
the exponent $+$ is associated to positive numbers $\{0 < a < +\infty\}$, for $a \in \mathbb{R}$ or $\mathbb{C}$,
$\mathbb{R}[\star]$ is the ring of polynomials in $\star$ with coefficients in $\mathbb{R}$,
$\mathbb{R}(\star)$ is the fields of rational fractions in $\star$ with coefficients in $\mathbb{R}$,
s denotes the Laplace variable
$\mathbb{R}[z][s] = \{\sum_{k=0}^{m} a_k(z)s^k : a_k(z) \in \mathbb{R}[z], m \in \mathbb{N}^+\}$,
$\mathbb{R}(s, z)$ is the field of rational functions in s and z with coefficients in $\mathbb{R}$,
$\Theta = \{p(s, z) = \frac{b(s,z)}{a(s)} \in \mathbb{R}(s, z) : b(s, z) \in \mathbb{R}[z][s] \ a(s) \in \mathbb{R}[s], \ \deg_s(a(s)) > \deg_s(b(s, z))$ and $p(s, z)$ is entire$\}$,
$\Theta[z]$ is the ring of polynomials in z with coefficients in Θ,
$\mathbf{F} = \{p(s, z) = \frac{b(s,z)}{a(s)} \in \Theta[z] : a(s)$ is monic and stable$\}$,
I_n denotes the $(n \times n)$ identity matrix,
$0_{i \times j}$ denotes the $(i \times j)$ zero matrix,
"T" denotes the transpose,
$\|.\|_\infty$ is the H_∞-norm defined by: $\|X(s)\|_\infty = \sigma_{\max}(X(j\omega))$; $\sigma_{\max}(X)$ denotes the maximum singular value of X, j is the imaginary number, ω denotes the frequency.
$\mathcal{C}[a, b]$ is the set of continuous functions $[a, b] \to \mathbb{R}^n$.

2. MODELING OF TIME–DELAY SYSTEMS

In this paper, we are only interested in continuous-time processes with delays of which models are continuous linear time-invariant given by:

$$\begin{cases} \dot{x}(t) &= \sum_{i=0}^{N} A_i x(t - ih) + \sum_{i=0}^{N} B_i u(t - ih) \\ y(t) &= \sum_{i=0}^{N} C_i x(t - ih) \end{cases} \tag{1}$$

where
  – $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ the known input vector, $y(t) \in \mathbb{R}^p$ the output vector
  – $h \in \mathbb{R}^+$ is the delay and N represents the maximal delay in state, input and output variable

– matrices A_i, B_i, C_i $(i = 0, \ldots, N)$ are real matrices of appropriate dimensions
– $x(t) = \varphi(t), t \in [-Nh, 0]$ is the functional initial condition of the time-delay system.

Moreover ring models of time-delay systems are also considered. Thus a linear, time-invariant system with delays in state, inputs and outputs, is in this case represented by [30, 43]:

$$\Sigma \begin{cases} \dot{x}(t) &=& A(\nabla)x(t) + B(\nabla)u(t) \\ y(t) &=& C(\nabla)x(t) \end{cases} \tag{2}$$

where

– ∇ is the delay operator defined for any continuous function $f(t)$ by $\nabla f(t) = f(t-h)$.
– matrices A, B, C are polynomial matrices in ∇ given by: $A(\nabla) = \sum_{i=0}^{N} \nabla^i A_i$, $B(\nabla) = \sum_{i=0}^{N} \nabla^i B_i$ and $C(\nabla) = \sum_{i=0}^{N} \nabla^i C_i$.

3. OBSERVABILITY OF TIME–DELAY SYSTEMS OVER RINGS

In the case of linear non-retarded systems observability is defined as:

Definition 1. A linear system $(\dot{x} = Ax + Bu; y = Cx)$ is completely observable if, given the control and the output over the interval $t_0 \leq t \leq T$, one can determine any initial state $x(t_0)$.

Note that this is equivalent to the state reconstruction of $x(T)$ and observability is characterized through its unobservable subspace i.e.

$$\bigcap_{i=1}^{n} \mathrm{Ker}\, C A^{i-1} = \{0\}.$$

In the case of time-delay systems lots of observability properties are defined, in particular depending on the model which is taken into account [25]. Of course each definition leads to different characteristics of state observation.
The basic extension of observability to time-delay systems is the initial observability, i.e.:

any initial state $(x(0), \varphi(t), t \in [-Nh, 0[)$ is observable if the output of the autonomous system is not identically zero on $[0, \infty)$.

Note that, when the initial function φ is assumed to be zero or a known function, this definition reduces to the $\mathbb{R}^n$-observability which is not useful in practice.

On the other hand, knowing the initial condition is not necessary for control purpose, what is important is to be able to reconstruct $x(t)$ at any time t. However, by opposition to linear systems without delay, the notion of initial observability is not equivalent to the reconstructibility of the state variables: indeed time-delay may have finite duration transient behaviour.

Therefore this paper does not consider these previous notions (initial and $\mathbb{R}^n$-observability). We are interested here in observability notions that are the most useful ones in the design of observers.

In this way, we briefly present some usual definitions of observability for ring models of time-delay systems.

The observability matrix of (2) is noted:

$$\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle = \left[\begin{array}{c} C(\nabla) \\ C(\nabla)A(\nabla) \\ \vdots \\ C(\nabla)A^{n-1}(\nabla) \end{array} \right].$$

The first presented notion is the dual one of the well-known strong controllability defined by [30, 43].

Definition 2. [25] The system (2) is strongly observable (i. e., observable over the ring $\mathbb{R}[\nabla]$) if it satisfies the following equivalent characterizations:

– the Smith form of $\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle$ is $\left(\begin{array}{c} I_n \\ 0 \end{array} \right)$

– $\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle$ has a left inverse over $\mathbb{R}[\nabla]$

– rank $\left[\begin{array}{c} sI_n - A(z) \\ C(z) \end{array} \right] = n, \quad \forall\ s,\ z \in \mathbb{C}.$

We will see later that this notion guarantees the reconstructibility of $x(t)$ at any time t.

The next notion corresponds to the extension of the Hautus condition to time-delay systems.

Definition 3. [25] The system (2) is spectrally observable if it satisfies the following characterization: rank $\left[\begin{array}{c} sI_n - A(e^{-sh}) \\ C(e^{-sh}) \end{array} \right] = n, \quad \forall\ s \in \mathbb{C}.$

When this condition is applied for $s \in \{s \in \mathbb{C} : Re(s) \geq \nu_0\}$, it is called ν_0-(spectral) detectability.

Finally, the last presented notion is the less restrictive characterization of observability and corresponds to the extension of the unobservable subspace definition to time-delay systems, i. e. in this case the unobservable submodule.

Definition 4. [25] The system (2) is weakly observable (i.e., observable over the field $\mathbb{R}(\nabla)$) if it satisfies the following equivalent characterizations:

- rank $\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle = n$

- the Smith form of $\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle$ has non zero elements on its diagonal

- $< \frac{C(\nabla)}{A(\nabla)} >$ has a left inverse over $\mathbb{R}(\nabla)$

- there exists no $x \in \mathbb{R}^n(\nabla)$ such that:
$C(\nabla)x = C(\nabla)A(\nabla)x = \ldots = C(\nabla)A^{n-1}(\nabla)x = 0$ and $x \neq 0$.

Remark 1. Note that Sontag [43] has claimed that controllability and observability looses duality for time-delay systems. In fact in his work, the considered definition for controllability was the strong controllability one, while the considered definition for observability was the extension of unobservable subspace definition, i.e. the weak one.

As pointed out in [25] the relationship between these definitions is:
Strong observability $\Rightarrow$ spectral observability $\Rightarrow$ weak observability.

The following simple example illustrates this comparison.

Example 1. Let us consider the following time-delay system:

$$\begin{cases} \dot{x}(t) & = & 0 \\ y(t) & = & x(t) - x(t - h) \end{cases}$$

Therefore

$$A(\nabla) = \begin{bmatrix} 0 \end{bmatrix}, \quad C(\nabla) = \begin{bmatrix} 1 - \nabla \end{bmatrix}.$$

Let us apply the previous criteria of observability.
The observability matrix is given by:

$$\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle = 1 - \nabla$$

- Strong observability: it is straightforward that the observability matrix has no left inverse over $\mathbb{R}[\nabla]$. Thus the system is not strongly observable.
- Spectral observability:

$$\begin{bmatrix} sI_n - A(z) \\ C(z) \end{bmatrix} = \begin{bmatrix} s \\ 1 - e^{-sh} \end{bmatrix}$$

which is of rank 1 $\forall \, s \in \mathbb{C}$ except for $s = 0$. Therefore the system is not spectrally observable.
- Weak observability: it is straightforward that the observability matrix has a left inverse over $\mathbb{R}(\nabla)$. Thus the system is weakly observable.

Note that taking $C(\nabla) = 1 + \nabla$ will give a spectrally observable system and then a weakly observable one.

4. OBSERVERS FOR TIME–DELAY SYSTEMS

In this part, a synthesis of the main contributions concerning the observer design is presented according to the observability assumption given in the previous section.

The understanding of the main constraints that come from the observability properties is important to get used with the methods which are provided in each case. The robustness issue will be considered in the further sections.

4.1. Strongly observable systems

This case does not present any specific difficulty. Indeed strong observability is dual to strong controllability for which it is well-known that pole placement (with polynomial poles) and coefficient assignment (with polynomial coefficients) can be solved. The design of observers is then straightforward by solving a pole placement problem for the dual system.

Let us recall that, for time-delay systems over a ring, coefficient assignment imply pole placement but the converse is not true as a characteristic polynomial may have non polynomial roots. This is because time-delay systems are systems over a ring. For systems over a field (classical linear systems), these problems are equivalent.

Concerning pole placement the problem has been solved by Sontag [43], Morse [30] and the reader is referred to the works of Lee and Zak [26], Eising [8] for some calculation procedures.

Concerning coefficient assignment, Lee and Lu [24] have provided a method to solve the problem by using some canonical form.

Therefore, the design of observers for strongly observable systems has been few considered in the literature. In [25] the dualization of Morse's result is presented to ensure the existence of some asymptotic observer. Pourboghrat and Chyung [37] have given a procedure to obtain a finite-time observer (exact) which is unusable when some parameter changes are considered as well as some external disturbances.

Finally Emre and Khargonekar [9] have proposed a polynomial approach to design observers by assigning the coefficient of the characteristic polynomial. The observer is constructed by solving some Bezout equations.

4.2. Spectrally canonical realizations: the factorization approach

The factorization approach is a very efficient tool to design robust observer for linear systems. This section is only devoted to the works of Yao et al [50, 51] where no robustness issue is provided: this will be considered in Section 7.

In [50] this approach is used to parameterize the set of all observers for time-delay systems, in a similar way to the results of Nobuyama and Kitamori [33] for controllers design. The results concern spectrally canonical (i. e. spectrally controllable and spectrally observable) systems. However, as, in general, a transfer function may not have a realization that is spectrally canonical, Kamen et al [22] have proved that the existence and construction of spectrally canonical realizations can be approached in terms of co-canonical realizations (i. e. weakly controllable and strongly observable systems). Indeed Kamen et al [22] have proved that assuming a spectrally canonical

realization for a time-delay system is equivalent to requiring that the co-canonical realization is spectrally controllable. The interest is that any transfer function has a co-canonical realization which can be computed in a finite number of steps.

Proper stable double Bezout factorizations are then constructed in a state-space framework as follows:

Lemma 1. [33] Consider the transfer function matrix:

$$G(s, z) = C(z) \left(sI_n - A(z) \right)^{-1} B(z) \tag{3}$$

associated with a co-canonical and spectrally controllable realization (2). The transfer function (3) can be factorized as follows

$$G(s, z) = N(s, z) M^{-1}(s, z) = \bar{M}^{-1}(s, z) \bar{N}(s, z) \tag{4}$$

where N, M, $\bar{M}$, $\bar{N}$ satisfy the following double Bezout equation

$$\begin{bmatrix} Y & X \\ -\bar{N} & \bar{M} \end{bmatrix} \begin{bmatrix} M & -\bar{X} \\ N & \bar{Y} \end{bmatrix} = \begin{bmatrix} I_m & 0_{m \times p} \\ 0_{p \times m} & I_p \end{bmatrix}. \tag{5}$$

The eight matrices in (5) are matrices over **F** and can be obtained as follows:

$$\begin{cases} M &=& I_m + F_e(sI_e - A_0)^{-1} B_e \\ \bar{M} &=& I_p + C_e(sI_e - \bar{A}_0)^{-1} K_e \\ N &=& C_e(sI_e - A_0)^{-1} B_e \\ \bar{N} &=& C_e(sI_e - \bar{A}_0)^{-1} B_e \\ Y &=& I_m - F_e(sI_e - \bar{A}_0)^{-1} B_e \\ \bar{Y} &=& I_p - C_e(sI_e - A_0)^{-1} K_e \\ X &=& F_e(sI_e - \bar{A}_0)^{-1} K_e \\ \bar{X} &=& F_e(sI_e - A_0)^{-1} K_e \end{cases} \tag{6}$$

where

$$\begin{aligned} A_0(s, z) &=& A_e(z) + B_e(z) F_e(z); \quad \bar{A}_0(z) = A_e(z) + K_e(z) C_e(z) \\ A_e(z) &=& \begin{bmatrix} A(z) & B(z) \\ 0_{m \times n} & -I_m \end{bmatrix} \quad B_e(z) = \begin{bmatrix} 0_{n \times m} \\ I_m \end{bmatrix} \\ C_e(z) &=& \begin{bmatrix} C(z) & 0_{p \times m} \end{bmatrix} \\ F_e(s, z) &=& \begin{bmatrix} F_1(s, z) & F_2(s, z) \end{bmatrix} \\ K_e(z) &=& \begin{bmatrix} K(z) \\ 0_{m \times p} \end{bmatrix} \quad I_e = \begin{bmatrix} I_n & 0_{n \times m} \\ 0_{m \times n} & 0_{m \times m} \end{bmatrix} \end{aligned}$$

$F_1(s,z)$, $F_2(s,z)$ are defined over the ring $\Theta[z]$ and $K(z)$ over the ring $\mathbb{R}[z]$. They can be chosen such that

$$\det \left[\begin{array}{cc} sI_n - A(z) & -B(z) \\ -F_1(s,z) & I_m - F_2(s,z) \end{array} \right] = \alpha(s)$$

$$\det[sI_n - A(z) - K(z)C(z)] = \beta(s)$$

$\alpha(s)$, $\beta(s) \in \mathbb{R}[s]$ are stable polynomials.

Using this lemma, all stable observers for a time-delay system can be parameterized depending on the double Bezout factorization.

Let $r(t) = E(\nabla)x(t) \in \mathbb{R}^{k \times 1}$, where $E(\nabla) \in \mathbb{R}[\nabla]^{k \times n}$, be the variable to be observed. An asymptotic observer of $r(t)$, is a dynamic system

$$\hat{r}(s) = U(s,z)\,u(s) + V(s,z)\,y(s) \tag{7}$$

where the entries of $U(s,z)$, $V(s,z)$ belong to $\Theta[z]$, with the property that for all $u(t)$:

$$\lim_{t \to \infty} (r(t) - \hat{r}(t)) = 0.$$

Remark 2. Note that by choosing $E = I_n$, $r(t)$ is the whole state $x(t)$ and (7) is therefore a classical observer. On the other hand, E can be chosen as a designed state feedback control F to be implemented trough an observer.

A parameterization of all stable observers of $r(t)$ based on the factorization (4) – (5) is given in the following Lemma.

Lemma 2. [50] Consider systems (2) and (7). The set of matrices $U(s,z)$, $V(s,z)$ such that system (7) is an observer of $r(s) = E(z)x(s)$ is given by

$$\left\{ \begin{array}{rcl} U(s,z) & = & P(s,z)\,Y(s,z) - Q(s,z)\,\bar{N}(s,z) \\ V(s,z) & = & P(s,z)\,X(s,z) + Q(s,z)\,\bar{M}(s,z) \end{array} \right. \tag{8}$$

where $P(s,z)$ (whose entries belong to $\Theta[z]$) is given by

$$\begin{array}{rcl} P(s,z) & = & E_e(z)\,(sI_e - A_0(s,z))^{-1}B_e(z) \\ E_e(z) & = & [E(z) \quad 0_{k \times r}] \end{array} \tag{9}$$

Y, $\bar{N}$, X, $\bar{M}$ are given by the double Bezout factorization (5) – (6) and $Q(s,z)$ is any matrix over $\Theta[z]$.

Remark 3. In [51] the case of delayed-input system is tackled; the same method as in [50] is used to obtain a functional observer and a state feedback as:

$$\hat{r}(s) = U(s)\,u(s) + V(s)\,y(s) \text{ and } u(s) = \hat{r}(s) + v(s).$$

A particular case of the previous parametrization (8) is obtained such that Loop Transfer Recovery is achieved, i.e. such that the closed-loop transfer function is the same as using $u(s)$ without the observer.

The factorization method is interesting because it considers a general form of observers with more freedom degrees to improve robustness of the designed observer. The way to design $Q(s)$ to ensure robustness is developed in Section 7.

4.3. Spectrally observable systems

4.3.1. A delay dependent method for spectrally observable systems
for any delay

The recent result of Tornambè [45] provides a delay dependent observer with point time-delay. The aim is the following:

– assume the spectral observability *for any value of time-delay*, and thus for zero delay.
– Decompose a delayed-state system as:

$$\begin{cases} \dot{x}(t) &= (A_0 + A_1)\,x(t) + A_1(x(t-h) - x(t)) + Bu(t) \\ y(t) &= Cx(t). \end{cases}$$

– Use the $(C, A_0 + A_1)$ observability assumption (i.e. spectral observability for zero delay value) to do linear state space transformation $(\zeta(t) = Mx(t))$, output space transformation $(w(t) = Ny(t))$ and an output injection $(Hw(t))$. This leads to some block canonical observer form given by:

$$\begin{cases} \dot{\zeta}(t) &= A^\circ \zeta(t) + A_1^\circ(\zeta(t-h) - \zeta(t)) + Hw(t) + B_0 u(t) \\ w(t) &= C^\circ \zeta(t) \end{cases}$$

where (C°, A°) is observable.
– Design the following observer:

$$\begin{cases} \dot{\hat{\zeta}}(t) &= A^\circ \hat{\zeta}(t) + A_1^\circ(\hat{\zeta}(t-h) - \hat{\zeta}(t)) + Hw(t) + B_0 u(t) \\ &\quad + K_0(C^\circ \hat{\zeta}(t) - w(t)) + K_0^1(C^\circ \hat{\zeta}(t-h) - w(t-h)) \end{cases}$$

i.e. the estimation error $\tilde{\zeta} = \hat{\zeta}(t) - \zeta(t)$:

$$\dot{\tilde{\zeta}}(t) = (A^\circ + K_0 C^\circ)\tilde{\zeta}(t) + (A_1^\circ + K_0^1 C^\circ)\,(\tilde{\zeta}(t-h) - \tilde{\zeta}(t))$$

where K_0 is chosen so that $(A^\circ + K_0 C^\circ)$ is stable.
– Tornambè [45] then uses some delay-dependent stability criterion to state that if $(A^\circ + K_0 C^\circ)$ is stable then the estimated error equation remains stable for some delay in $[0, h^\star)$, where $h^\star$ is some maximal delay.

This design method seems interesting but no procedure is actually given to compute a solution and this requires the restrictive assumption of spectral observability for any delay.

4.3.2. Finite spectrum assignment method

Many studies have considered this case and the corresponding case of spectrally controllable systems. Apart the historical work of Salamon [39] using spectral decomposition, some procedure to design observers have been proposed by Fiagbedzi and Pearson [17], Pearson and Fiadbedzi [34] under the spectral detectability assumption. However their method needs to calculate the unstable poles of the system and some matrix F solution to $F = A_0 + A_1 e^{-sF}$ using some extended Jordan's form. In fact this seems to be rather difficult to solve.

Some of the first key results have been obtained by Watanabe [47], Watanabe and Ouchi [48]. Their results do not need the calculation of the open loop spectrum of the system which was not the case in [39].

Thus Watanabe and Ouchi [48] have considered systems with a single non delayed output and with delays in state and inputs. Under the spectral observability assumption, they have provided a schematic design procedure of observers. This procedure is based on the canonical controllable form of the dual system (with a single input) $(A^T(\nabla), c^T)$.

In [47] the more general case of systems with delays in state, inputs and outputs is tackled (with m inputs, r outputs). It is proved that spectral controllability is equivalent to finite spectrum assignment for multivariable systems with delays. The proposed method is some extension of the famous Wonham [49]'s result: it has been shown in [47] that, for a spectrally controllable system, there exists a state feedback such that the closed-loop system is spectrally controllable through a single input. Then the spectrum assignment is made on the single input system. This result has then been used to design an observer that includes distributed time-delays as:

$$
\begin{aligned}
\dot{\hat{x}}(t) \;=\; & A(\nabla)\hat{x}(t) + B(\nabla)u(t) + K_0(\nabla)[C(\nabla)\hat{x}(t) - y(t)] \\
& + K_x(\nabla)[C_i(\nabla)\hat{x}(t) - y_i(t)] \\
& + \int_{-N_3 h}^{0} \phi(\tau)[C_i(\nabla)\hat{x}(t+\tau) - y_i(t+\tau)]\,\mathrm{d}\tau + \int_{-N_4 h}^{0} \Psi(\tau)v_0(t+\tau)\,\mathrm{d}\tau
\end{aligned}
$$

where i is a preassigned integer, C_i is the ith row of C and y_i the ith element of y. Moreover $K_0(\nabla) \in \mathbb{R}^{n \times r}[\nabla]$, $K_x(\nabla) \in \mathbb{R}^n[\nabla]$, $\phi(.) \in L_2([-N_3 h, 0], \mathbb{R}^n)$, $\Psi(.) \in L_2([-N_4 h, 0], \mathbb{R}^n)$, N_3 and N_4 are appropriate positive integers.

In this case the characteristic equation is:

$$
\det \left[\begin{array}{cc} sI_n - A^T(z) - C^T(z)K_0^T(z) & -C_i^T(z) \\ -K_a^T(s, z) & (1 - K_b(s)) \end{array} \right] \tag{10}
$$

with $K_a(s, e^{-sh}) = K_x(e^{-sh}) + \int_{-N_3 h}^{0} \phi(\tau)e^{s\tau}\,\mathrm{d}\tau$ and $K_b(s) = \int_{-N_4 h}^{0} \Psi(\tau)e^{s\tau}\,\mathrm{d}\tau$.

Then the spectral observability assumption (i. e. spectral controllability of $(A^T(\nabla)$, $C^T(\nabla))$) allows to guarantee a finite spectrum for (10).

4.4. Weakly observable systems

Very few result concern this case. This is because this assumption cannot guarantee to get a stable spectrum assignment in closed-loop. In the dual case of weakly

controllable systems Lee and Lu [24], Sename et al [42] have shown that the coefficients of the characteristic polynomial cannot be assigned arbitrarily. In fact the coefficients must satisfy some constraints in terms of their polynomial form.

Concerning weak observable systems, Picard et al [36] have defined the constructible state submodule $\mathcal{H}(\nabla)$ from the Smith form of the observability matrix, which is shown to correspond to some finite-time observer. Their result points out that the best that can be done without anticipation (i. e. without using $y(t+\epsilon)$) is to construct $\mathcal{H}(\nabla)x(t)$. This state reconstruction is then interpreted using observability indices relative to weak observability.

In conclusion it clearly appears today that the weak observability assumption is too weak to insure the existence of an asymptotic observer.

Remark 4. Let us add here few comments about the so-called separation principle. For systems over rings this still works as mentioned by Habets [18]. Let us also cite the recent result of Fattouh et al [14] where this is proved for systems with point and distributed time-delays using a memoryless state feedback implemented through an observer including point and distributed time-delays.

5. UNKNOWN INPUT OBSERVERS (UIO) FOR TIME–DELAY SYSTEMS

The results of this section have been presented by the author in [10, 13, 41]. Let us recall that, for linear systems, unknown input observers are designed such that the state estimation is efficient in presence of some external input (disturbances). This kind of observers is widely used for Fault Detection and Isolation purpose. Some of the classical references on this subject are Darouach et al [5], Hou and Müller [19].

5.1. Problem statement

Let us consider the following ring model of time-delay systems.

$$\Sigma \begin{cases} \dot{x}(t) &=& A(\nabla)x(t) + B(\nabla)u(t) + E(\nabla)d(t) \\ y(t) &=& C(\nabla)x(t) \end{cases} \tag{11}$$

where $d(t) \in \mathbb{R}^q$ the unknown input (or disturbance) and E is a polynomial matrix in ∇.

We assume that $p \geq q$ and, without loss of generality:

$$\operatorname{rank} E(\nabla) = q \quad \text{and} \quad \operatorname{rank} C(\nabla) = p.$$

In [41] the design of UIO has been generalized to time-delay systems through full-order Luenberger state observers given as:

$$\begin{cases} \dot{z}(t) &=& N(\nabla)z(t) + M(\nabla)u(t) + L(\nabla)y(t) \\ \hat{x}(t) &=& z(t) + H(\nabla)y(t) \end{cases} \tag{12}$$

where $z(t) \in \mathbb{R}^n$ is the state of the full-order observer, $N(\nabla), M(\nabla), L(\nabla)$ are matrices of appropriate dimensions which have to be designed for achieving unknown input decoupling.

$z(t) = \psi(t), t \in [-Nh, 0]$ is the initial condition of the observer.

Let us recall the following definition of an Unknown Input Observer [1].

Definition 5. System (12) over $\Re[\nabla]$ is said to be an Unknown Input Observer for system (2) over $\Re[\nabla]$ if its state estimation vector $e(t) \, (:= x(t) - \hat{x}(t))$ approaches zero asymptotically, regardless of the presence of the unknown input in the system.

Using the estimation error $e(t) = x(t) - \hat{x}(t)$ it has been proved that, in a classical way, the following equations are used for (12) to be an observer for system (2) and for achieving disturbance (unknown input) decoupling:

$$
\begin{aligned}
\text{a)} \quad & N(\nabla) && \text{is stable} \\
\text{b)} \quad & T(\nabla)\, E(\nabla) && = \quad 0 \\
\text{c)} \quad & T(\nabla)\, A(\nabla) - N(\nabla)\, T(\nabla) && = \quad L(\nabla)\, C(\nabla) \\
\text{d)} \quad & T(\nabla)\, B(\nabla) && = \quad M(\nabla)
\end{aligned}
\tag{13}
$$

where $T(\nabla) = I_n - H(\nabla)C(\nabla)$.

Defining $K(\nabla) = L(\nabla) - N(\nabla)H(\nabla)$, the problem is then reduced to the choice of matrix $K(\nabla)$ such that $N(\nabla) = T(\nabla)A(\nabla) - K(\nabla)C(\nabla)$ is stable, where $T(\nabla)$ satisfies (13b). Two statements have then to be solved.

1. Is there a polynomial matrix $H(\nabla)$ such that:
   $[I_n - H(\nabla)C(\nabla)]E(\nabla) = 0$?

2. Is there a polynomial matrix $K(\nabla)$ such that:
   $(T(\nabla)A(\nabla) - K(\nabla)C(\nabla))$ is stable?

5.2. A partial solution to the design of UIO for time-delay systems

Let us first recall that, in the case of systems *without* delay, the first statement is solved as soon as $\operatorname{rank}(CE) = \operatorname{rank} E$.

In the case of systems with delays, this last condition is not sufficient for a polynomial matrix $H(\nabla)$ to be a solution. Indeed this may lead to a rational predictive (i.e. unrealistic) solution.

The first step is to rewrite equation (13b) as:

$$
\underbrace{\left[\begin{array}{cc} H_1(\nabla) & H_2(\nabla) \end{array} \right]}_{H(\nabla)} \underbrace{\left[\begin{array}{c} M_1(\nabla) \\ M_2(\nabla) \end{array} \right]}_{C(\nabla)E(\nabla)} = E(\nabla),
$$

or as the linear diophantine equation

$$
H_1(\nabla)\, M_1(\nabla) + H_2(\nabla)\, M_2(\nabla) = E(\nabla) \tag{14}
$$

where $H_1(\nabla)$, $H_2(\nabla)$, $M_1(\nabla)$ and $M_2(\nabla)$ are of respective dimension $n \times q$, $n \times (p-q)$, $q \times q$ and $(p-q) \times q$.

The results of Kučera [23] concerning polynomial diophantine equations can then be used.

Now, following Habets [18], the dual problem of detectability (i.e. the existence of a stable observer) is here solved by (only) a necessary condition which leads to the theorem:

Theorem 1. [41] System (12) is an unknown input observer (UIO) for system (2) if the following conditions are satisfied:

(i) A necessary and sufficient condition to solve (14) ($[I_n - H(\nabla)C(\nabla)]E(\nabla) = 0$) is that the greatest common right divisor of matrices $M_1(\nabla)$ and $M_2(\nabla)$ is a right divisor of $E(\nabla)$.

(ii) A necessary condition for finding a polynomial matrix $K(\nabla)$ such that $(T(\nabla)A(\nabla) - K(\nabla)C(\nabla))$ is stable is:
System (2) is detectable with respect to the stability domain $\mathcal{D}$ i.e.:
$\begin{pmatrix} sI_n - A(\nabla) \\ C(\nabla) \end{pmatrix}$ is left invertible over $\mathbb{R}_{\mathcal{D}}(s)$

where $\mathcal{D}$ is the stability domain:
$\mathcal{D} = \{p(s, \nabla) \in \mathbb{R}[s, \nabla] | p(s, \nabla) \text{ is monic in s} \text{ and } p(s, e^{-sh}) = 0 \Rightarrow Re(s) < 0\}$
$\mathbb{R}_{\mathcal{D}}(s) = \{\frac{p(s,\nabla)}{q(s,\nabla)} \in \mathbb{R}[\nabla](s) | p \in \mathbb{R}[\nabla][s], \ q(s, \nabla) \in \mathcal{D}\}$.

Note that the detectability condition here used corresponds to spectral detectability and is less restrictive than strong observability but more than the weak one [25].

Note that, at this time, we cannot ensure that condition (ii) is (or not) a sufficient one. Moreover this theorem does not provide any method to obtain $K(\nabla)$ such that $(T(\nabla)A(\nabla) - K(\nabla)C(\nabla))$ is stable. Finally, in the case of systems with delays, it has been proved in [18] that stabilization using pole placement by static state feedback is a too strong property to be obtained. Following Habets, dynamic feedback compensators is a more adequate tool to solve stabilizability problems. In order to solve the problem of UIO design though a necessary and sufficient condition, the definition of Dynamic UIO has been provided as shown in the next section.

5.3. Dynamic UIO

The idea of the Dynamic UIO is quite similar to the well known integral control. The aim is to consider a dynamic time-delay system whose input is the output estimation error and to inject it in the dynamic equation of the UIO, as (see also Figure 1):

$$\begin{cases} \dot{z}(t) &= N(\nabla)z(t) + M(\nabla)u(t) + L(\nabla)y(t) + y_0(t) \\ \hat{x}(t) &= z(t) + H(\nabla)y(t) \end{cases} \tag{15}$$

where $z(t) \in \mathbb{R}^n$ and $\hat{x}(t) \in \mathbb{R}^n$ are the estimator state and the estimated state vectors respectively, N, M, L and H are polynomials matrices in ∇ of appropriate

dimensions and $y_0(t) \in \mathbb{R}^n$ is the output of the dynamic gain given by the following system:

$$\begin{cases} \dot{x}_0(t) & = & A_0(\nabla)x_0(t) + B_0(\nabla)(y(t) - \hat{y}(t)) \\ y_0(t) & = & C_0(\nabla)x_0(t) + D_0(\nabla)(y(t) - \hat{y}(t)) \end{cases} \tag{16}$$

where $x_0(t) \in \Re^v$ and $\hat{y}(t) = C(\nabla)\hat{x}(t)$ are the dynamic gain state and the estimated output vectors respectively and A_0, B_0, C_0 and D_0 are polynomials matrices in ∇ of appropriate dimensions.
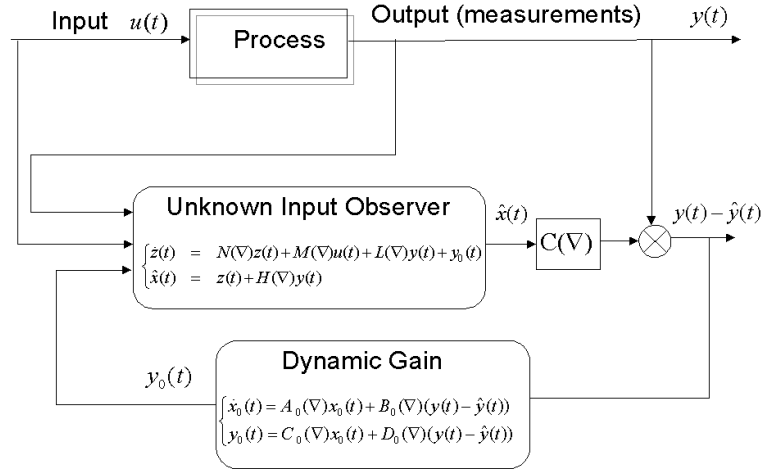

Fig. 1. Dynamic Unknown Input Observer Scheme.

In this framework, let us introduce the following definition.

Definition 6. System $(15) - (16)$ over $\Re[\nabla]$ is said to be a Dynamic Unknown Input Observer for system (2) over $\Re[\nabla]$ if its state estimation vector $e(t)$ $(:= x(t) - \hat{x}(t))$ approaches zero asymptotically, regardless of the presence of the unknown input in the system.

Now the estimated error, $e(t) := x(t) - \hat{x}(t)$, satisfies:

$$\begin{aligned} \begin{bmatrix} \dot{e}(t) \\ \dot{x_0}(t) \end{bmatrix} & = \begin{bmatrix} N - D_0 C & -C_0 \\ B_0 C & A_0 \end{bmatrix}(\nabla) \begin{bmatrix} e(t) \\ x_0(t) \end{bmatrix} + \begin{bmatrix} TB - M \\ 0_{v \times r} \end{bmatrix}(\nabla)u(t) \\ & + \begin{bmatrix} T(\nabla)E(\nabla) \\ 0_{v \times q} \end{bmatrix}d(t) + \begin{bmatrix} TA - NT - LC \\ 0_{v \times n} \end{bmatrix}(\nabla)x(t) \end{aligned}$$

where

$$T(\nabla) = I_n - H(\nabla)\, C(\nabla). \tag{17}$$

Proposition 1. Consider system (2) over $\mathbb{R}[\nabla]$. If the following statements are satisfied:

$$
\begin{array}{rll}
\text{a)} & \left[\begin{array}{cc} N - D_0 C & -C_0 \\ B_0 C & A_0 \end{array} \right] (\nabla) & \text{is asymptotically stable} \\[3ex]
\text{b)} & M(\nabla) \quad = \quad T(\nabla)B(\nabla) & \\[1ex]
\text{c)} & T(\nabla)E(\nabla) \quad = \quad 0 & \\[1ex]
\text{d)} & T(\nabla)A(\nabla) - N(\nabla)T(\nabla) \quad = \quad L(\nabla)C(\nabla) &
\end{array}
\tag{18}
$$

then system (15) – (16) over $\mathbb{R}[\nabla]$ is a dynamic unknown input observer for (2).

Conditions (18, b – d) ensure the rejection of the effects of the control input, the unknown input and the state respectively from the estimated error while (18, a) ensures the asymptotic convergence of the estimated error to zero. These conditions extend to the dynamic gain case the conditions given by Sename [41].

Unlike the static gain (part 5.1) and the non retarded cases, for which conditions (18.a) and (18.d) are reduced to a pole placement problem, here condition (18.a) merely allows to solve the pole placement problem.

Therefore, if $T(\nabla)$ has been found such that (18.c) is satisfied, (18, b) and (18, d) can be solved by taking:

$$
M(\nabla) = T(\nabla)B(\nabla), \ \ N(\nabla) = T(\nabla)A(\nabla) \ \text{ and } \ L(\nabla) = T(\nabla)A(\nabla)H(\nabla).
$$

Thus the problem of designing a full order dynamic unknown input observer is equivalent to:

(i) There exists a matrix $T(\nabla)$ over $\mathbb{R}[\nabla]$ such that $T(\nabla)E(\nabla) = 0$.

(ii) There exists a dynamic gain time-delay systems (A_0, B_0, C_0, D_0) such that

$$
\left[\begin{array}{cc} T(\nabla)A(\nabla) - D_0(\nabla)C(\nabla) & -C_0(\nabla) \\ B_0(\nabla)C(\nabla) & A_0(\nabla) \end{array} \right]
\tag{19}
$$

is asymptotically stable.

Now, statement (i) is solved in Theorem 1. Moreover, by duality with dynamic state feedback stabilization, statement (ii) is solved as soon as $(T(\nabla)A(\nabla), C(\nabla))$ is spectrally detectable (see [18]).

Nevertheless this means that one needs to calculate $T(\nabla)$ before testing this condition. Of course this is not satisfactory as it does depend on the observer parameters and cannot be checked a priori. An improvement of these results has been obtained by Fattouh [10] where new equivalent necessary and sufficient conditions that can be checked a priori (and only depend on the system parameters) have been proposed.

The necessary and sufficient conditions to the Dynamic UIO deisgn problem are:

Theorem 2. [10] Consider system (2) over $\mathbb{R}[\nabla]$. System (15)−(16) over $\mathbb{R}[\nabla]$ is a Dynamic Unknown Input Observer for (2) (according to Definition 6) if and only if

    a) The greatest common right divisor of matrices $M_1(\nabla)$ and $M_2(\nabla)$ is a right divisor of $E(\nabla)$, where

$$\left[\begin{array}{c} M_1(\nabla) \\ M_2(\nabla) \end{array}\right] := C(\nabla)E(\nabla).$$

    b) The matrix

$$\left[\begin{array}{cc} sI_n - A(e^{-sh}) & E(e^{-sh}) \\ C(e^{-sh}) & 0 \end{array}\right]$$

    is left-invertible over $\mathbb{R}_{\mathcal{D}}(s)$.

Notice that these conditions are the extension of the classical results for non-delayed systems [5].

Design procedure: Given a system of the form (2), one can follow the following steps in order to construct a dynamic unknown input observer of the form (15)−(16):

Step 1: Check whether condition (a) and (b) of Theorem 2 are satisfied or not. If yes, proceed to Step 2, if not stop and declare that there is no solution.

Step 2: Find $H(\nabla)$ solution to (14) (see [23]) then deduce
$T(\nabla) = I_n - H(\nabla)\,C(\nabla)$, $M(\nabla) = T(\nabla)\,B(\nabla)$, $N(\nabla) = T(\nabla)\,A(\nabla)$
and $L(\nabla) = T(\nabla)\,A(\nabla)\,H(\nabla)$.

Step 3: Find $(A_0(\nabla), B_0(\nabla), C_0(\nabla), D_0(\nabla))$ such that (19) is asymptotically stable.
   This can be solved using the algorithm proposed by Habets [18] which can be summarized in the following steps:
   First a left inverse of $\left[\begin{array}{c} sI_n - T(\nabla)A(\nabla) \\ C(\nabla) \end{array}\right]$ allows to find a closed-loop characteristic polynomial $\varphi(\nabla, s) \in \mathcal{D}$. Next find $Q(\nabla, s) \in \mathbb{R}_{\mathcal{D}}{}^{n \times n}(s)$ and $P(\nabla, s) \in \mathbb{R}_{\mathcal{D}}^{p \times n}(s)$ solution to the following diophantine equation:

$$(sI_n - A^t(\nabla)T^t(\nabla))Q(\nabla, s) + C^t(\nabla)P(\nabla, s) = \varphi(\nabla, s)\,I_n. \tag{20}$$

Finally, find a realization $(A_0(\nabla), B_0(\nabla), C_0(\nabla), D_0(\nabla))$ of the transfer function matrix $(P(\nabla, s)Q^{-1}(\nabla, s))^t$ (it is always possible [18]).

6. H_∞ OBSERVERS FOR TIME–DELAY SYSTEMS

In this section some methods to design H_∞ observers for time-delay systems are presented. The robustness issue (i. e. for systems including uncertainties) is analyzed in the next section.

We are interested here in the case of delayed-state systems, for which arise some difficulties in using H_∞ approach. For sake of simplicity, we only consider single delayed-state systems, even if some results are directly extended to the case of multiple time-delay systems.

The system under consideration is then:

$$
\begin{cases}
\dot{x}(t) &= A_0 x(t) + A_1 x(t-h) + Ew(t) + Bu(t) \\
y(t) &= Cx(t) + Fw(t)
\end{cases}
\tag{21}
$$

where $w(t) \in R^q$ is the square-integrable disturbance vector.

In this section, we consider Luenberger-type observer for system (21) given by the following dynamical system:

$$
\dot{\hat{x}}(t) = A_0 \hat{x}(t) + A_1 \hat{x}(t-h) + Bu(t) - L(C\hat{x}(t) - y(t))
\tag{22}
$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state of $x(t)$ and L is the $n \times p$ constant observer gain matrix to be designed.

The estimated error, defined as $e(t) := x(t) - \hat{x}(t)$, satisfies the following dynamical system:

$$
\dot{e}(t) = (A_0 - LC)\, e(t) + (A_1)\, e(t-h) + (E - LF)\, w(t).
\tag{23}
$$

The transfer function $T_{ew}(s)$ between $w(t)$ and $e(t)$ is then given by:

$$
T_{ew}(s) = (sI_n - A_0 + LC - A_1 e^{-sh})^{-1}(E - LF).
$$

The problem of designing H_∞ observers for time-delay systems is presented in the following definition.

Definition 7. Let us consider a time-delay system (21). System (22) is said to be an H_∞ observer for the associated system (21) if the solution of the functional differential equation (23) with $w(t) \equiv 0$ converges to zero asymptotically and, under zero initial condition, the H_∞ norm of the transfer function between the disturbance and the estimated error is bounded, that is

$$
\lim_{t \to \infty} e(t) \to 0 \quad \text{for} \quad w(t) \equiv 0 \qquad \text{and} \qquad \|T_{ew}(s)\|_\infty \leq \gamma
$$

where $\|.\|_\infty$ is the H_∞-norm and $\gamma > 0$ is a disturbance attenuation level.

The main contributions to this approach are based on stabilization method by memoryless state feedback in an H_∞ framework. The aim is to design the gain L such that the estimated error equation is stable, which corresponds to the control problem of finding L such that the closed-loop system (23) is stable.

Two approaches have then been provided in the literature for such control problem. The first one is based on the Krasovskii theorem and leads to delay-independent result (see Lee et al [27]) as the second one uses the Razumikhin theorem and leads to delay-dependent result (see Su [44]).

6.1. Lyapunov–Krasovskii approach

The considered contributions are the results of Choi and Chung [2], Fattouh et al [12].

In both papers, some slight modification of the results of Lee et al [27] has been used. Now in [2] the problem is to design an observer-based controller and two cases were considered:

    – some delay is included in the observer and the control law is memoryless.

    – the observer as well as the control law are memoryless.

In [12] only the design of observers is tackled.

In order to compare both results of [2, 12], only the first of the previous cases is here analyzed. To avoid some long presentation these papers are here illustrated trough an example and only the main result of each paper is given. Those readers interesting by the complete designed methods to find the solution are referred to the papers mentioned above.

6.1.1. Choi and Chung [2]'s method

In [2] the observer-based controller is of the form:

$$\begin{cases} \dot{\hat{x}}(t) & = & A_0\hat{x}(t) + A_1\hat{x}(t-h) + Bu(t) - L(C\hat{x}(t) - y(t)) + EG\hat{x}(t) \\ u(t) & = & K\hat{x}(t). \end{cases} \tag{24}$$

This method aims to design an observer-based controller. Note that the observer form is specific to their works and includes a specific term $EG\hat{x}(t)$ that represents the coupling with the observer and the control.

The observer and the control are obtained through two coupled Riccati equations including 7 parameters (2 matrices and 5 constants). Moreover the estimation error stability is guaranteed for closed-loop systems only and no performance (in terms of H_∞ gain) is ensured. On the other hand, such a performance is obtained for the closed-loop system.

The main result given below guarantees some H_∞ disturbance attenuation level for the closed-loop system(but not for the state estimation error):

Theorem 3. Consider the time-delay system (21) (with y as the controlled output and $F = I_p$) and the observer-based controller (24). If both following algebraic Riccati equations

$$\begin{aligned} A_0^T P_c + P_c A_0 - \tfrac{1}{\epsilon_c} P_c \left(BB^T - \tfrac{1}{\delta_c} A_1 A_1^T - \tfrac{1}{\gamma^2} EE^T \right) P_c & \\ + \epsilon_c(\delta_c I_n + C^T C + Q_c) &= 0 \\[2mm] (A_0 + EG)P_0 + P_0(A_0 + EG)^T - \tfrac{1}{\epsilon_0} P_0 \left(C^T C - \tfrac{1}{\gamma^2} K^T K - \tfrac{\delta_0}{\gamma^2} I_n \right) P_0 & \\ + \epsilon_0 \left(\tfrac{\gamma^2}{\delta_0} A_1 A_1^T + EE^T + Q_0 \right) &= 0 \end{aligned} \tag{25}$$

have symmetric positive definite solutions P_c and P_0 for some positive constants γ, ϵ_c, δ_c, ϵ_0 and δ_0, some positive-definite matrices Q_c and Q_0, and with the control parameters:

$$K = -\frac{1}{\epsilon_c} B^T P_c, \quad G = \frac{1}{\gamma^2 \epsilon_c} E^T P_c$$

then, for all h:

1. Observer (24) is asymptotically stable and the observer gain is:

$$L = \frac{1}{\epsilon_0} P_0 C^T$$

2. the closed-loop system is asymptotically stable and such that: $\|y\|_2 \leq \gamma \ \|w\|_2$.

Let us illustrate this result on the following example which will be considered along all the designed methods of H_∞ observers.

Example 2.

$$\dot{x}(t) \ = \ \begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x(t - h) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w(t)$$

$$y(t) \ = \ \begin{bmatrix} 0 & 7 \end{bmatrix} x(t) + w(t)$$

$$x(t) = \phi(t); \quad t \in [-h, 0].$$

The observability matrix is given by:

$$\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle = \begin{bmatrix} 0 & 7 \\ 7\nabla + 14 & 0 \end{bmatrix}.$$

The above system is weakly observable as rank $\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle = 2$. However, it is not strongly observable as $\left\langle \frac{C(\nabla)}{A(\nabla)} \right\rangle$ is not invertible over $\mathbb{R}[\nabla]$. Now

$$\text{rank} \begin{bmatrix} sI_2 - A(e^{-sh}) \\ C(e^{-sh}) \end{bmatrix} = \text{rank} \begin{bmatrix} s + 3 & -4 \\ -e^{-sh} - 2 & s \\ 0 & 7 \end{bmatrix}$$

which is always of rank 2. Then the system is spectrally observable.

Using the method of Choi and Chung [2] we can obtain the following result. Let us choose the following parameters: $\epsilon_c = 0.01$, $\epsilon_0 = 0.01$, $\delta_c = 2$, $\delta_0 = 2$ and $Q_0 = Q_c = I_2$. For $\gamma = 1.5$ (for the closed-loop system), the ARE (25) have the following solution and the corresponding observer gain:

$$P_c = \begin{bmatrix} 0.0054 & 0.0042 \\ 0.0042 & 0.0404 \end{bmatrix}, \quad K = \begin{bmatrix} -1.3831 & -8.5082 \end{bmatrix}, \quad G = \begin{bmatrix} 0.4283 & 1.9839 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0.0901 & 0.0748 \\ 0.0748 & 0.0648 \end{bmatrix}, \quad L = \begin{bmatrix} 52.3402 \\ 45.3570 \end{bmatrix}.$$

The estimated error responses are shown in Figure 2 for $h = 0.3$ sec.

For simulation purpose an initial value at time $t = 0$ sec. is used to generate a functional initial condition on $t \in [0, -0.3]$. The observer acts at $t = 1$ sec. A step disturbance is applied at $t = 3$ sec.

Moreover the maximum singular value of $T_{ew}(j\omega)$ is traced versus the frequency on Figure 2. It shows that $\|T_{ew}(j\omega)\|_\infty \leq \gamma$ for all $\omega \in \mathbb{R}$.
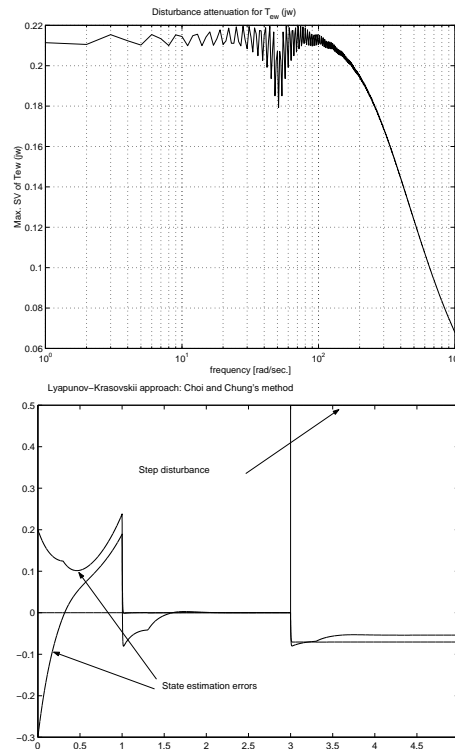

Fig. 2. Maximum SV of $T_{ew}(jw)$ and state estimation errors – Choi and Chung [2].

Note that, even if no robust property is guaranteed for the observer in this case, one can see that the disturbance attenuation for the state estimation error is quite good. However the gain L in the observer is large and the convergence of the observer is guaranteed only for state feedback controlled-loop systems.

6.1.2. Fattouh et al [12]'s method

This method allows to design an observer independently of the control. The following result ensures some H_∞ disturbance attenuation level for the observer.

Theorem 4. Consider the time-delay system (21) and the observer (22). If the following algebraic Riccati equation

$$A_0^T P + P A_0 + 2P \left(\gamma^2 A_1 A_1^T + E E^T \right) P - \frac{2}{\epsilon} C^T \left(I_p - \frac{1}{\epsilon} F F^T \right) C + \frac{2}{\gamma^2} = 0 \quad (26)$$

has symmetric positive definite solution P for some positive constants γ, and ϵ then, for all h, system (22) is an H_∞ observer according to Definition 7 with the disturbance attenuation level γ and the observer gain:

$$L = \frac{1}{\epsilon} P^{-1} C^T.$$

This method only concerns H_∞ observer design. First the dynamical system (22) has a more usual form than the previous one in (24). Then only 2 parameters are here necessary to obtain an observer for which stability as well as H_∞ performance are guaranteed.

Let us illustrate these results.

Example 3. In order to construct an observer of the form (22) for the system given previously in Example 2, the algorithm allows to solve one ARE (26). For $\epsilon = 2.5$ and $\gamma = 0.45$, the ARE (26) has the following solution and the corresponding observer gain:

$$P = \begin{bmatrix} 26.6201 & -23.7295 \\ -23.7295 & 22.1317 \end{bmatrix}, \quad L = \begin{bmatrix} 15.9351 \\ 17.8763 \end{bmatrix}$$

The estimated error responses are shown in Figure 3 for $h = 0.3$ sec. For simulation purpose an initial value at time $t = 0$ sec. is used to generate a functional initial condition on $t \in [0, -0.3]$. The observer acts at $t = 1$ sec. A step disturbance is applied at $t = 3$ sec.

In this case some fixed disturbance attenuation level is guaranteed (here $\gamma = 0.45$) and the convergence of the observer is similar to the previous case (see Figures 3 and 2). Moreover, in Figure 3, the disturbance attenuation is better in a frequential representation. Note that state estimation error on x_1 has less important steady-state error due to the disturbance than in Choi and Chung's case (-0.0386 instead of -0.0540).

Note that a sufficient condition in terms of some observability property can be given for the resolution of the ARE (26). However this condition does not depend only on the system matrices but also on the parameters (ϵ, γ) that appear in (26).

As a conclusion it appears that the recent results obtained in the design of observers are more usable than those obtained previously (for observer-based controller design) although they are easier to obtain. In Fattouh et al [12] only one Riccati equation has to be solved (with two parameters) than in [2] two coupled Riccati equations have to be solved, including seven parameters. Furthermore the observer
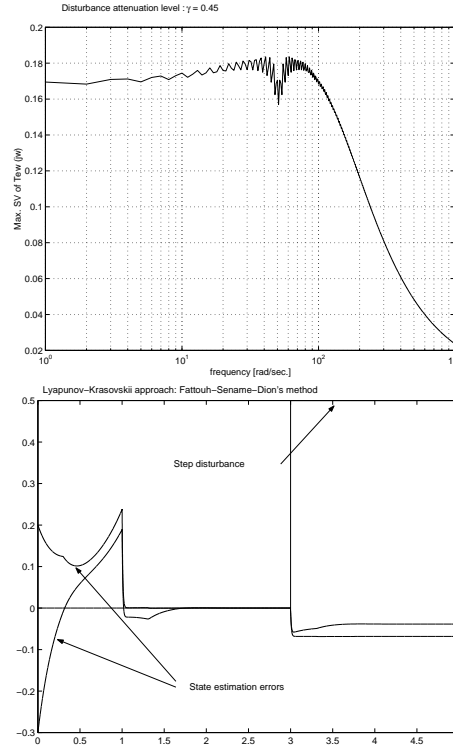
Fig. 3. Maximum SV of $T_{ew}(jw)$ and state estimation errors – Fattouh et al [12].

designed by Fattouh et al [12] can then be used to performed advanced robust control laws. The one developed by [2] is linked to a state feedback controlled loop system where the state feedback is obtained by solving the Riccati equation given in Theorem 3 (see (25)).

Of course the result in [2] guarantees some H_∞ disturbance attenuation level for the output of the closed-loop system which was not tackled in Fattouh et al [12].

6.2. Lyapunov–Razumikhin approach

Apart the delay-independent methods, some results have been obtained using the Razumikhin theorem. We present here the contribution of Fattouh et al [11] where some design procedure is provided as well as a weak sufficient condition for the existence of the observer. The results of this part are based on stabilization results which can be found in Li and de Souza [28], Niculescu et al [32], Su [44].

In order to design the observer, the equation (23) can be rewritten as follows:

$$\dot{e}(t) = (A_0 - LC)\,e(t) + A_1 e(t - h) + \bar{D}\bar{w}(t) \tag{27}$$

where $\bar{w}(t) := [w^T(t) \quad \bar{\gamma}w^T(t)]^T$, $\bar{\gamma}$ is some positive constant and $\bar{D} := [E \quad -\frac{1}{\bar{\gamma}}LF]$.

Consider now the following output:

$$z(t) := Pe(t) \qquad (28)$$

where P is a $n \times n$ symmetric positive definite (i.e. $P = P^T > 0$) matrix to be determined.

After some algebraic manipulations, one can easily find that: if $\|z\|_2 \leq \|\bar{w}\|_2$, then $\|e\|_2 \leq \|P^{-1}\| \ \sqrt{1 + \bar{\gamma}^2} \|w\|_2$.

Let us now consider the following Lemma which is obtained through some slight modification of [44]'s result.

Lemma 3. Consider the system:

$$\begin{cases} \dot{x}(t) & = & Ax(t) + A_1 x(t - h) + Dw(t) \\ z(t) & = & Cx(t). \end{cases} \qquad (29)$$

Given a scalar $\delta > 0$, system (29) is asymptotically stable and $\|z\|_2 \leq \delta\|w\|_2$ for any constant time-delay $0 \leq h < \bar{h}$ if there exist two positive definite matrices P and Q satisfying the following algebraic Riccati equation:

$$(A + A_1)^T P + P(A + A_1) + C^T C + \frac{1}{\delta^2} PDD^T P + Q = 0$$

where $\bar{h}$ is given by:

$$\bar{h} = \|Q^{-\frac{1}{2}}[\beta_1 P A_1 A P^{-1} A^T A_1^T P + \beta_2 A_1 A_1 P^{-1} A_1^T A_1^T P + (\beta_1^{-1} + \beta_2^{-1})P]Q^{-\frac{1}{2}}\|^{-1}$$

for any positive real numbers β_1 and β_2.

Applying the above Lemma on the modified system $(27) - (28)$ for $\delta = 1$ leads to the following Theorem.

Theorem 5. Consider the time-delay system (21) and the observer (22). If the following algebraic Riccati equation

$$(A_0 + A_1)^T P + P(A_0 + A_1) + P(EE^T + (1+\epsilon)I_n)P - (C)^T \{2I_p - \frac{1}{\bar{\gamma}^2} FF^T\}(C) = 0 \quad (30)$$

has a positive definite solution P for some positive constants $\bar{\gamma}$ and ϵ, then, for all $0 \leq h < \bar{h}$, system (22) is an H_∞ observer according to Definition 7 with the disturbance attenuation level γ:

$$\gamma = \|P^{-1}\| \ \sqrt{1 + \bar{\gamma}^2},$$

the maximal delay

$$\begin{aligned} \bar{h} & = & \left\| (\epsilon PP)^{-\frac{1}{2}} \Big[\beta_1 P A_1 \bar{A}_0 P^{-1} \bar{A}_0^T A_1^T P + \beta_2 P A_1 A_1 P^{-1} A_1^T A_1^T P \right. \\ & & \left. + \Big(\frac{1}{\beta_1} + \frac{1}{\beta_2} \Big) P \Big] (\epsilon PP)^{-\frac{1}{2}} \right\|^{-1}, \end{aligned}$$

with $\bar{A}_0 = A_0 - LC_0$ and for any positive real numbers β_1 and β_2, and the observer gain:

$$L = P^{-1}(C_0 + C_1)^T$$

In [11], an algorithm is proposed in order to minimize γ and the following sufficient condition is provided.

Proposition 2.   For a given time-delay system of the form (21), if the pair $[(C),(A_0 + A_1)]$ is detectable, then there exists an H_∞ observer of the form (22) with certain constants ϵ, $\bar{\gamma}$ and certain disturbance attenuation level. This corresponds to spectral detectability for zero-value delay.

It should be noted that the results in [11] are, at our knowledge, the only ones provided using the Razumikhin theorem. Moreover, as in Theorem 4, only 2 parameters are required to solve the H_∞ observer design problem. Note also that β_1 and β_2 may allow to obtain a suboptimal bound of $\bar{h}$ [31].

Example 4.   Consider the same example as in the previous part.
Note that the pair $[(C_0),(A_0 + A_1)]$ is detectable as

$$\left[\begin{array}{c} sI_2 - A(z) \\ C(z) \end{array} \right] = \left[\begin{array}{cc} s+3 & -4 \\ -z-2 & s \\ 0 & 7 \end{array} \right]$$

is of rank 2 for $z = 1$ and all $s \in \mathbb{C}^+$, $Re(s) \geq 0$.
In order to construct an observer of the form (22) for the above system, our algorithm allows to solve the ARE (30) by iteration on $\bar{\gamma}$ in order to get a minimal γ. For $\epsilon = 1$ and $\bar{\gamma} = 0.67$, the ARE (30) has the following solution and the corresponding observer gain:

$$P = \left[\begin{array}{cc} 2.4128 & -1.5548 \\ -1.5548 & 4.9996 \end{array} \right], \quad L = \left[\begin{array}{c} 1.1284 \\ 1.7510 \end{array} \right]$$

with the maximal delay $\bar{h} = 0.366\,\text{sec}$ (for $\beta_1 = 0.2$ and $\beta_2 = 0.6$) and the minimal attenuation level $\gamma_{\min} = 1.0701$.

The estimated error responses are shown in Figure 4 for $h = 0.3\,\text{sec}$. For simulation purpose an initial value at time $t = 0\,\text{sec}$ is used to generate a functional initial condition on $t \in [0, -0.3]$. The observer acts at $t = 1\,\text{sec}$. A step disturbance is applied at $t = 3\,\text{sec}$.

Moreover the maximum singular value of $T_{ew}(j\omega)$ is traced versus the frequency. It shows that $\|T_{ew}(j\omega)\|_\infty \leq \gamma_{\min}$ for all $\omega \in \mathbb{R}$.

In this particular case where $h = 0.3\,\text{sec}$, the results obtained using a Lyapunov–Razumikhin approach are better than those obtained using a Lyapunov–Krasovskii one.
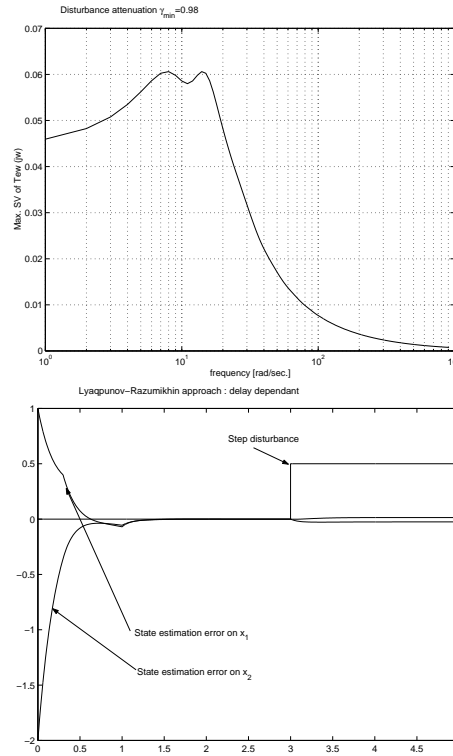
Fig. 4. Maximum SV of $T_{ew}(jw)$ and state estimation errors.

It should also be noted that the maximum delay $\bar{h}$ obtained above for which one guarantees the asymptotic stability of the observer and the H_∞ norm bound is quite conservative and simulation results point out that the observer retains its properties for some values of h larger than $\bar{h}$.

7. ROBUST OBSERVERS FOR TIME–DELAY SYSTEMS

The design of robust observers for time-delay systems has received few attention in the past ten years. We present here two approaches to solve this problem: H_∞ approach and factorization one.

7.1. H_∞ approach

In this section the case of uncertain time-delay systems is considered. We consider here three contributions using the Lyapunov–Krasovskii approach. In all of them, the observer is a delay free system. In Choi and Chung [3, 46] the case of norm-bounded uncertainties on each of the system matrices has been tackled. In both

papers, the uncertainties on each system matrix M are assumed to be of the form:

$$\Delta M(.) = HF(.)N$$

where (.) means that this could be time-varying uncertainties, H and N are known constant matrices, and F, which may be time-varying, is unknown and meets:

$$F^T F \leq I.$$

In [3] the design of robust observer-based H_∞ control is obtained through the resolution of two coupled Riccati equations, which extends the results of [2] to uncertain systems. Of course both Riccati equations are then more complicated in [3] than in [2]. In the same way, in [46], the solution is constructed by solving two Riccati equations. In fact, if both contributions are interesting, numerical problems may arise in solving these complicated Riccati equations.

In order to overcome this problem the paper of de Souza et al [6] provides an interesting solution in terms of $\mathcal{LMI}$s.

In [6], linear continuous-time systems with multiple time-varying state delays are considered. In the single delay case these are given by:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t - h(t)) + B w(t) \\ y(t) = C_0 x(t) + C_1 x(t - h(t)) + D w(t) \end{cases}$$

$$z(t) = L_0 x(t) + L_1 x(t - h(t)) + T w(t)$$

$$x(t) = \phi(t), \ \forall t \in [-h, 0) \tag{31}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^r$ is the measurement, $w(t) \in \mathbb{R}^m$ is the noise signal and $z(t) \in \mathbb{R}^p$ is the signal to be estimated, and $h(t)$ is the time-varying bounded delay satisfying:

$$0 < h(t) \leq h, 0 \leq \dot{h}(t) \leq \tau < 1.$$

The systems matrices are assumed to be unknown but belonging to a known complex compact set of polytopic type $\mathcal{D}$ where

$$(A_0, A_1, B, C_0, C_1, D, L_0, L_1, T) = \sum_{i=1}^{l} \alpha_i (A_{0i}, A_{1i}, B_i, C_{0i}, C_{i1}, D_i, L_{0i}, L_{1i}, T_i)$$

with $\alpha_i \geq 0; \sum_{i=1}^{l} \alpha_i = 1$.

The robust filtering problem consists in obtaining an estimate $\hat{z}(t)$ of the signal $z(t)$ such that a prespecified H_∞ performance is guaranteed in a filtering error sense. In [6] the filter is assumed to be a linear time-invariant system of the form:

$$\begin{cases} \dot{\hat{x}}(t) = A_f \hat{x}(t) + B_f y(t) \\ \hat{z}(t) = C_f \hat{x}(t) + D_f y(t). \end{cases} \tag{32}$$

The robust H_∞ filtering problem with time-varying state delays is: determine a linear asymptotically stable filter (32) such that the filtering error system, with $\tilde{x}(t) = [x^T(t) \quad \hat{x}^T(t)]^T$ as augmented state vector, is stable independently of delays and ensures a prescribed level γ of H_∞ attenuation, namely, under zero-initial conditions and for any non-zero $w(t) \in \mathcal{L}_2$:

$$\|\tilde{z}\|_2 \leq \gamma \|w\|_2$$

where $\tilde{z}(t) = z(t) - \hat{z}(t)$.

In [6], this problem has been solved by a sufficient condition in terms of $\mathcal{LMI}$s, and the matrices in (32) depend on the solution of these $\mathcal{LMI}$s.

7.2. The factorization approach

In this section, we consider the parametrization of observers given previously (Section 4.2.2). Our aim is here to design the parametrization matrix $Q(s,z)$ such that the problem can be solved for the real plant, i.e. the model including uncertainties. This problem has been solved by Fattouh et al [16] but some minor changes are here provided. We are interested here in the following forms of uncertainties:
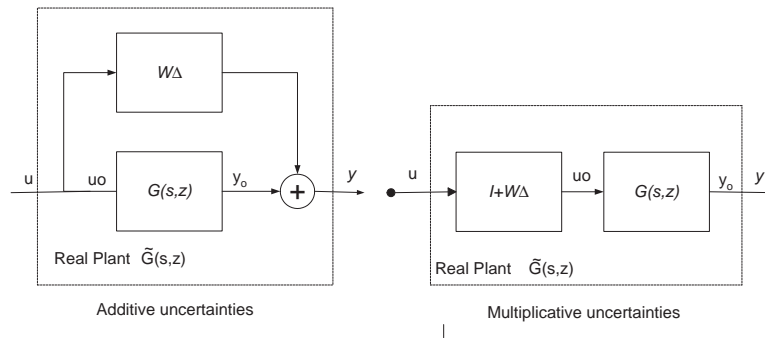

Fig. 5. Uncertainties.

where
  – $W(s)$ is a fixed stable transfer function matrix and $\Delta(s,z)$ is assumed to be variable stable transfer function matrices with $\|\Delta(s,z)\|_\infty \leq 1$,
  – The real plant is denoted by $\tilde{G}(s,z)$ and the nominal model by $G(s,z)$,
  – u_0 and y_0 are the input and the output of the nominal plant respectively, u and y are the input and the output of the real plant respectively, where u and u_0 are assumed to be norm-bounded.

The aim of the proposed method is to design a robust observer among the set of observers (7) – (8), by selecting a parameterized matrix $Q(s,z)$ such that the difference between the real estimation and the nominal one is minimal (see Figure 6, i.e.:
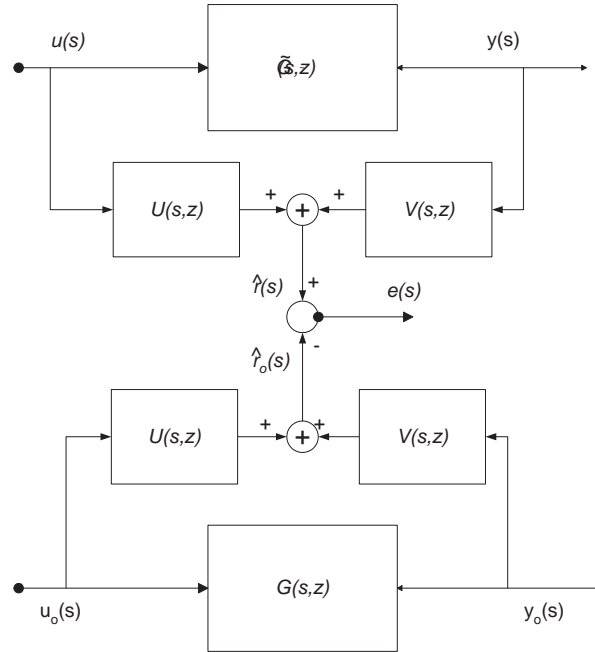
Fig. 6. Estimated error $e(s)$ due to system uncertainty.

(i) $\lim_{t \to \infty}(r(t) - \hat{r}_0(t)) = 0,$

(ii) $\|e(s)\|_2 := \|\hat{r}(s) - \hat{r}_0(s)\|_2$ is minimized,

where $r(t)$ is the variable to be estimated, $\hat{r}_0(t)$ is the nominal estimation given by:

$$\hat{r}_0(s) = U(s, z)u_0(s) + V(s, z)\,y_0(s),$$

and $\hat{r}(t)$ is the estimation obtained when applying the observer $(7) - (8)$ on the real plant.
This problem is now formulated as an optimization problem.

7.2.1. Additive uncertainty

In this case $\tilde{G} = G + W\Delta$ and $y(s) = y_0(s) + W\Delta u_0(s)$. The real estimation is given by

$$\begin{aligned}
\hat{r}(s) &= U(s, z)\,u_0(s) + V(s, z)\,y(s) \\
&= (U + VW\Delta)\,u_0(s) + V\,y_0(s).
\end{aligned}$$

From the above equation,

$$
\begin{aligned}
e(s) &= \hat{r}(s) - \hat{r}_0(s) = VW\Delta u_0(s) \\
&= (PXW + Q\bar{M}W)\,\Delta u_0(s).
\end{aligned}
$$

Now, as W and Δ are stable then $e(s)$ is stable. Moreover, as $u_0(s)$ is bounded then $e(s)$ is bounded.

To minimize the difference between both estimation $\hat{r}(s)$ and $\hat{r}_0(s)$, i.e. in order to guarantee that $\hat{r}(t)$ is a good estimation of $Ex(t)$ in the real configuration, one has to solve:

$$
\min_Q \|(PXW + Q\bar{M})\Delta\|_\infty.
$$

As $\|\Delta(s,z)\|_\infty \le 1$, then the following optimization problem has to be solved:

$$
\min_Q \|T_1 + QT_2\|_\infty
$$

where

$$
\begin{cases}
T_1(s,z) &= P(s,z)\,X(s,z)\,W(s) \\
T_2(s,z) &= \bar{M}(s,z)\,W(s).
\end{cases}
\tag{33}
$$

7.2.2. Input multiplicative uncertainty

In this case $\tilde{G} = G(I+W\Delta)$ and $u_0(s) = (I+W\Delta)u(s)$. In a same way as previously,

$$
\begin{aligned}
e(s) &= \hat{r}(s) - \hat{r}_0(s) = -UW\Delta u(s) \\
&= -(PYW - Q\bar{N}W)\,\Delta u(s).
\end{aligned}
$$

Now, as W and Δ are stable then $e(s)$ is stable. Moreover, as $u(s)$ is bounded then $e(s)$ is bounded.

To get a good estimations one has to solve $\min_Q \|T_1 + QT_2\|_\infty$ where

$$
\begin{cases}
T_1(s,z) &= P(s,z)\,Y(s,z)\,W(s) \\
T_2(s,z) &= -\bar{N}(s,z)\,W(s).
\end{cases}
\tag{34}
$$

From both above cases one can conclude that in order to minimize the effect of the model uncertainty on the estimated states one has to solve the optimization problem:

Finding $Q(s,z)$ such that:

$$
\min_Q \|T_1(s,z) + Q(s,z)T_2(s,z)\|_\infty
\tag{35}
$$

where $T_1(s,z)$ and $T_2(s,z)$ are given by (33) or (34) according to the model uncertainty.

7.2.3. A suboptimal solution

Here the following suboptimal problem is solved:

$$\|T_1(s,z) + Q(s,z)T_2(s,z)\|_\infty \leq \gamma$$

where γ is some positive scalar.

This problem is tackled as a model matching problem.

First, any matrix $T(s,z)$ over $\mathbf{F}$ can be written as $T(s,z) = \sum_{i=0}^{m} T_i(s)z^i$ where $T_i(s)$ are proper stable rational transfer function matrices and $m \in \mathbb{N}^+$.

Let

$$T_1(s,z) = \sum_{i=0}^{m_1} T_{1i}(s)z^i \text{ and } T_2(s,z) = \sum_{j=0}^{m_2} T_{2j}(s)z^j \tag{36}$$

where $T_{1i}(s)$ and $T_{2j}(s)$ are proper stable transfer function matrices and m_1, $m_2 \in \mathbb{N}^+$.

Since $\deg_z(T_1(s,z)) \geq \deg_z(T_2(s,z))$ (see part 4.2) then $m_1 \geq m_2$.

Let the parametrized matrix Q be of the form:

$$Q(s,z) := \sum_{k=0}^{(m_1-m_2)} Q_k(s)z^k.$$

This means that $(m_1 - m_2 + 1)$ $Q_k(s)$-matrices have to be obtained. Now the $(m_1 + 1)$ equations given in $T_1(s,z) + Q(s,z)T_2(s,z)$ can always be reduced to a set of $(m_1 - m_2 + 1)$ model matching problems to be solved, which leads to

$$\|T_1 + QT_2\|_\infty \leq \sum_{i=0}^{(m_1-m_2)} := \gamma$$

where γ is the suboptimal uncertainties attenuation level.

Remark 5. To illustrate this suboptimal problem, and the way to reduce $m_1 + 1$ equations to $(m_1 - m_2 + 1)$ ones, let us consider both following simple examples.

   – Let:
$$T_1(s,z) = T_{10}(s) + zT_{11}(s) \text{ and } T_2(s,z) = T_{20}(s) + zT_{21}(s).$$

   Now $Q(s,z) = Q_0(s)$. Therefore one can solve:

$$\|T_1(s,z) + Q(s,z)\,T_2(s,z)\|_\infty$$
$$\leq \| \begin{bmatrix} T_{10}(s) & T_{11}(s) \end{bmatrix} - Q_0(s) \begin{bmatrix} T_{20}(s) & T_{21}(s) \end{bmatrix} \|_\infty.$$

   – Let:
$$T_1(s,z) = T_{10}(s) + zT_{11}(s) \text{ and } T_2(s,z) = T_{20}(s).$$

   Now $Q(s,z) = Q_0(s) + zQ_1(s)$. Therefore one can solve:

$$\|T_1(s,z) + Q(s,z)T_2(s,z)\|_\infty \leq \|T_{10}(s) - Q_0(s)T_{20}(s)\| + \|T_{11}(s) - Q_1(s)T_{20}(s)\|_\infty.$$

A suboptimal solution has then been proposed for both cases of unstructured uncertainties.

8. AND THE FUTURE?

In this paper, the recent methods available for observers design have been presented. We have focused our attention on three methods that are efficient for this objective. First a method has been presented to obtain unknown input observers for time-delay systems: it appears that the use of dynamic feedback allows to solve the problem which is not necessary for systems without delays. Then the H_∞ approach has been presented in the Lyapunov–Krasovskii and Lyapunov–Razumikhin frameworks; in these cases the results obtained by the authors allows to get robust observers that guarantee good disturbance attenuation properties. Finally the factorization approach (i. e. parametrization of the observers) solved in a robust framework allows to obtain robust observers for spectrally co-canonical time-delay systems including unstructured uncertainties (additive and input multiplicative uncertainties). This result has recently been improved in Fattouh et al [15] where the use of LMI allows relaxes the spectrally co-canonical assumption.

An interesting (and natural) extension of these works concern systems with point and distributed time-delays. In this more general case, different approaches have been used to some control problems. It clearly appears today that using LMI allows to get a solution to some control problems, which was almost impossible five years ago. In the case of observer design some preliminary and interesting works have been done by Fattouh et al [14].

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