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NONLINEAR CONTROL OF A PLANAR MULTIAXIS SERVOHYDRAULIC TEST FACILITY USING EXACT LINEARIZATION TECHNIQUES[1]

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This paper presents a nonlinear control concept of a planar multi-axis servohydraulic test facility. Based on nonlinear model equations including servohydraulic actuator dynamics and test table and payload mechanics a global nonlinear diffeomorphism is derived which maps the model equations into nonlinear canonical form. A nonlinear control law is derived using exact linearization techniques. The lengthy controller expressions are calculated by applying symbolic computer languages.

1. INTRODUCTION

High quality multi-axis servohydraulic test facilities are widely used for testing of critical components of industrial equipment and of future spacecraft [1]. Theoretical investigations of multi-axis test facilitiy control concepts are usually based on simplified linear model equations [1, 2]. However, the exact model equations of such systems are highly nonlinear and strongly coupled. This paper provides a nonlinear control concept for a planar multi-axis servohydraulic test facility (cf. 1) based on

Fig. 1. Computer drawing of a multi-axis servohydraulic test facility driven by six actuators.

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nonlinear model equations. The nonlinear controller design is based on input state linearization of the control plant equations [3, 4]. For this purpose two questions must be answered:

1. Is the system considered exact linearizable?
2. How can this be achieved?

These two questions are answered subsequently in several steps:

– definition of smooth scalar output functions which depend on all state variables,
– calculation of the relative degree of the selected output functions in connection with the control plant equations,
– redefinition of the output functions, if the total relative degree is less than the system order n.

The output functions selected provide both a transformation of the model equations into nonlinear canonical form and an input state linearization together with the nonlinear control law [3, 4, 5].

2. NONLINEAR PLANT MODELING

The planar servohydraulic test facility model considered has been reduced from a spatial test facility model of six spatial degrees of freedom used in industry (cf. 1). The dynamic model of the control plant includes the planar equations of motion of the rigid test table with a rigidly attached rigid payload and the hydraulic actuator dynamics. In order to describe the planar motion of the test facility the following notations are used (cf. 2):

Fig. 2. Planar multi-axis test facility with three actuators.

R	inertial frame with origin 0,
L	body fixed frame with origin P,
K_i	joint fixed frames with origins $Q_i (i = 1, 2, 3)$,
θ	angle of L with repect to R,
β_i	angles of $K_i (i = 1, 2, 3)$ with respect to R,
m	mass of the rigid body (test table and payload),
J^L_{Cy}	moment of inertia with respect to the center of gravity C, represented in frame L,

$r_P^R \quad := \quad \left[x_P^R,\, z_P^R\right]^T$ Vector from 0 of R to P of L,
$r_{Qi0}^R \quad := \quad \left[x_{Qi0}^R,\, z_{Qi0}^R\right]^T$ Vectors from 0 of R to $Q_i\,(i = 1,2,3)$,
$r_{PiP}^L \quad := \quad \left[x_{PiP}^L,\, z_{PiP}^L\right]^T$ Vectors from 0 of L to $P_i\,(i = 1,2,3)$,
$r_{CP}^L \quad := \quad \left[x_{CP}^L,\, z_{CP}^L\right]^T$ Vector from P of L to C.

The *planar mechanical equations* of motion of the test facility are

$$M(x)\cdot\ddot{x} + J_x^T(x)\cdot d_k\cdot J_x(x)\cdot\dot{x} + q_C(x,\dot{x}) + q_G(x) = J_x^T(x)\cdot A_k\cdot p \tag{1}$$

where (cf. 2)

$$x := \left[x_P^R,\quad z_P^R,\quad \theta\right]^T,\quad p := \left[p_1,\quad p_2,\quad p_3\right]^T \quad \in I\!R^3 \tag{2}$$

are the position and orientation vector of the planar test facility degrees of freedom (of point P and of frame R) and the vector of the actuator pressure differences, respectively, $M(x)$ is the inertia matrix of the test facility,

$$M(x) := \begin{bmatrix} \begin{bmatrix} m, & 0 \\ 0, & m \end{bmatrix}, & m\cdot\begin{bmatrix} c\theta, & s\theta \\ -s\theta, & c\theta \end{bmatrix}\cdot\hat{r}_{CP}^L \\ m\cdot\left(\hat{r}_{CP}^L\right)^T\begin{bmatrix} c\theta, & -s\theta \\ s\theta, & c\theta \end{bmatrix}, & J_{C_y}^L + m\cdot\left[\left(x_{CP}^L\right)^2 + \left(z_{CP}^L\right)^2\right] \end{bmatrix} \tag{3}$$

$$\text{where}\quad c\theta := \cos\theta \quad,\quad s\theta := \sin\theta \quad\text{and}\quad \left(\hat{r}_{CP}^L\right)^T := \left[\,z_{CP}^L,\ -x_{CP}^L\,\right], \tag{4}$$

$J_x^T(x)$ and $J_x(x)$ are nonlinear transformation matrices which map the test table forces and the velocities from joint fixed frames K_i to the degree of freedom representation x

$$J_x(x) = \begin{bmatrix} \cos\beta 1, & -\sin\beta 1, & \left(\hat{r}_{P1P}^L\right)^T\cdot\begin{bmatrix} c\theta, & -s\theta \\ s\theta, & c\theta \end{bmatrix}\cdot\begin{pmatrix} \cos\beta 1 \\ -\sin\beta 1 \end{pmatrix} \\ \sin\beta 2, & \cos\beta 2, & \left(\hat{r}_{P2P}^L\right)^T\cdot\begin{bmatrix} c\theta, & -s\theta \\ s\theta, & c\theta \end{bmatrix}\cdot\begin{pmatrix} \sin\beta 2 \\ \cos\beta 2 \end{pmatrix} \\ \sin\beta 3, & \cos\beta 3, & \left(\hat{r}_{P3P}^L\right)^T\cdot\begin{bmatrix} c\theta, & -s\theta \\ s\theta, & c\theta \end{bmatrix}\cdot\begin{pmatrix} \sin\beta 3 \\ \cos\beta 3 \end{pmatrix} \end{bmatrix} \tag{5}$$

where

$$\sin\beta 1 = \frac{-AA_1}{\sqrt{AA_1^2 + BB_1^2}} \quad,\quad \cos\beta 1 = \frac{BB_1}{\sqrt{AA_1^2 + BB_1^2}},$$

$$\sin\beta i = \frac{-BB_i}{\sqrt{AA_i^2 + BB_i^2}} \quad,\quad \cos\beta i = \frac{AA_i}{\sqrt{AA_i^2 + BB_i^2}} \quad (i = 2,3),$$

$$AA_i := z_P^R - \sin\theta\cdot x_{PiP}^L + \cos\theta\cdot z_{PiP}^L - z_{Qi0}^R,$$

$$BB_i := x_P^R + \cos\theta\cdot x_{PiP}^L + \sin\theta\cdot z_{PiP}^L - x_{Qi0}^R$$

and

$$\left(\hat{r}_{PiP}^L)\right)^T := \left[\,z_{PiP}^T,\ -x_{Pip}^L\,\right],\quad (i = 1,2,3),$$

$q_C(x, \dot{x})$ is the nonlinear vector of the centifugal forces

$$g_c(x, \dot{x}) := \begin{bmatrix} -m \cdot \begin{bmatrix} c\theta, & s\theta \\ -s\theta, & c\theta \end{bmatrix} \cdot r_{CP}^{L} \cdot \dot{\theta}^2 \\ 0 \end{bmatrix}, \tag{6}$$

$q_G(x)$ is nonlinear vector of gravitational force and torque

$$qG(x) := \begin{bmatrix} -(0,\, m \cdot g)^T \\ -(\hat{r}_{CP}^{L})^T \cdot \begin{bmatrix} c\theta, & -s\theta \\ s\theta, & c\theta \end{bmatrix} \cdot \begin{pmatrix} 0 \\ m \cdot g \end{pmatrix} \end{bmatrix}, \tag{7}$$

A_k and d_k are the diagonal matrices of the actuator piston areas and of the actuator damping coefficients, respectively,

$$A_k := \operatorname{diag}\left(A_{k1}, \quad A_{k2}, \quad A_{k3}\right) \quad , \quad d_k := \operatorname{diag}\left(d_{k1}, \quad d_{k2}, \quad d_{k3}\right). \tag{8}$$

The *actuator dynamics* are modeled as:

$$\dot{p} = C_H^{-1}\left[Q_x K_V u + Q_p p - A_k J_x(x)\, \dot{x}\right] \quad , \quad u = [u_1, u_2, u_3]^T \quad \in I\!R^3 \tag{9}$$

where u is the vector of servovalve command inputs and

$$C_H \quad := \quad \operatorname{diag}\left(C_{H1}, C_{H2}, C_{H3}\right), \quad Q_p := \operatorname{diag}\left(Q_{p1}, Q_{p2}, Q_{p3}\right), \tag{10}$$

$$Q_x \quad := \quad \operatorname{diag}\left(Q_{x1}, Q_{x2}, Q_{x3}\right), \quad K_v := \operatorname{diag}\left(K_{V1}, K_{V2}, K_{V3}\right), \tag{11}$$

are the matrices of actuator hydraulic capacities, servovalve pressure coefficients, displacement coefficients and gain factors, respectively. The state space representation of equations (1) and (9) is

$$\underline{\dot{x}} = f\left(\underline{x}\right) + \sum_{j=1}^{3} b_j u_j; \quad \underline{x} \in I\!R^9 \quad , \quad \underline{u} \in I\!R^3 \tag{12}$$

where

$$
\begin{aligned}
\underline{x} &:= \left[p^T, \quad x_1^T, \quad x_2^T\right]^T, \\
x_1 &:= \left[x_{11}, \quad x_{12}, \quad x_{13}\right]^T := \left[x_P^R, \quad z_P^R, \quad \theta\right]^T = x, \\
x_2 &:= \left[x_{21}, \quad x_{22}, \quad x_{23}\right]^T := \left[\dot{x}_P^R, \quad \dot{z}_P^R, \quad \dot{\theta}\right]^T, \\
f(\underline{x}) &:= \left[f_1^T(\underline{x}), \quad f_2^T(\underline{x}), \quad f_3^T(\underline{x})\right]^T, \\
f_1(\underline{x}) &:= \left[f_{11}, \quad f_{12}, \quad f_{13}\right]^T = \tilde{Q}_P \cdot p - \tilde{A}_k \cdot J_{x_1}(x_1) \cdot x_2, \\
f_2(\underline{x}) &:= \left[f_{21}, \quad f_{22}, \quad f_{23}\right]^T = x_2, \\
f_3(\underline{x}) &:= \left[f_{31}, \quad f_{32}, \quad f_{33}\right]^T = \tilde{\alpha}(x_1, x_2) + \tilde{\beta}(x_1) \cdot p, \\
\tilde{A}_k &:= C_H^{-1} \cdot A_k, \quad \tilde{Q}_P := C_H^{-1} \cdot Q_P, \\
\tilde{\beta}(x_1) &:= M^{-1}(x_1) \cdot J_{x_1}^T(x_1) \cdot A_k, \quad J_{x_1}^T(x_1) := J_x^T(x), \\
\tilde{\alpha}(x_1, x_2) &:= -M^{-1}(x_1) \cdot \left\{J_{x_1}^T(x_1) \cdot d_k \cdot J_{x_1}(x_1) \cdot x_2 + q_C(x, \dot{x}) + q_G(x)\right\}, \\
b_1 &:= \left[b_{11}, \quad 0 \quad, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0\right]^T, \\
b_2 &:= \left[0, \quad b_{22}, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0\right]^T, \\
b_3 &:= \left[0, \quad 0, \quad b_{33}, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0\right]^T,
\end{aligned}
$$

and

$$
B_1 = \begin{bmatrix} b_{11}, & 0, & 0 \\ 0, & b_{22}, & 0 \\ 0, & 0, & b_{33} \end{bmatrix} := C_H^{-1} \cdot Q_x \cdot K_V.
$$

$f(\underline{x})$ and $b_j (j = 1, 2, 3)$ in (13) are smooth vector fields.

3. NONLINEAR CONTROLLER DESIGN

3.1. Derivation of the nonlinear diffeomorphism

The derivation of a nonlinear control law for system (12) using exact linearization techniques is closely related to the construction of a nonlinear diffeomorphism which transforms the plant into a nonlinear canonical form. To obtain an input state linearization the following theorem is used [3]:

Theorem. (*Exact linearization via state feedback.*)
System (12) with output functions

$$
y_i = h_i(\underline{x}) = h_i\left(p, \quad x_1, \quad x_2\right)^T \in I\!R^1, \quad (i = 1, 2, 3) \tag{13}
$$

is exactly state linearizable if and only if the following conditions hold:

1.
$$
L_{b_j} L_f^k h_i(\underline{x}) = 0 \tag{14}
$$

for $1 \le i, j \le 3$, for $k \le r_i - 1$ and for all $\underline{x}$ in a neighborhood of $\underline{x}_0$.

2.
$$E(\underline{x}) = \begin{bmatrix} L_{b_1} L_f^{r_1-1} h_1(\underline{x}), & L_{b_2} L_f^{r_1-1} h_1(\underline{x}), & L_{b_3} L_f^{r_1-1} h_1(\underline{x}) \\ L_{b_1} L_f^{r_2-1} h_2(\underline{x}), & L_{b_2} L_f^{r_2-1} h_2(\underline{x}), & L_{b_3} L_f^{r_2-1} h_2(\underline{x}) \\ L_{b_1} L_f^{r_3-1} h_3(\underline{x}), & L_{b_2} L_f^{r_3-1} h_3(\underline{x}), & L_{b_3} L_f^{r_3-1} h_3(\underline{x}) \end{bmatrix} \tag{15}$$
is nonsingular at $\underline{x} = \underline{x}_0$.

3.
$$\sum_{i=1}^{3} r_i = n. \tag{16}$$

Relations (14) and (15) define the vector relative degree $(r_1, \quad r_2, \quad r_3)$ of the system (12) and (13) [3]. $L_f^k h_i(\underline{x})$ are the Lie derivatives of $h_i(\underline{x})$ with respect to f .

This theorem essentially depends on the output functions $h_i(\underline{x})$. These output functions can be choosen arbitrarily provided there exists an invertible and bijective transformation which maps the arbitrarily choosen output functions $h_i(\underline{x})$ to the physical relevant system outputs (measurements). Using this interpretation the subject of the subsequent steps is to find those formal output functions which yield a maximum relative degree of the system considered (hopefully a relative degree (3,3,3) in the case considered). If such output functions can be found a nonlinear diffeomorphism can be constructed which maps the given system equations into nonlinear form where the new state vector is closely related to the selected output functions $h_i(\underline{x})$ (comp. (32)). The procedure of finding suitable output functions is summarized in the following steps (cf. 3):

(i) Start with arbitrarily selected output functions $h_i(\underline{x})$ depending on the state variables in a most general form,

(ii) calculation of the vector relative degree of the system (12) and those functions $h_i(\underline{x})$ (conditions 1 and 2 of the above theorem),

(iii) test of condition 3 of the above theorem,

(iv) stepwise redefining of the output functions $h_i(\underline{x})$ which don't satisfy condition 3,

(v) repetition of steps (ii) to (iv) until condition 3 holds.

Fig. 3. Flow diagram for selection of the output functions h_i.

If equations (13) satisfy conditions (17) then all selected output functions $h_i\left(\underline{x}\right)$ lead to components $r_i \geq 1$ of the vector relative degree (r_1, r_2, r_3) (first step).

$$L_{b_j} L_f^0 h_i\left(\underline{x}\right) \stackrel{!}{=} 0; \quad i, j = 1, 2, 3 \quad . \tag{17}$$

The Lie derivatives of (17) are calculated according to relations

$$L_{b_j} L_f^0 h_i\left(\underline{x}\right) := L_{b_j} h_i\left(\underline{x}\right) = \nabla h_i\left(\underline{x}\right) \cdot b_j = \frac{\partial h_i\left(\underline{x}\right)}{\partial p_j} \cdot b_{jj}; \quad i, j = 1, 2, 3. \tag{18}$$

If the partial derivatives in (18) are zero then (17) is satisfied for $b_{jj} \neq 0$ (comp. (12)). Then the output functions are independent on p_j $(j = 1, 2, 3)$. As a consequence the output functions are redefined in a second step as

$$y_i = h_i\left(\underline{x}\right) = h_i\left(x_1, \quad x_2\right); \quad (i = 1, 2, 3). \tag{19}$$

If equations (19) satisfy the following conditions (20), then all selected output functions $h_i\left(\underline{x}\right)$ lead to components $r_i \geq 2$ of the vector relative degree (r_1, r_2, r_3).

$$L_{b_j} L_f^1 h_i\left(\underline{x}\right) \stackrel{!}{=} 0; \quad i, j = 1, 2, 3. \tag{20}$$

The Lie derivatives in (20) are calculated as

$$L_{b_j} L_f^1 h_i\left(\underline{x}\right) = \frac{\partial L_f^1 h_i}{\partial p_j} \cdot b_{jj}; \quad i, j = 1, 2, 3. \tag{21}$$

where

$$L_f^1 h_i = \nabla h_i \cdot f = \sum_{j=1}^{3} \left(\frac{\partial h_i}{\partial x_{1j}} \cdot f_{2j} + \frac{\partial h_i}{\partial x_{2j}} \cdot f_{3j} \right); \quad i = 1, 2, 3. \tag{22}$$

For $b_{jj} \neq 0$ the partial derivatives in (21) must be zero due to (20). This condition ist satisfied if equations (21) are idependent of $p, (j = 1, 2, 3)$. However, (22) are functions of p due to f_{3j} (compare (12)). Then the following partial derivatives hold

$$\frac{\partial h_i}{\partial x_{2j}} \stackrel{!}{=} 0; \quad i, j = 1, 2, 3, \tag{23}$$

i. e., the output functions are independent of x_2(j=1,2,3). As a consequence the output functions are redefined in a third step as

$$y_i = h_i\left(\underline{x}\right) = h_i\left(x_1\right) \quad , \quad (i = 1, 2, 3). \tag{24}$$

The simplest candidate for (24) is

$$y_i = h_i\left(\underline{x}\right) = h_i\left(x_1\right) = x_{1i} \quad i = 1, 2, 3. \tag{25}$$

Then the following relation (26) is sufficient for the selected output functions $h_i(x)$ (comp. (25)) to have components of the vector relative degree which are $r_i = 3$.

$$L_{b_j} L_f^2 h_i = \nabla L_f^2 h_i \cdot b_j \overset{!}{\neq} 0; \quad i,j = 1,2,3 \tag{26}$$

where

$$L_{b_j} L_f^2 h_i = \nabla L_f^2 h_i \cdot b_j = \frac{\partial f_{3i}}{\partial p_j} \cdot b_{jj}; \quad i,j = 1,2,3. \tag{27}$$

If one of the partial derivatives in (27) is not zero, then (26) holds. The partial derivatives in (27) provide the matrix

$$\left[\frac{\partial f_{3i}}{\partial p_j}\right]_{i,j=1,2,3} = \tilde{\beta}(x_1) = M^{-1}(x_1) \cdot J_{x_1}^T(x_1) \cdot A_k. \tag{28}$$

Equation (27) together with (15) implies:

$$E(\underline{x}) = \left[\frac{\partial f_{3i}}{\partial p_j} \cdot b_{jj}\right]_{i,j=1,2,3} = M^{-1}(x_1) \cdot J_{x_1}^T(x_1) \cdot A_k \cdot B_1. \tag{29}$$

From (3), (8) and (12) follows that matrices $M(x)$, A_k and B_1 have rank 3. Then the regularity condition 3 of the preceding theorem is satisfied if

$$\det J_{x_1}^T(x_1) \neq 0 \tag{30}$$

holds. This regularity condition of the strongly nonlinear matrix (comp. (5)) can be physically discussed as follows:

A calculation of J_{x1} with respect to an operating point x_{1c} where the actuators are in parallel or orthogonal to the axes of the inertial frame R (cf. 2 and cf. 4(a)) yield the relation:

$$J_{x_1}(x_{1c}) = T_d = \begin{bmatrix} 1, & 0, & z_{P1P} \\ 0, & 1, & -x_{P2P} \\ 0, & 1, & -x_{P3P} \end{bmatrix}. \tag{31}$$

Fig. 4. Drawings of the test facility locations with singular matrix J_{x_1}.

Matrix (31) is singular for e. g. collinear or parallel actuator configurations (cf. 4(b) to 4(d)). If these pathological actuator attachment configurations are excluded, system (12) together with (25) has relative degree (3, 3, 3) and hence satisfies condition (16). As a consequence the model equations of the planar test facility are *input state exact linearizable*. The nonlinear state transformation of system (12) into a normal form is defined as

$$\underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}}_{:=z} = \underbrace{\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix}}_{:=\Phi(\underline{x})} = \begin{bmatrix} L_f^0 h \\ L_f^1 h \\ L_f^2 h \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \tilde{\alpha}\left(x_1, x_2\right) + \tilde{\beta}\left(x_1\right) \cdot p \end{bmatrix}, z_i \in I\!R^3, (i = 1, 2, 3)\,. \tag{32}$$

Due to (3), (8) and (30) the inverse vector function of (32) is

$$\underline{x} = \begin{bmatrix} p \\ x_1 \\ x_2 \end{bmatrix} = \Phi^{-1}\left(z\right) = \begin{bmatrix} \tilde{\beta}^{-1}\left(z_1\right) \cdot \left\{z_3 + \cdot\tilde{\alpha}\left(z_1, z_2\right)\right\} \\ z_1 \\ z_2 \end{bmatrix}. \tag{33}$$

Equations (32) and (33) are smooth vector fields and define a global diffeomorphism of the model equations (12) [4, 5] with respect to the actuator configurations considered. Here z_1, z_2 and z_3 are vectors of the positions (orientation), velocities and accelerations of the planar test facility, respectively. These signals are available in practical applications both, as command input signals and as measurement signals [1].

3.2. Construction of the nonlinear control law

Equations (34) to (36) represent a canonical form of the model equations (12) calculated by using nonlinear coordinate transformations (32) and (33)

$$\dot{z}_1 = z_2 \quad , \quad \dot{z}_2 = z_3 \quad , \quad \dot{z}_3 = \alpha\left(z_1, z_2\right) + \beta\left(z\right) \cdot u \tag{34}$$

where

$$\alpha\left(z_1, z_2\right) := \left[\tilde{\beta}\left(x_1\right) \cdot f_1\left(\underline{x}\right) + \frac{\partial \Phi_3}{\partial x_1} \cdot f_2\left(\underline{x}\right) + \frac{\partial \Phi_3}{\partial x_2} \cdot f_3\left(\underline{x}\right)\right]_{\underline{x}=\Phi^{-1}(z)} \tag{35}$$

and

$$\beta\left(z_1\right) := M^{-1}\left(x_1\right) \cdot J_{x_1}^T\left(x_1\right) \cdot A_k \cdot B_1 \quad , \quad x_1 = z_1 \tag{36}$$

turn out to be extreme lengthy formal expressions.

The partial derivatives in (35) have been calculated by using the symbolic algebra program *MACSYMA*. The nonlinear controller used has the form

$$u = \beta^{-1}\left(z_1\right) \cdot \left[\nu - \alpha\left(z_1, z_2\right)\right] \tag{37}$$

where

$$\nu := \dot{z}_{3d} - K \cdot e, \quad e := \left(e_1,\ \dot{e}_1,\ \ddot{e}_1\right)^T, \quad e_1 := z_1 - z_{1d}, \tag{38}$$

$$z_{1d} := x_d = \left(x^R_{Pd},\ z^R_{Pd},\ \theta_d \right)^T \quad (d\text{: desired) and} \tag{39}$$

$$K = [\, K_1,\ K_2,\ K_3 \,], \quad K_i = \mathrm{diag}\,(\, k_{i1},\ k_{i1},\ k_{i1} \,), \quad (i = 1, 2, 3)\,. \tag{40}$$

The control law (37) provides an input state linearization of (12) and a stable tracking error e in case of disturbances using a suitable feedback K. Matrices (29) and (36) are identical and are called decoupling matrices. Equation (37) is called nonlinear decoupling law [4]. If $\dot{z}_{3d}$ (the desired jerk of the test table) in (37) is not available, the input state linearization or the decoupling can still be achieved. This yields a non ideal tracking behaviour.

4. COMPUTER SIMULATION RESULTS

The closed loop nonlinear control system shown in Fig. 5 has been simulated without using the jerk input signals $\dot{z}_{3d}$. Typical transient test signals for vibration testing of space structures are shown in Fig. 6. Figure 7 and Fig. 8 show computer simulation results using system (12) based on linear control concepts [2]. The simulation of system (34) using the nonlinear control law (37) without $\dot{z}_{3d}$ is shown in Fig. 9. A comparsion of the simulation results of the nonlinear control plant (12) controlled by linear controllers (cf. 7 and 8) with the simulation results of the nonlinear plant using the nonlinear control concept (37) (cf. 9) demonstrates the power of the nonlinear control concept. The remaining nonideal tracking behaviour of the control system with nonlinear decoupling controller is caused by the fact that the input jerk signal has been omitted.

Fig. 5. Block diagram of the normal form planar multi-axis test facility model controlled by nonlinear controllers.

Fig. 6. Set point transient test signals ($\ddot{x}_d$).

Fig. 7. Simulation results using linear position controllers.

Fig. 8. Simulation results using linear multi sensor controllers.

Fig. 9. Simulation results using nonlinear decoupling controllers.

5. CONCLUSIONS

The nonlinear control algorthim for a planar multi-axis servohydraulic test facility derived in this paper is based on exact state linearization techniques. The nonlinear diffeomorphism which maps the state equations into nonlinear normal form is derived by choosing suitable output functions. The simulation results show sophisticated tracking and decoupling performance for given transient input signals. The control laws are derived by extensively using symbolic computer algebra languages. The extreme lengthy control algorithms are systematically condensed and reduced by using recursive substitution steps. In subsequent investigations the robustness of the closed loop system has to be considered. An interesting question with respect to industrial applications is the robustness of the closed loop system in the presence of structured and unstructured uncertainties (uncertain system parameters, as e. g. test table mass or moment of inertia; uncertain system order, as e. g. unmodeled servovalve dynamics and elastic modes of the payload).

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