

On Distributions of Order Statistics for Absolutely Continuous Copulas with Applications to Reliability

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Abstract: Performance of coherent reliability systems is strongly connected with distributions of order statistics of failure times of components. A crucial assumption here is that the distributions of possibly mutually dependent lifetimes of components are exchangeable and jointly absolutely continuous. Assuming absolute continuity of marginals, we focus on properties of respective copulas and characterize the marginal distribution functions of order statistics that may correspond to absolute continuous and possibly exchangeable copulas. One characterization is based on the vector of distribution functions of all order statistics, and the other concerns the distribution of a single order statistic.

Keywords: coherent system; order statistic; copula; exchangeable distribution; absolute continuous distribution; absolute continuous copula;

AMS Subject Classification: 60E05; 62G30; 62H05; 62N05;

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