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# Asymptotic Behaviour of a BIPF Algorithm with an Improper Target

Claudio Asci; Mauro Piccioni

*Abstract*: The BIPF algorithm is a Markovian algorithm with the purpose of simulating certain probability distributions supported by contingency tables belonging to hierarchical log-linear models. The updating steps of the algorithm depend only on the required expected marginal tables over the maximal terms of the hierarchical model. Usually these tables are marginals of a positive joint table, in which case it is well known that the algorithm is a blocking Gibbs Sampler. But the algorithm makes sense even when these marginals do not come from a joint table. In this case the target distribution of the algorithm is necessarily improper. In this paper we investigate the simplest non trivial case, i. e. the $2 \times 2 \times 2$ hierarchical interaction. Our result is that the algorithm is asymptotically attracted by a limit cycle in law.

*Keywords:* log-linear models; marginal problem; null Markov chains;

*AMS Subject Classification:* 60J05; 65C40; 62F15;

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