

On Exact Null Controllability of Black - Scholes Equation

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Abstract: In this paper we discuss the exact null controllability of linear as well as nonlinear Black-Scholes equation when both the stock volatility and risk-free interest rate influence the stock price but they are not known with certainty while the control is distributed over a subdomain. The proof of the linear problem relies on a Carleman estimate and observability inequality for its own dual problem and that of the nonlinear one relies on the infinite dimensional Kakutani fixed point theorem with L^2 topology.

Keywords: Black-Scholes equation; volatility; controllability; observability; Carleman estimates;

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